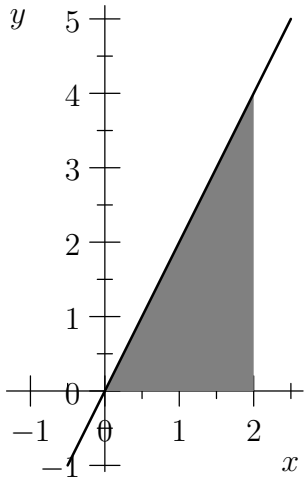
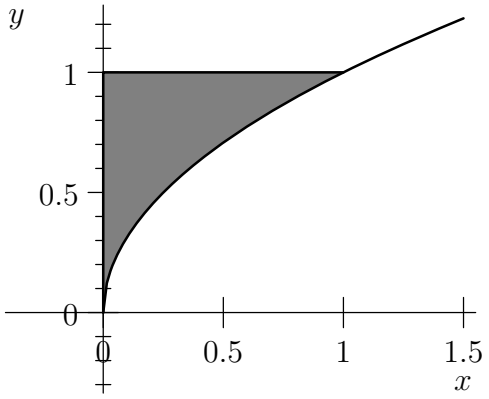
Integration Intuition

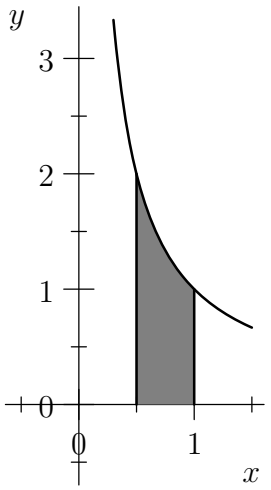
When calculating areas, it's a good idea to check your answer against a rough visual estimate of the region's area. For each graph shown below, select the value that's closest to the shaded area.



Approximate area: a) 2 b) 4 c) 8 d) 16



Approximate area: a) $\frac{1}{4}$ b) $\sqrt{2}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$



Approximate area: a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{3}{2}$ d) 2

Solutions

Graph 1: The shaded region is a triangle with base 2 and height 4, so its area is $\frac{1}{2} \cdot 2 \cdot 4 = 4$ and the answer is (b). The scale on the x and y-axes is important in estimating the area of a region.

Graph 2: Drawing a diagonal between $(0, 0)$ and $(1, 1)$ we see that the shaded region fits inside a triangle with area $\frac{1}{2}$. This rules out answers (b) and (d). (Don't be intimidated by numbers like $\sqrt{2}$ and π. Learn their values, rounded off for use in estimation.) Either of answers (a) and (c) is reasonable. By comparing the shaded area to that of a square with area $\frac{1}{4}$, we might conclude that (c) is the best answer.

Graph 3: This region is comparable to a rectangle of area $\frac{1}{2}$ topped by one of area $\frac{1}{4}$, so the best answer appears to be (b). A common error in this problem is to consider only the height of the region, assuming that the width of its base is 1.

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