

**PROFESSOR:** Welcome back to recitation. In this video what we'd like to do is to solve a certain differential equation. And I'm not even giving you initial condition. So we just want to find a  $y$ , such that  $x$  times  $dy/dx$  is equal to  $x$  squared plus  $x$  times  $y$  squared plus 1. So I'll give you a minute to think about it and then I'll be back.

Welcome back. I'm going to use the technique of separation of variables to solve this differential equation problem. Hopefully you thought about doing that as well. Because it's really set up nicely for separation of variables.

So let me let me first get all of the values, or terms that involve  $y$  on the left-hand side. And then I'm going to move the  $dx$  to the right-hand side and get all the terms that involve  $x$  to the right-hand side. So I'm going to write  $dy$  divided by  $y$  squared plus 1 is equal to-- so I'm going to multiply through by  $dx$  divided by  $x$ . So I'm going to get  $x$  squared plus  $x$  over  $x$   $dx$ .

Now if  $x$  is 0, I have a little problem but we're going to ignore that for the moment. So I can rewrite this as  $x$  plus 1  $dx$ . And now I'm totally set up with my next step in separation of variables and so I can integrate both sides.

So let's see what happens when I integrate the left-hand side. OK? What is an antiderivative to  $1$  over  $y$  squared plus 1? Well that's arctangent, that's arctangent of  $y$ . So the derivative of arctangent of  $y$  is  $1$  over  $y$  squared plus 1. And so on the left-hand side I get  $\arctan y$ .

If you wrote tangent to the minus 1  $y$ , that's the same thing, the inverse tangent of  $y$ , that's the same thing. And on the right-hand side what do I get? Well this is a nice easy thing to integrate. When I integrate  $x$ , I get  $x$  squared over 2. And here I just get an  $x$ . And then I should add in my constants. So  $x$  squared over 2 plus  $x$  plus a constant.

So this is, so far I'm doing everything OK. Now I need to figure out how to isolate the  $y$ . Well  $\arctan y$  is the inverse of the tangent function. So if I want to isolate  $y$  I have to take the tangent of both sides. So when I take tangent of arctangent of  $y$  I just get  $y$ . Then over here, I get tangent of  $x$  squared over 2 plus  $x$  plus  $C$ .

And now because I didn't give you any initial conditions, we can't say anything about  $C$ . But if I gave you some initial conditions then we could evaluate and solve for  $C$ . Now how do I go about checking to make sure that this works? Well, what I do is actually take the derivative. So you may want to take the derivative of the right-hand side, take  $dy/dx$ . Evaluate what that is

when you have  $y$  equals tangent of  $x$  squared over 2 plus  $x$  plus  $C$  and see if you in fact get the relationship you're supposed to get. But I'll stop there.