Exploiting Derivative Rules

Every differentiation rule $F'(x) = f(x)$ corresponds to a rule for finding the anti-derivative $F(x)$ of some function f.

a) Find an anti-derivative rule that is the inverse of the sum rule $(f+g)'(x) = f'(x) + g'(x)$.

b) Find an anti-derivative rule that is the inverse of the product rule $(f \cdot g)'(x) = f(x)g'(x) + f'(x)g(x)$.

Solution

a) Find an anti-derivative rule that is the inverse of the sum rule $(u+v)' = u' + v'$.

If F and G are the anti-derivatives of f and g, respectively, then the anti-derivative of:

$$f(x) + g(x)$$

is:

$$F(x) + G(x).$$

We can check this result by differentiating.

b) Find an anti-derivative rule that is the inverse of the product rule $(u \cdot v)' = u\,v' + u'\,v$.

If F and G are the anti-derivatives of f and g, respectively, then the anti-derivative of:

$$F(x)g(x) + f(x)G(x)$$

is:

$$F(x) \cdot G(x).$$

Later we will study a technique called "integration by parts", which is closely related to this anti-differentiation formula. It relies upon the fact that the anti-derivative of $F(x)g(x)$ is equal to $F(x)G(x)$ minus the anti-derivative of $G(x)f(x)$.

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