PROFESSOR: Right now, we're finishing up with the first unit, and I'd like to continue in this lecture, lecture seven, with some final remarks about exponents. So what I'd like to do is just review something that I did quickly last time, and make a few philosophical remarks about it. I think that the steps involved were maybe a little tricky, and so I'd like to go through it one more time. Remember, that we were talking about this number a_k, which is $(1 + 1/k)^k$. And what we showed was that the limit as k goes to infinity of a_k was e.

So the first thing that I'd like to do is just explain the proof a little bit more clearly than I did last time with fewer symbols, or at least with this abbreviation of the symbol here, to show you what we actually did. So I'll just remind you of what we did last time, and the first observation was to check, rather than the limit of this function, but to take the log first. And this is typically what's done when you have an exponential, when you have an exponent. And what we found was that the limit here was 1 as k goes to infinity.

So last time, this is what we did. And I just wanted to be careful and show you exactly what the next step is. If you exponentiate this fact; you take e to this power, that's going to tend to e^1, which is just e. And then, we just observe that this is the same as a_k. So the basic ingredient here is that $e^{\ln a} = a$. That's because the log function is the inverse of the exponential function. Yes, question?

STUDENT: [INAUDIBLE]

PROFESSOR: So the question was, wouldn't the log of this be 0 because a_k is tending to 1. But a_k isn't tending to 1. Who said it was? If you take the logarithm, which is what we did last time, logarithm of a_k is indeed $k * \ln(1 + 1/k)$. That does not tend to 0. This part of it tends to 0, and this part tends to infinity. And they balance each other, 0 times infinity. We don't really know yet from this expression, in fact we did some cleverness with limits and derivatives, to figure out this limit. It was a very subtle thing. It turned out to be 1. All right?

Now, the thing that I'd like to say - I'm sorry I'm going to erase this aside here - but you need

to go back to your notes and remember that this is what we did last time. Because I want to have room for the next comment that I want to make on this little blackboard here. What we just derived was this property here, but I made a claim yesterday, and I just want to emphasize it again so that we realized what it is that we're doing. I looked at this backwards. One way you can think of this is we're evaluating this limit and getting an answer. But all equalities can be read both directions. And we can write it the other way: e equals the limit, as k goes to infinity, of this expression here.

So that's just the same thing. And if we read it backwards, what we're saying is that this limit is a formula for e. So this is very typical of mathematics. You want to always reverse your perspective all the time. Equations work both ways, and in this case, we have two different things here. This e was what we defined as the base, which when you graph e^x, you get slope 1 at 0. And then it turns out to be equal to this limit, which we can calculate numerically.

If you do this on your calculators, you, of course, will have a way of programming in this number and evaluating it for each k. And you'll have another button available to evaluate this one. So another way of saying it is it that there's a relationship between these two things. And all of calculus is a matter of getting these relationships. So we can look at these things in several different ways. And indeed, that's what we're going to be doing at least at the end of today in talking about derivatives. A lot of times when we talk about derivatives, we're trying to look at them from several perspectives at once.

Okay, so I have to keep on going with exponents, because I have one loose end. One loose end that I did not cover yet. There's one very important formula that's left, and it's the derivative of the powers. We actually didn't do this - well we did it for rational numbers r. So this is the formula here. But now we're going to check this for all real numbers, r. So including all the irrational ones as well. This is also good practice for using base e and using logarithmic differentiation.

So let me do this by our two methods that we can use to handle exponential type problems. So method one was base e. So if I just rewrite this base e again, that's just this formula over here. x^r = (e^ln x)^r, which is e^r ln x.

Okay, so now I can differentiate this. So I get that d/dx (x^r), now I'm going to use prime notation, because I don't want to keep on writing that d/dx here; (e^(r ln x))'. And now, what I can do is I can use the chain rule. The chain rule says that it's the derivative of this times the

derivative of the function. So the derivative of the exponential is just itself. And the derivative of this guy here, well I'll write it out once, is $(r \ln x)'$. So what's that equal to? Well, $e^{(r \ln x)}$ is is just x^r. And this derivative here is-- Well the derivative of r is 0. This is a constant. It just factors out. And $\ln x$ now has derivative-- What's the derivative of $\ln x$? $1/x$, so this is going to be times r/x.

And now, we rewrite it in the customary form, which is r, we put the r in front, $x^{(r-1)}$. Okay? So I just derived the formula for you. And it didn't matter whether r was rational or irrational, it's the same proof.

Okay so now I have to show you how method two works as well. So let's do method two, which we call logarithmic differentiation. And so here I'll use a symbol, say u, for x^r, and I'll take its logarithm. That's $r \ln x$. And now I differentiate it. I'll leave that in the middle, because I want to remember the key property of logarithmic differentiation. But first I'll differentiate it. Later on, what I'm going to use is that this is the same as u'/u. This is one way of evaluating a logarithmic derivative. And then the other is to differentiate the explicit function that we have over here. And that is just, as we said, r/x. So now, I multiply through, and I get $u' = ur/x$ which is just $x^r r/x$, which is just what we did before. It's $r x^{(r-1)}$. Again, you can now see by comparing these two pieces of arithmetic that they're basically the same. Pretty much every time you convert to base e or you do logarithmic differentiation, it'll amount to the same thing, provided you don't get mixed up. You generally have to introduce a new symbol here. On the other hand, you're dealing with exponents there. It's worth it to know both points of view.

All right, so now I want to make one last remark before we finish with exponents. And, I'll try to sell this to you in a lot of ways as the course goes on, but one thing that I want to try to emphasize is that the natural logarithm really is natural. So, I claim that the natural log is natural. And the example that we're going to use for this illustration is economics. Okay?

So let me explain to why the natural log is the one that's natural for economics. If you are imagining the price of a stock that you own goes down by a dollar, that's a totally meaningless statement. It depends on a lot of things. In particular, it depends on whether the original price was a dollar or 100 dollars. So there's not much meaning to these absolute numbers. It's always the ratios that matter. So, for example, I just looked up an hour ago, the London Exchange closed, and it was down 27.9, which as I said, is pretty meaningless unless you know what the actual total of this index is. It turns out it was 6,432. So the change in the price, divided by the price, which in this case is 27.9 / 6,432, is what matters. And, in this case, it

happens to be .43%. All right? That's what happened today. And similarly, if you take the infinitesimal of this, people think of days as being relatively small increments when you're investing in a stock, you would be interested in the infinitesimal sense, you would be interested in p'/p. The derivative of p divided by p. That's just (ln p)'. So this is the - let me put a little box around it - the formula of logarithmic differentiation. But let me just emphasize that it has an actual significance, and it's the one that's used by economists and people who are modeling prices of things all the time. They never use absolute prices when there are large swings. They always use the log of the price. And there's no point in using log base 10, or log base 2. Those give you junk. They give you an extra factor of log 2. It's the natural log that's the obvious one to use. It's completely straightforward that this is a simpler expression than using log base 10 and having a factor of natural log of 10 there, which would just mess everything up.

All right, so this is just one illustration. Anything that has to do with ratios is going to encounter logarithms. All right, so that's pretty much it. That's all I want to say for now anyway. There's lots more to say, but we'll be saying it when we do applications of derivatives in the second unit. So now, what I'd like to do is to start a review. I'm just going to run through what we did in this unit. I'll tell you approximately what I expect from you on the test that's coming up tomorrow. And, well, so let's get started with that.

So this is a review of Unit One. And I'm just going to put on the board all of the things that you need to think about, anyway, keep in your head. And there are what are called general formulas for derivatives. And then there are the specific ones. And let me just remind you what the general formulas are. There's what you do to differentiate a sum, a multiple of a function, the product rule, the quotient rule. Those are several general formulas. And then there's one more, which is the chain rule, which I'm going to say just a little bit more about. So the derivative of a function of a function is the derivative of the function times the derivative of the other function. So here, I've abbreviated u is u(x). Right, so this is one of two ways of writing it. The other way is also one that you can keep in mind and you might find easier to remember. It's probably a good idea to remember both formulas.

And then the last type of general formula that we did was implicit differentiation. Okay? So when you do implicit differentiation, you have an equation and you don't try to solve for the unknown function. You just put it in its simplest form and you differentiate. So, we actually did this, in particular, for inverses. That was a very, very key method for calculating the inverses of

functions. And it's also true that logarithmic differentiation is of this type. This is a transformation. We're differentiating something else. We're transforming the equation by taking its logarithm and then differentiating. Okay, so there are a number of different ways this is applied. It can also be applied, anyway, these are two of them. So maybe in parenthesis. These are just examples.

All right. I'll try to give examples of at least a few of these rules later. So now, the specific functions that you know how to differentiate: well you know how to differentiate now x^r thanks to what I just did. We have the sine and the cosine functions, which you're responsible for knowing what their derivatives are. And then other trig functions like tan and secant. We generally don't bother with cosecants and cotangents, because everything can be expressed in terms of these anyway. Actually, you can really express everything in terms of sines and cosines. But what you'll find is that it's much more convenient to remember the derivatives of these as well. So memorize all of these.

All right, and then we had e^x and ln x. And we had the inverses of the trig functions. These were the two that we did: the arctangent and the arcsine. So those are the ones you're responsible for. You should have enough time, anyway, to work out anything else, if you know these. All right, so basically the idea is you have a bunch of special formulas. You have a bunch of general formulas. You put them together, and you can generate pretty much anything.

Okay, so let's do a few examples before going on with the review. Okay, so I do want to do a few examples in sort of increasing level of difficulty in how you would combine these things together. So first of all, you should remember that if you differentiate the secant function, that's just - oh I just realized that I wanted to say something else before - so forget that. We'll do that in a second. I wanted to make some general remarks.

So there's one rule that you discussed in my absence, which is the chain rule. And I do want to make just a couple of remarks about the chain rule now to remind you of what it is, and also to present some consequences. So, a little bit of extra on the chain rule.

The first thing that I want say is that we didn't really fully explain why it's true. And I do want to just explain it by example, okay? So imagine that you have a function which is, say, 10x + b. All right? So y = 10x + b. Then obviously, y is changing 10 times as fast as b, right? The issue is this number here, dy/dx, is 10. And now if x is a function of something, say t, shifted by some

other constant here, then dx/dt = 5.

Now all the chain rule is saying is that if y is going 10 times as fast as t, I'm sorry as x, and x is going 5 times as fast as t, then y is going 50 times as fast as t. And algebraically, all this means is if I plug in and substitute, which is what the composition of the two functions amounts to, 10(5t + a) + b and I multiply it out, I get 50t + 10a + b. Now these terms don't matter. The constant terms don't matter. The rate is 50. And so the consequence, if we put them together, is that dy/dt = 10*5, which is 50. All right, so this is in a nutshell why the chain rule works. And why these rates multiply.

The second thing that I wanted to say about the chain rule is that it has a few consequences that make some of the other rules a little easier to remember or possibly to avoid. The messiest rule in my humble opinion is the quotient rule, which is kind of a nuisance to remember. So let me just remind you, if you take just the reciprocal of a function, and you differentiate it, there's another way of looking at this. And it's actually the way that I use, so I want to encourage you to think about it this way too. This is the same as $(v^{(-1)})'$. . And now instead of using the quotient rule, which we could've used, we can use the chain rule here with the power -1, which works by the power law. So what is this equal to? This is equal to $-v^{(-2)}$ v'.

So here, I've applied the chain rule rather than the quotient rule. And similarly, suppose I wanted to derive the full quotient rule. Well, now this may or may not be easier. But this is one way of remembering what's going on. If you convert it to $uv^{(-1)}$ and you differentiate that, now I can use the product rule on this. Of course, I have to use the chain rule and this rule as well. So what do I get? I get u', the inverse, plus u, and then I have to differentiate the v inverse. That's the formula right up here. That's $-v^{(-2)}$ v'. So that's one way of doing it. This actually explains the funny minus sign when you differentiate v in the formula. The other formula, the other way that we did it, was by putting this over a common denominator. The common denominator was v^2. This comes from this v $v^{(-2)}$. And then the second term is -uv'. And the first term, we have to multiply by an extra factor of v, because we have a v^2 in the denominator. So it's u'v. All right, so this is the quotient rule as we wrote it down in lecture, and this is just another way of remembering it or deriving it without remembering it, if you just remember the chain rule and the product rule. Okay, so you'll find that in many contexts, it's easier to do one or the other.

Okay, so now I'm ready to differentiate the secant and a few such functions. So we'll do some

examples here here. So here's the secant function, and I want to use that formula up there for the reciprocal. This is the way I think of it. This is the cosine function to the power -1. And so, the formula here is just what? It's just $-(\cos x)^{(-2)}$ times $-\sin x$. So now this is usually written in a different fashion, so that's why I'm doing this for a reason actually. Which is although there are several formulas for things, with trig functions, there are usually five ways of writing something. So I'm writing this one down so that you know what the standard way of presenting it is.

So what happens here is that we have two minus signs cancelling. And we get $\sin x / \cos^2 x$. That's a perfectly acceptable answer, but there's a customary way in which is written. It's written $(1 / \cos x)(\sin x / \cos x)$. And then we get rid of the denominators by rewriting it in terms of secant and tangent, so $\sec x \tan x$. So this is the form that's generally used when you see these formulas written in textbooks. And so you know, you need to watch out, because if you ever want to use this kind of calculus, you'll have not be put off by all the secants and tangents.

All right, so getting slightly more complicated, how about if we differentiate $\ln(\sec x)$? If you differentiate the natural log, that's just going to be $(\sec x)' / \sec x$. And plugging in the formula that we had before, that's $\sec x \tan x / \sec x$, which is $\tan x$. So this one also has a very nice form. And you might say that this is kind of an ugly function, but the strange thing is that the natural log was invented before the exponential by a guy named Napier, exactly in order to evaluate functions like this. These are the functions that people cared about a lot, because they were used in navigation. You wanted to multiply sines and cosines together to do navigation. And the multiplication he encoded using a logarithm. So these were invented long before people even knew about exponents. And it was a surprise, actually, that it was connected to exponents. So the natural log was invented before the log base 10 and everything else, exactly for this kind of purpose.

Anyway, so this is a nice function, which was very important, so that your ships wouldn't crash into the reef. Okay, let's continue here.

So there's another kind of function that I want to discuss with you. And these are the kinds in which there's a choice as to which of these rules to apply. And I'll just give a couple of examples of that. There usually is a better and a worse way, so let me illustrate that.

Okay, yet another example. I hope you've seen some of these before. Say $(x^{10} + 8x)^6$. So

it's a little bit more complicated than what we had before, because there were several more symbols here. So what should we do at this point? There's one choice which I claim is a bad idea, and that is to expand this out to the 6th power. That's a bad idea, because it's very long. And then your answer will also be very long. It will fill the entire exam paper, for instance. Yeah?

STUDENT: Can you use the chain rule?

PROFESSOR: Chain rule. That's it. We use the chain rule. So fortunately, this is relatively easy using the chain rule. We just think of this box as being the function. And we take 6 times this guy to the 5th, times the derivative of this guy, which is $10x^9 + 8$. And this is, filling this in, it's $x^{10} + 8x$. And that's it. That's all you need to do differentiate things like this. The chain rule is very effective.

STUDENT: [INAUDIBLE]

PROFESSOR: That's a good question. So I'm not really willing to answer too many questions about what's going to be on the exam. But the question that was just asked is exactly the kind of question I'm very happy to answer. Okay, the question was, in what form is-- what form is an acceptable answer? Now in real life, that is a really serious question. When you ask a computer a question and it gives you 500 million sheets of printout, it's useless. And you really care what form answers are in, and indeed, somebody might really care what this thing to the 6th power is, and then you would be forced to discuss things in terms of that other functional form. For the purposes of this exam, this is okay form. And, in fact, any correct form is an okay form. I recommend strongly that you not try to simplify things unless we tell you to.

Sometimes it will be to your advantage to simplify things. Sometimes we'll say simplify. It takes a good deal of experience to know when it's really worth it to simplify expressions. Yes?

STUDENT: [INAUDIBLE]

PROFESSOR: Right, so turning to this example. The question is what is this derivative? And here's an answer. That's the end of the problem. This is a more customary form. But this is answer is okay. Same issue. That's exactly the point. Yes?

STUDENT: [INAUDIBLE]

PROFESSOR: The question is, do you have to show the work? Do you have to show the work? Well if I ask

you what is d/dx of sec x, then if you wrote down this answer or you wrote down this answer showing no work, that would be acceptable. If the question was derive the formula for this from the formula for the derivative of the cosine or something like that, then it would not be acceptable. You'd have to carry out this arithmetic.

So, in other words, typically this will come up, for instance, in various contexts. You just basically have to follow directions. Yes?

STUDENT: [INAUDIBLE]

PROFESSOR: The next question is, are you expected to be able to prove what the derivative of the sine function is? The short answer to that is yes. But I will be getting to that when I discuss the rest of the material here. We're almost there.

Okay, so let me just finish these examples with one last one. And then we'll talk about this question of things like the derivative of the sine function, and deriving it. So the last example that I'd like to write down is the one that I promised you in the first lecture, namely to differentiate $e^{(x \tan^{-1} x)}$. Basically you're supposed to be able to differentiate any function. So this is the one that we mentioned at the beginning. So here it is. Let's do it.

So what is it? Well, it's just equal to - I have to differentiate. I have to use the chain rule - it's equal to the exponential times the derivative of this expression here. That's the chain rule. That's the first step. And now I have to apply the product rule here. So I have $e^{(x \tan^{-1} x)}$. And I differentiate the first factor, so I get $\tan^{-1} x$. Add to it what happens when I differentiate the second factor, leaving alone the x. So that's $x / (1+x^2)$. And that's it. That's the end of the problem. It wasn't that hard. Of course, it requires you to remember all of the rules, and a lot of formulas underlying them.

So that's consistent with what I just told you. I told you that you wanted to know this. I told you that you needed to know this product rule, and that you needed to know the chain rule. And I guess there was one more thing, the derivative of e^x came into play there. So of these formulas, we used four of them in this one calculation.

Okay, so now what other things did we talk about in Unit One? So the main other thing that we talked about was the definition of a derivative. And also there was sort of a goal which was to get to the meaning of the derivative. So these are things - so we had a couple of ways of looking at it, or at least a couple that I'm going to emphasize right now. But first, let me remind

you what the formula is. The derivative is the limit as delta x goes to 0 of (f(x + delta x) - f(x)) / delta x. So that's it, and this is certainly going to be a central focus here. And you want to be able to recognize this formula in a number of ways.

So, how do we use this? Well one thing we did was we calculated a bunch of these rates of change. In fact, more or less, they're the ones which are written right over here. This list of functions here. Now, which ones did we start out with just straight from the definition here? Which of these things? There were a whole bunch of them. So we started out with a function 1/x. We did x^n. We did sine x. We did cosine x. Now there was a little bit of subtlety with sine x and cosine x. We got them using something else. We didn't quite get them all the way. We got them using the case x = 0. We got them from the derivative at x = 0, we got the formulas for the derivatives of sine and cosine. But that was an argument which involved plugging in sin (x + delta x), and running through. So that's one example.

We also did a^x. And that may be it. Oh yeah, I think that's about it. That may be about it. No. It isn't. Okay, so let me make a connection here which you probably haven't yet made, which is that we did it for (u v)'. And we also did it for (u / v)'. So sorry, I shouldn't write primes, because that's not consistent with the claim there. I differentiated the product; I differentiated the quotient using the same delta x notation. I guess I forgot that because I wasn't there when it happened.

So look, these are the ones that you do by this. And, of course, you might have to reduce them to other things. These involve using something else. This one involves using the slope of this function at 0, just the way the sine and the cosine did. This one involves the slopes of the individual functions, u and v. And this one also involves the individual-- So, in other words, it doesn't get you all the way through to the end, but it's expressed in terms of something simpler in each of these cases.

And I could go on. We didn't do these in class, but you're certainly-- e^x is a perfectly okay one on one of the exams. We ask you for 1/x^2. In other words, I'm not claiming that it's going to be one on this list, but it certainly can be any one of these. But we're not going to ask you to go all the way through to the beginning in these formulas.

There are also some fundamental limits that I certainly want you to know about. And these you can derive in reverse. So I will describe that now. So let me also emphasize the following thing: I want to read this backwards now. This is the theme from the very beginning of this lecture.

Namely, if you're given the function f, you can figure out its derivative by this formula here. That is the formula for this in terms of what's on the right hand side. On the other hand, you can also use the formula in that direction, and if you know the slope of something, you can figure out what the limit is. For example, I'll use the letter x here, even though it's cheating. Maybe I'll call it delta x so it's clearer to you. Maybe I'll call it u. Suppose you look at this limit here. Well, I claim that you should recognize that is the derivative with respect to u of the function e^u at u = 0, which of course we know to be 1.

So this is reading this formula in reverse. It's recognizing that one of these limits - let me rewrite this again here - one of these so-called difference quotient limits is a derivative. And since we know a formula for that derivative, we can evaluate it.

And lastly, there's one other type of thing which I think you should know. These are the ones you do with difference quotients. There are also other formulas that you want to be able to derive. You want to be able to derive formulas by implicit differentiation. In other words, the basic idea is to take whatever equation you've got and simplify it as much as possible, without insisting that you solve for y. That's not necessarily the most appropriate way to get the rate of change. The much simpler formula is sin y = x. And that one is easier to differentiate implicitly. So I should say, do this kind of thing. So that's, if you like, a typical derivation that you might see.

And then there's one last type of problem that you'll face, and it's the other thing that I claim we discussed. And it goes all the way back to the first lecture. So the last thing that we'll be talking about is tangent lines. All right? The geometric point of view of a derivative. And we'll be doing more of this in next the unit. So first of all, you'll be expected to be able to compute the tangent line. That's often fairly straightforward. And the second thing is to graph y' , the derivative of a function. And the third thing, which I'm going to throw in here, because I regard it in a sort of geometric vein, although it's got an analytical aspect to it. So this is a picture. This is a computation. And if you combine the two together, you get something else. And so this is to recognize differentiable functions.

Alright, so how do you do this? Well, we really only have one way of doing it. We're going to check the left and right tangents. They must be equal. So again, this is a property that you should be familiar with from some of your exercises. And the idea is simply, that if the tangent line exists, it's the same from the right and from the left. Okay, now I'm going to just do one example here from this sort of qualitative sketching skill to give you an example here. And

what I'm going to do is I'm going to draw a graph of a function like this. And what I want to do underneath is draw the graph of the derivative. So this is the function y = f(x), and here I'm going to draw the graph of the function y = f'(x) right underneath it.

So now, let's think about what it's supposed to look like. And the one step that you need to make in order to do this, is to draw a few tangent lines. I'm just going to draw one down here. And I'm going to draw one up here. Now, the tangent lines here - notice that the slope of these tangent lines are all positive. So everything I draw down here is going to be above the x-axis.

Furthermore, as I go further to the left, they get steeper and steeper. So they're getting higher and higher. So the function is coming down like this. It starts up there. Maybe I'll draw it in green to illustrate the graph here. So that's this function here. As we get farther out, it's getting flatter and flatter. So it's leveling off, but above the axis like that.

So one of the things to emphasize is, you should not expect the derivative to look like the function. It's totally different. It's keeping track at each point of its tangent line. On the other hand, you should get some kind of physical feel for it, and we'll be practicing this more in the next unit. So let me give you an example of a function which does exactly this. And it's the function y = ln x. If you differentiate it, you get y' = 1/x. And this plot above is, roughly speaking, the logarithm. And this plot underneath is the function 1/x.

We still have time for one question. And so, fire away. Yes?

STUDENT: [INAUDIBLE]

PROFESSOR: The question is, can you show how you derive the inverse tangent of x. So that's in a lecture. I'm happy to do it right now, but it's going to take me a whole two minutes. So, here's how you do it. y = tan^(-1) x. And now this is hopeless to differentiate, so I rewrite it as tan y = x. And now I have to differentiate that. So when I differentiate it, I get the derivative of tan y with respect to x-- with respect to y. That's 1 / (1 + y^2) times y'.

So this is a hard step. That's the chain rule. And on the left side I get 1. So I'm doing this super fast because we have thirty seconds left. But this is the hard step right here. And it needs for you to know that d/dy tan y is equal to one over-- Oh, bad bad bad, secant squared. I was ahead of myself so fast. So here's the identity. So you need have known this in advance. And that's the input into this equation.

So now, what we have is that y' = 1 / sec^2 y y, which is the same thing as cos^2 y.

Now, the last bit of the problem is to rewrite this in terms of x. And that you have to do with a right triangle. If this is x and this is 1, then the angle is y, because the tangent of y is x. So this expresses the fact that the tangent of y is x. And then the hypotenuse is the square root of $1 + x^2$. And so the cosine is 1 divided by that. So this thing is 1 divided by the square root of $1 + x^2$, the quantity squared. So, and then the last little bit here, since I'm racing along, is that it's $1 / (1 + x^2)$, which I incorrectly wrote over here. Okay, so good luck on the test. See you tomorrow.