PRACTICE EXAM 2

(1) (10 points) Find

$$\lim_{h\to 0} \frac{\int_0^{1+h} e^{t^2}\,dt - \int_0^1 e^{t^2}\,dt}{h(3+h^2)}.$$

(If you're using a theorem, state the theorem you're using.)

(2) (10 points) Find $(f^{-1})'(0)$ where $f(x) = \int_0^x \cos(\sin t))dt$ is defined on $[-\pi/2, \pi/2]$.

(3) (10 points) In each case below, assume f is continuous for all x. Find $f(2)$.

$$\int_0^x f(t)\,dt = x^2(1+x); \quad \int_0^{f(x)} t^2 dt = x^2(1+x).$$

(4) (15 points) Give an example of a function $f(x)$ defined on $[-1,1]$ such that
- f is continuous and differentiable on $[-1,1]$
- f' is not continuous for at least one value of $x \in [-1,1]$.

(5) (15 points) Let $f(x)$ be continuous on $[0,1]$ such that $f(0) = f(1)$. Show that for any $n \in \mathbb{Z}^+$ there exists at least one $x \in [0,1]$ such that $f(x) = f(x+1/n)$.