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18.01 Single Variable Calculus  
Fall 2006

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**Problem 1.** (10 pts.) Find the tangent line to  $y = \frac{1}{3}x^2$  at  $x = 1$

**Problem 2.** Find the derivative of the following functions:

a. (7 pts.)  $\frac{x}{\sqrt{1-x}}$

b. (8 pts.)  $\frac{\cos(2x)}{x}$

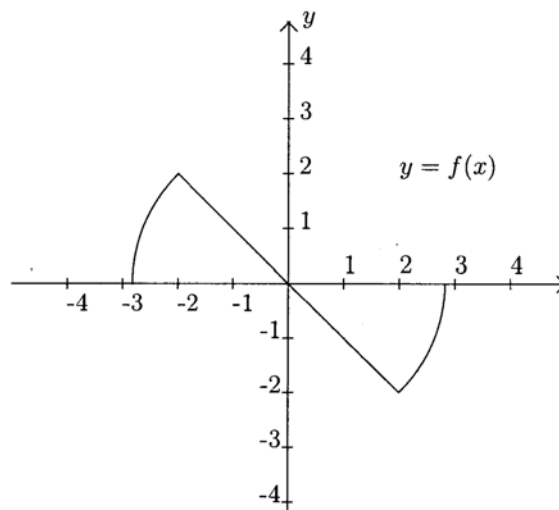
c. (5 pts.)  $e^{2f(x)} = g(x)$

d. (5 pts.)  $\ln(\sin x)$

**Problem 3.** (15 pts.) Find  $\frac{dy}{dx}$  for the function for the function  $y$  defined implicitly by

$$y^4 + xy = 4 \text{ at } x=3, y=1$$

**Problem 4.** (15 pts.) Draw the graph of the derivative of the function (qualitatively accurate) directly under the graph of the function.



**Problem 5.** (15 pts) Let

$$f(x) = \begin{cases} ax + b & x < 1 \\ x^4 + x + 1 & x \geq 1 \end{cases}$$

Find all  $a$  and  $b$  such that the function  $f(x)$  is differentiable.

**Problem 6.** Evaluate these limits by relating them to a derivative.

a. (5 pts.) Evaluate  $\lim_{x \rightarrow 0} \frac{(1+2x)^{10} - 1}{x}$

b. (5 pts.) Evaluate  $\lim_{x \rightarrow 0} \frac{\sqrt{\cos x} - 1}{x}$

**Problem 7.** (10 pts.) Derive the formula  $\frac{d}{dx} a^x = M(a)a^x$  directly from the definition of the derivative, and identify  $M(a)$  as a limit.