TREE HEIGHT

(1) DIMENSIONAL ARGUMENT:

RELEVANT VARIABLES ARE:

TREE MASS : DENSITY, ρ
GRAVITATIONAL ACC'N, g
DIAMETER, d
HEIGHT, h
ALSO : YOUNG'S MODULUS, E

THESE CAN BE FORMED INTO A DIMENSIONLESS GROUP:

$$\frac{Ed^2}{\rho g \, h^3} \qquad \left[\frac{F}{L^2} \; L^2 \; \frac{L^3}{F} \; \frac{1}{L^3} \right] = [-]$$

$$\Rightarrow \frac{Ed^2}{\rho g h^3} = \text{CONSTANT}$$

$$\Rightarrow \quad d \propto h^{3/2}$$

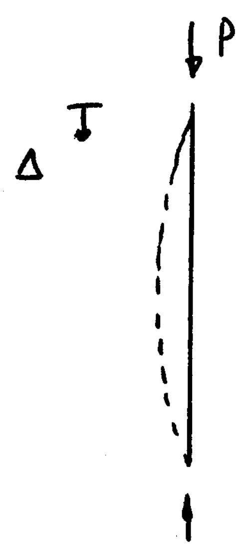
(2) WHAT CONTROLS TREE HEIGHT? BUCKLING ARGUMENT

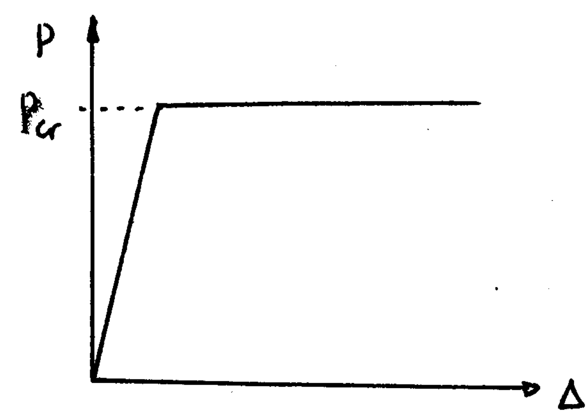
- TREE LOADED BY ITS OWN MASS, m

$$m \propto \rho g d^2 h$$

d = trunk diameter
h = trunk height
ρ = density
g = grav. acc'n.

- COLUMN BUCKLING (EULER)





INITIALLY COLUMN COMPRESSES SLIGHTLY & $P \propto \Delta$ (HOOKE'S LAW)

AT SOME CRITICAL LOAD, COLUMN BUCKLES & Δ INCREASES DRAMATICALLY

$$P_{cr} = \frac{\pi^2 E I}{l^2}$$

E = Young's Modulus
I = moment of inertia
l = column length

TREE HEIGHT: BUCKLING

FOR A TREE: $P_{cr} \propto$ MASS, M.

$$\therefore \quad m \propto \frac{EI}{h^2}$$

$$\rho g d^2 h \propto \frac{E d^4}{h^2}$$

$$\frac{\rho g}{E} \propto \frac{d^2}{h^3}$$

FOR WOODS OF DIFFERENT DENSITIES (e.g. oak, balsa)
$E \propto \rho$ (ie. ρ / E = CONSTANT)

$$\therefore \quad d \propto h^{3/2}$$

LOG-LOG PLOT OF DIAMETER VS. HEIGHT: SLOPE 3/2.

TREE HEIGHT

$d \propto h^{3/2}$

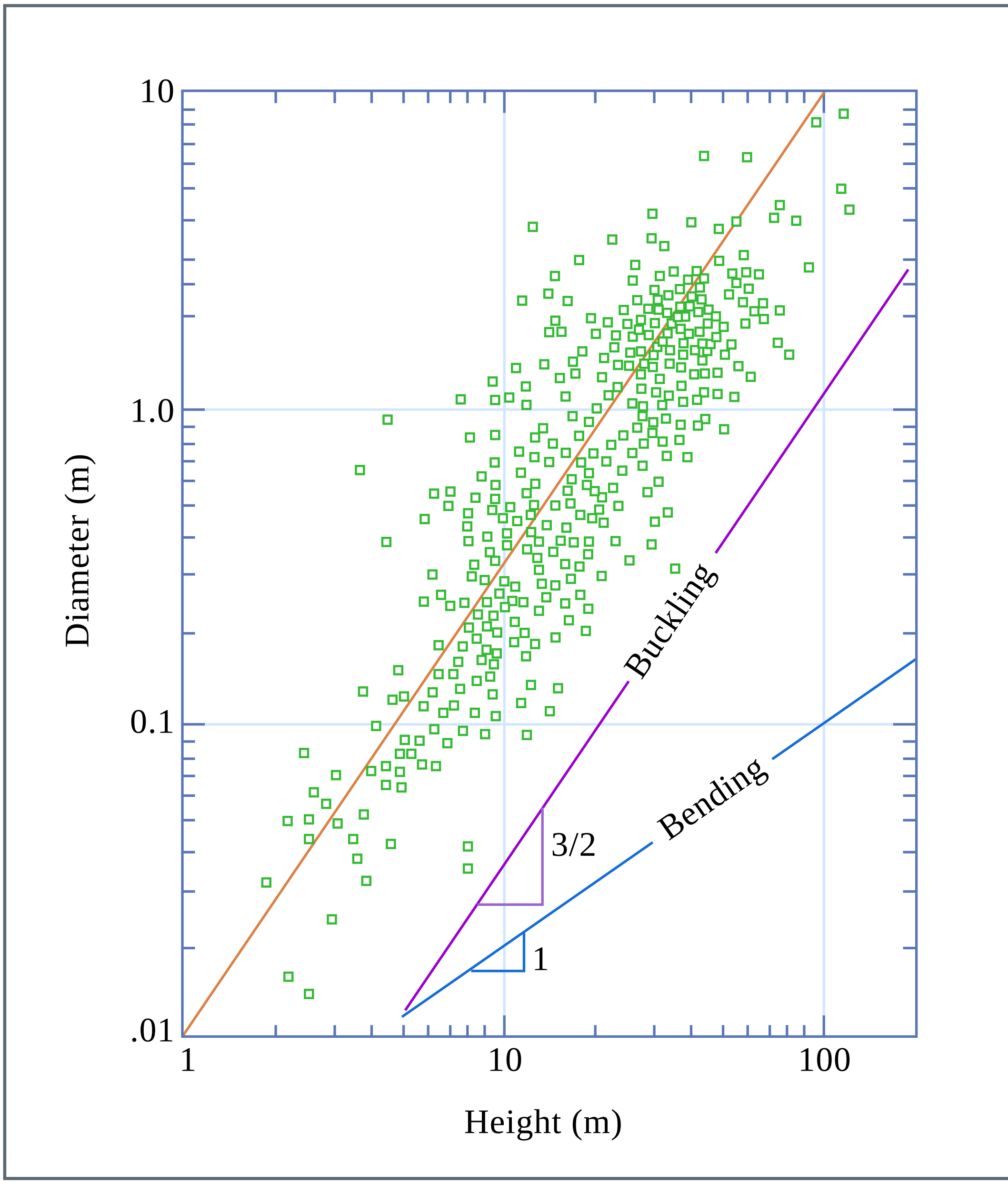


Figure by MIT OCW. After Bonner and McMahon (1983).