PROFESSOR: This, as you know, is the last lecture of thermal tensor properties of crystals for the semester. Come on, can't somebody go, aww, just to make me feel good? At least nobody said yay, so that makes me feel good, too. So I guess on balance I come out feeling pretty happy.

What I'm going to do today is to talk a little bit more about forthright tensor properties, which we've not said much about. And I'm going to look at some scalar moduli and look at them for a couple of different symmetries and restrictions on the compliances. The surfaces that you might expect turn out to be really, really weird. Because these will be fourth order variation with the direction cosines. So when you take trigonometric functions and raise them to the fourth power, you get severe anisotropies for a great many of the scalar moduli.

I'm sorry to say, years ago when I made the acquaintance of a computer programmer in course six who was looking for something challenging to do in connection with material science and engineering, I said, wow, have I got something for you. How about doing some computer graphics? And this was a few years ago when such products were fairly rare. And how about making a program that would let us see visually how some of these scalar moduli will vary with direction and provide the provision so you can turn them over in space, just as you would pick up a model in your hands and rotate it around till you finally came to appreciate it?

And then for something like a triclinic crystal, where there are almost a couple of dozen different moduli, how about allowing people to scale up or scale down the value of one of the elastic constants and let them see how the shape changes? And oh, that was-- he went to work on it for a semester and came back. And it was a gorgeous thing.

You could take a monoclinic representation of three dimensions of Young's modulus. Say you want to change S31. And it would change by a factor of 10. You'd see a big nose blow out on the surface and then contract down again. And you could turn around. Anyway, the program having told you how marvelous it is, it's no longer supported because of the operating system that it operated under having done defunct. But anyway, that was a fun thing to play with.

AUDIENCE: What operating system?

PROFESSOR: I don't even remember anymore. It was a fairly obscure one. Was not only the system, but it worked through a software package which operated on that system. All right.

So we're going to talk today about fourth rank tensor properties. The most important-- in fact, probably the only ones you've ever heard of, are the elastic, stiffnesses, and compliances, which are ways of representing strain in terms of stress or stress in terms of strain. I think I'll sort of see how the time plays out. If I finish just about everything that I'm hoping to say by on the hour, I think I'll just go an extra five or 10 minutes.

And then comes a great, grand, and traditional event at MIT. All semester long I've been grilling you with problem sets and quizzes, and you've had to dance through your steps just because I told you to. Now how's this for reciprocity-- you get to evaluate me. You get to grade me. MIT, at the end of every semester, has a course evaluation. It's submitted anonymously by you, so you could really vent your spleen without ever having called to task for it.

So when we finish, either for our break or at the end of a slightly extended first session, I will bow, to thunderous applause, I hope, and exit. And then you will be left alone to fill out the course evaluation. Corinne will collect them from you, and she will deliver them over to department headquarters. The results will be tabulated, and so will the comments. But they will not be in your hand, all of which I recognize, having spent hour after hour grading your quizzes. So they will be submitted to me in a completely sanitized fashion. I won't know who said what.

A final question that I will answer before you ask it, what about the quizzes? How are you doing on them? Well, I have come to hate them. I think I will give one-question quizzes from now on the future. It takes me about five to seven minutes to grade each question. And there are about 25 of you. So it takes me, to grade one question, somewhere between two and 1/2 and three hours. And there are 10 questions on the two quizzes remaining to be graded. And so I let you do the arithmetic, and you can see what sort of torment I've been living in the last few days in return for the torment I put you through for two brief hours. So wow. I will leave it to you to decide who's getting the worst of this deal.

OK, fourth rank tensors. Elasticity's probably the only one you can think of. Let me give you an example in this handout, an example of another forthright tensor property. And this is the piezoresistive effect, the way electrical resistance of the material changes in response to an applied stress. It's a pretty exotic property. I had never heard of it until I was at a meeting of the American Crystallographic Association and somebody, believe it or not, from Texas Instruments-- so you better bet that this property is useful in technology-- fellow named Ahmed Amin got up and gave a talk on the piezoresistive effect. And he had marvelous slides at the outset that defined the effect

So what you have here on these few sheets is a few pages that define the piezoresistance effect. We're not going to do anything with it. It's a fourth rank tensor, so it will proceed to transform like other forthright tensors, such as elastic stiffness and compliance. I should warn you about these following pages, which were slides that he used at his talk. He seems to have done the lettering with a magic marker. Because all of the I's and J's are indistinguishable, which makes interpretation of what he's saying here a little bit challenging.

And then the other thing that throws one off is that he uses x to represent stress. I don't know what field or in what discipline the elements of stress are called xij, but this is what he does. And once you figure that out, the interpretation of what he has said here is fairly apparent if you think about it.

Anyway, he takes a state of zero stress and strain and expands it as a series, and then picks off different coefficients. And one of them is a set of moduli which he calls pi, pi subscript ijkl, which relate stress, xkl in his notation, to a change in the density, delta row ij. So here's a fourth rank tensor property that will indeed have to conform to all the restrictions for the moduli that we have defined for the elastic stiffnesses and moduli.

All right. Let me then turn to stress and strain. We introduced the relation between stress and strain, but didn't really go into detail on the bizarre absorptions of factors of two or four that have to be done in order to make this come out in a nice, clean matrix form. So let me remind you of where these silly factors came in.

We took our stress tensor, sigma 1 1, sigma 1 2, sigma 1 3, 2 1, 2 2, and 2 3, sigma 3 1, sigma 3 2, and sigma 3 3. The tensor had to be symmetric. The off-diagonal term sigma ij had to be equal to the term sigma ij. And this was for the reason of mechanical equilibrium. These off-diagonal terms, sigma ij and sigma ij, we saw, exerted a torque on a body. And unless they were equal, the body would undergo an angular acceleration.

And then we renumbered these according to the convention that as we would march down the diagonal of the tensor and take pairs of subscripts, 1 1, 2 2, 3 3, represent them by a single 1, 2, and 3, then march up the side to define a sigma 4, sigma 5, and a sigma 6. If you remember that little algorithm you can always keep straight what these reduced subscripts represent.

So with that definition, then, the stress tensor reduced to sigma 1, sigma 6, sigma 5, sigma 6, sigma 2, sigma 4, sigma 5, sigma 4, sigma 3. And that takes a tensor and degrades it to a matrix. Because there is no law of transformation for this representation of the elements of stress.

OK, then we do something very similar with the strain tensor, epsilon 1 1, 1 2, 1 3, epsilon 2 1, epsilon 2 2, epsilon 2 3, epsilon 3 1, epsilon 3 2, epsilon 3 3. And we saw that for physical reasons this tensor also had to be symmetric. We had to have epsilon ij identical to epsilon ji, the reason being that if these off-diagonal shear

strains were not equal, the state that we would be defining was one of actual deformation combined with rigid body rotation. So unless this was the case, the definition performed by the tensor epsilon iij would define rigid body notation as well.

OK. We use the same process of renumbering to convert this from a tensor to a matrix. And we got this into a form, epsilon 1, epsilon 6, epsilon 5, epsilon 6, epsilon 2, epsilon 4, epsilon 5, epsilon 4, epsilon 3. But in order to do that, we saw we had to consume a factor of two. So there is a factor of two built into the off-diagonal terms. And what we have put in here is we converted this to actually epsilon 1, 1/2, epsilon 6, 1/2, epsilon 5.

And I should have written that this way to begin with. So all these factors of two appear so that when we combine these epsilons we get a nice, clean matrix relation between stress and strain that doesn't involve factors of two. And that's a great convenience if we're going to be doing all of our deformation within the framework of one coordinate system.

OK. So if we now use these definitions to write the stress in terms of strain, we will set up our fourth rank tensor property. If we do this first for the stress in terms of strain, you would have a tensor element of stress sigma 1 1, which would be equal to C1 1 1 1 times epsilon 1 1 plus C1 1 2 2, epsilon 2 2 plus C1 1 3 3, epsilon 3 3. And then we would have, in addition, off-diagonal elements of strain. We'd have C1 1 2 3 times epsilon 2 3 plus C1 1 3 2 times epsilon 3 2 plus C1 1 2 1, epsilon 2 1 plus C1 1 1 2, epsilon 1 2 plus C1 1-- and I should have made this 1 3 so that the other integers come out right. And C1 1 1 2 times an epsilon 1 1 1 2 plus a C 1 1 2 1 times an epsilon 2 1.

So that is one line of the relation between stress and strain. And when we condense this to a matrix form, it is surprising, once we expand it once more, at how cumbersome this expression is. Now, why do I remind you of all this? Isn't it nice to work in the form of a fixed coordinate system where we can use a matrix for the Cijkl's? The answer is, fine, unless you want to change the coordinate system. And you might want to do that for practical reasons such as cutting out a specimen

which makes it convenient to define the stiffnesses and compliances relative to a coordinate system taken along the logical directions in the specimen.

And the other reason for doing it, and that is what I want to do a little bit of this afternoon, is to derive the symmetry restrictions on the stiffnesses and compliances. You cannot transform a matrix representation of the stiffness or the compliance. You can only do this for the full-blown tensor arrangement. So having collapsed down, which we'll do momentarily just to remind you of how it goes, we will have to expand again to derive symmetry restrictions. Do not exit at this point. We're not going to do any of these calculations in their full glory detail. We'll just set up the problem and then jump immediately to the outset.

So how does this pay out if we try to make the condensation? I remind you again that the symbol C stands for stiffness, and the symbol S, which we'll see later, the Sijkl's are call compliances. And the perversity of that semantic description of these tensor elements is perverse for reasons that I have never really been able to understand.

So if we go down to matrix form, we'll write instead of sigma 1 1, simply sigma 1 1. We'll call this C1 1 times epsilon 1 plus C1 2 times epsilon 2. So far so good. C1 3 times epsilon 3. And now we have problems, because epsilon 2 3 was defined as 1/2 of epsilon 5. Now we'll have a C1 5 here, and then a C1 3 2 is C1 5 again. And this also is 1/2 of epsilon 5. And then similarly, this is C1-- I'm sorry. This is C1 4. This is C1 5 times 1/2 of epsilon 5 plus C1 5 times 1/2 of epsilon 5 plus C1 6 times 1/2 of epsilon 6 plus C1 6 times 1/2 of epsilon 6.

So this, given the way in which we had defined matrix strain, initially in connection with piezoelectricity when we entered the realm of third rank tensors, this will play out OK. This says that sigma 1 is simply C1 epsilon 1 plus C1 2 times epsilon 2 plus C1 3 times epsilon 3. And now the 1/2 gets absorbed, and it's simply C1 4 times epsilon 4, and so on.

If we look at another line, one that involves some of the off-diagonal terms and strain, if we, for example, look at sigma 2 3 and we write this down as sigma 2 3

times 1 1 times epsilon 1 1 plus C2 3 2 2 times epsilon 2 2 plus C2 3 3 3 times epsilon 3 3 plus C-- and I'll just write down a few of these additional terms. This would be C2 3 3 2 times epsilon 3 2 plus C2 3 2 3, epsilon 2 3, and then other terms for additional terms, and epsilon ij with i not equal to j, which are going to behave the same way.

So if we convert this, we go to sigma, and we would call this sigma 4. This would be C4 1 in matrix notation times epsilon 1 plus C4 2 time epsilon 2 2 plus C4 3 times epsilon 3. And now this matrix element would be C4 4. And in place of the tensor element of strain epsilon 3 2, we would write 1/2 of epsilon 4. And the next term is again a C4 4 times a 1/2 epsilon 4 and the terms in epsilon 5 and epsilon 6 would behave the same way. So you can see that the factor 2 is absorbed, and this becomes simply C4 4 times epsilon 4.

So all the factors of 2 have disappeared. And we can say, provided we remember the way we have condensed the elements of tensor strain, we can say with complete confidence that in our matrix notation, sigma i is Cij times epsilon j where i goes 1, 2, and 3, and j goes 1, 2, 3 all the way up to six.

AUDIENCE: [INAUDIBLE]

PROFESSOR: No. No, no, these are the-- yeah, I'm sorry. Yeah, inj. So it's a six by six matrix, right. Right you are. And this is a six by six. And in here are 36 businesses stiffnesses. OK, any comments other than, yuck? And again, if you stay in one coordinate system it's not so bad. In fact, instead of having nine by nine 81 terms you have 36 stiffnesses. And that's a great convenience.

Unfortunately, things are not quite so simple if we work with the compliances represented by the symbol S. And if we attempt to write strain in terms of stress, we'll have an S1 1 1 1 times sigma 1. We'll have an S1 1 2 2 times sigma 2 2 plus an S1 1 3 3 times a sigma 3 3 plus an S1 1 2 3 times a sigma 2 3 plus an S1 1 3 2 times a sigma 3 2, and an S1 1 2 3 3 1 times sigma 3 1 plus an S1 1 3 times a sigma 1 3, and then other off-diagonal terms which I won't bother to mention.

So if we convert this to matrix form, epsilon 1 1 would be replaced by the matrix term epsilon 1. S1 1 1 1 would become S1 1, and this would be sigma 1, and an S1 2 times a sigma 2, plus and S1 3 times sigma 3.

And now we have a problem, Houston. Because we would have an S1 4 times a sigma 4 plus an S1 4 times sigma 4. And now in the sigmas there is no factor 1/2 in the matrix representation of stress as there was with strain. Because we ate the factor of 2 in defining strain. But now there is no factor of 2 and stress. So we're going to have to do something with that. So this would give us some additional terms, S1 5 times sigma 5, and then an S1 5 times a sigma 5 again.

So what are we going to do? Again, we're stuck. We can either say that epsilon i is equal to Sij times sigma j if j is not equal to 4, 5, or 6. Or we can absorb, again, a factor of 2 in the definition of the compliances. And that, again, since we will usually be working in one, and the same coordinate system, is the convenient thing to do. So what we will do is to define this as S1 4 times sigma 4. And in so doing, we have to combine the factor of 2 into the definition of S1 4. So S1 4 would be equal to 1/2 of S1 1 2 3 plus S1 1 3 2.

So it's going to be 1/2 of one of these equal terms that involve one on-diagonal subscript and one off-diagonal subscript. So this is the only way we're going to be able to write a nice, clean matrix representation.

Things get worse.

AUDIENCE: Sir?

PROFESSOR: Yes?

AUDIENCE: You're sure of your [INAUDIBLE]?

PROFESSOR: Yeah, this would be equal to 2S1 4. I'm sure it's 1/2. I'm not sure of what term would go in front. So I would have 2S1 4 times sigma 4. And if I want to write this-- I'm sorry. Let me go back to the full matrix expression. I would have S1 1 2 3 times sigma 2 3 plus S1 1 3 2 times sigma 3 2. I know that sigma 2 3 is equal to sigma 3

2, so I could write this as S1 1 2 3 plus S1 1 3 2 just simply times sigma 2 3, one of them.

Now what I would really like to do is to write this as S1. Let's change this to sigma 4. I would like to write this as S1 4. So it follows then that S1 4 is equal to S1 2 3 plus S1 1 3 2. And that is the other way around, isn't it? Thank you. I knew there was a factor of 1/2, but it goes in here.

AUDIENCE: And then there's no 2 at all. If you wanted to say that S1 1 2 3 is equal to S, 1 1 3 2, then you would have S1 4 equal to 2S1 1 3 2.

PROFESSOR: OK, you're right, you're right. So it's this. Let's leave it at that. But if these are equal, we could say-- and we did show that the compliance tensor is symmetric, so we could say that this is equal to 2S1 1 2 3 or 2S1 1 3 2 as they're equal. I told you it was going to get bad, but I didn't think I would be contributing to it the extent that I am.

I don't know if you really want to see this, but this gets even worse when we deal with something like epsilon 2 3, where this is a term that would be replaced by epsilon 4. And then all of our absorbed factors come back to haunt us. Let me go through this quickly. This would be S2 3 1 1 times sigma 1 1 plus S2 3 2 2 times sigma 2 2 plus S2 3 3 3 times sigma 3 3. And then we'll have these terms, off-diagonal terms S2 3 3 1, sigma 3 1 plus S2 3 1 3 times sigma 1 3, and so on, other terms.

OK, and going from epsilon 2 3 to epsilon 4, we have to put on a factor 1/2. And that says that that 1/2 is going to create problems in the first term in this expansion. We'd call this sigma 1 1. We'd like to call this S4 1. But what is S4 1 from what we have defined here? It's the sum of two tensor elements. So we will have to define, now, again, S4 1 equals the same as we did before. It's going to be equal to the term S2 3 1 1 plus S3 2 1 1 1. And therefore if I put a 1/2 in this definition, then that will cancel, so far.

But then in the terms that come down in the lower quadrant of the matrix-- and I will

just right in two terms. We've got an S2 3 31 that we would write as S4 4 times sigma 4. And then I would have another term, S4 4 times sigma 4. But the S4 4 really is a sum of two tensor elements. So my definition of an S4 4 is that it should be--

AUDIENCE: Isn't that sigma 5 [INAUDIBLE]?

PROFESSOR: You're right, you're right. Yeah, this is 5. So this was right, 3 1, and this is sigma 5. You're right. And I go to S4 5, yeah. No, sigma-- OK, yeah. OK, let me cut to the chase. What we're going to have to do is to put in not the factor 1/2 that we had here, but there's going to be a factor 4 that-- something like S-- which one are we dealing with? Things like S2 3 3 1, something like S4-- and we're dealing with 5. S4 5 is going to be defined as S1, S2 3 1 3 plus S2 3 3 1 plus S3 2 1 3 plus S3 2 3 1. So we're going to have to put in a factor of 1/4 in front of the S4 5 in order to accommodate for those terms.

So our definition, then, in going from tensor compliances to matrix compliances is much more complicated. And the rule-- and just to summarize this, I gave you a handout last time. The necessary conventions to absorb these factors of 2 and 4 is that-- and I didn't bring it with me. OK, I just simply did not bring it with me. So I wouldn't attempt to summarize it.

All right. So hopefully I've convinced you of nothing other than that these conversions are messy. But the summary is that for the compliances, the Sijkl, you take this equal to Slm if l and m are equal to 1, 2, or 3. Then you have to replace Sijkl by 1/2 of Slm if i, j, or kl is 4, 5, or 6. And then you have to write Sijkl as 1/4 of Slm if i, j, and kl are 4, 5, or 6. But once you're into the coordinate system, which will be fixed in matrix notation, things are simple, since you don't have to worry about these factors of 2 or 3.

The reason I did this is I would like to now talk a little bit about how one can define important scalar moduli that describe a particular phenomenon. If you ask the question, how do mechanical properties vary with direction, we've got six unique elements of stress. And we've got six independent elements of strain that can work.

And if you wanted to show how each one of those elements varied with the other one, you'd need 36-- 6 times 6, 36-- representation surfaces and this is not going to be fruitful.

So just as we did for piezoelectricity where we could define a scalar modulus which represented the component of polarization normal to a very thin plate-- making it a thin plate means you're going to measure primarily the charge on the large surface, because polarization is charge per unit area. So if you looked at the surface of a plate with x1 normal to the plate, you are going to have a specimen that primarily gives you a measure of P1, or the charge on a surface normal to x1. And then you could apply any of six different states of simple stress.

And one thing that you could do is squeeze it with the tensile stress in the same direction as the normal to the surface. And that was the longitudinal piezoelectric modulus. Then on the quiz you looked at another modulus that related a component of polarization on another surface in response to a tensile stress that was not parallel to it.

There are a number of such moduli and mechanical properties. Probably the most important one is something called Young's modulus, which I'm sure you've all heard of. And Young's modulus involves taking a very long rod of the material and hanging a weight on it. And that weight will induce a strain. And if we take this as the direction of x1, we will, with Young's modulus, relate the change of length to the initial length of the rod.

And my question now is, rhetorically, do we want to do this in terms of stiffnesses or compliances? Does it make any difference? Yeah, makes a lot of difference in terms of the simplicity of the result that you get. Suppose we wanted to do this in terms of the compliances. What we'd be then applying would be a sigma 1. And this would be given by the compliance C1 1 times the strain E1. That looks like exactly what we want.

This delta l over l for this one-dimensional specimen with a one-dimensional directional applied stress, this would be simply epsilon 1. And that's force per unit

area here. This would be sigma 1 1. And it looks as though what we want is a C1 1 1 that relates a sigma 1 1 to an epsilon 1 1.

Is this going to be a definition that I would want to make for describing this? My colleague here shakes his head very seriously. No. Do you want to share your reservation?

AUDIENCE: Because as I recall, Young's modulus, once strain is weaker output [INAUDIBLE] in terms of stress you're going to want a compliance.

PROFESSOR: That's one answer.

AUDIENCE: You could theoretically get either one [INAUDIBLE].

PROFESSOR: If I could restate your objection, we can't impose a strain to get a stress. We really stresses the independent variable, practically speaking. We can stress it, but we can't instantly say, [INAUDIBLE] develop a epsilon 1. Yeah?

AUDIENCE: [INAUDIBLE]

PROFESSOR: You betcha. That's the reason. Added onto this is not only this term, but there will be a C1 2 times epsilon 2 plus a C1 3 times an epsilon 3, and so on. So we're not going to be able to get a nice, tidy relation between the tensile strain that we're measuring and the tensile uniaxial stress that is produced there. So the relation of stress in terms of strain that involves the stiffnesses is just not going to work.

But if we would instead express epsilon1 in terms of a compliance, a matrix compliance S1 1 times sigma 1, that's all that she wrote. We're measuring this by design by selecting an elongated specimen for which we'll primarily be seeing epsilon 1. This is not to say these other strains exist. There will be lateral strains here that will be epsilon 2 and epsilon 3. There will be shear strains.

Sounds mind-boggling. You pull the sample in this direction, and what it does is shear. Well, if this were a single crystal, that would happen. And these would be other components of deformation. But if we measure a pi sigma 1 and measure epsilon 1, then this is the way we have to define it. And the definition of Young's

modulus is that 1 over S1 1 is sigma 1 over epsilon 1. And this is Young's modulus.

I would hasten to observe that it is unfortunate that this very practical modulus involves the S's, which have all these complicated factors of 2 and 4. And therefore, if we try to go to a particular symmetry and ask the question, how does Young's modulus change as we cut this rod in different orientations from the single crystal. So I would like to illustrate for you how we would do this. I'm going to take a very, very simple example.

If you look in the table of symmetry restrictions that I passed out a couple of times ago, prior to the quiz, for an isotropic the tensor has a different form than it does for a cubic crystal. Cubic crystals are elastically anisotropic. But the form of the stiffness tensor for an isotropic material was S1 1, S1 2, S1 2, 0, 0, 0, S1 2, S1 1, S1 2, 0, 0, 0. Then the diagonal terms, if the material was isotropic, was 2s1 1 minus S1 2 for this term, 0, 0, 0, 0, 0, 0, and again, a 2S1 1 minus S1 2 plus a 0, and then 0, 0, 0, 0, 2S1 1 minus S1 2.

OK. Let me now derive Young's modulus as a function of direction and show that, in fact, this says that regardless how you orient the rod, the value of Young's modulus will stay the same. Let me also do something else first, though, which is something we should have scratched our head over when we first encountered it. Here's something curious. When we looked at symmetry constraints on single crystals, we found that for a cubic crystal, C6 6 had to be equal to 1/2 of C1 1 minus C1 2 if the stiffnesses were to be independent for all symmetry transformations.

For the compliances, however, we found that S6 6 had to be equal to 2 times S1 1 minus S1 2. Two very different equalities between tensor elements to make the elastic behavior be invariant to the symmetry transformations of a cubic crystal. How come? How come these are so different? Well, they have to be, in terms of the tensor, the same sort of a quality.

And the reason they don't look the same is because of our different absorption of the factors of 2 and 4 in defining the stiffnesses an in defining the compliances. Remember that for the Cijkl's there was no factor of 2 or 4 introduced in defining the

matrix elements. So this one is OK. This is a true equality between tensor elements, for any tensor whatsoever of fourth rank for a cubic crystal.

For these terms, these are off-diagonal. This is an off-diagonal compliance. And our definition is that Sijkl is equal to Smn, for m and n not equal to 4, 5, or 6. Here's this crazy thing again. It's equal to 1/2 of Smn for m or n equal to 4, 5, or 6. And it's equal to 1/4 of Smn for m and n equal to 4, 5, or 6. So S6 6 here is actually 4S1 2 1 2. And that's supposedly equal to 2S1 1, which is really S1 1 1, minus S1 1 2 2. And this says that 1/2 of S1 2 1 2 is equal to S1 1 1 1 minus S1 1 2 2.

So this is exactly the same constraint on tensor elements, when we take out these factors of 2 and 4 that have been absorbed. So this, in fact, although it looks very different with those factors absorbed, is exactly this same relation. So the symmetry constraint is the same.

AUDIENCE: How'd you get 1/2 there?

PROFESSOR: Hmm? How did I get 1/2 here? It's 4S1 2 2 had to be equal to 2S1 1 minus S1 2. And this was the equality. I'm working up this way. So this was the statement in the matrix forms of the compliances that I handed out. Looks different from the term on the left. So I wrote this now in terms of the matrix elements. And the factor of 4 comes in here. The 2 was here in the equality that had to be held that the tensors stay invariant. And if I bring this 2 over on the left-hand side, it's exactly the same as when--

AUDIENCE: Shouldn't it be 2 now? [INAUDIBLE]. [INAUDIBLE] the right side.

PROFESSOR: No, this is just replacement of the matrix terms with the definition of the tensor.

AUDIENCE: Right, but [INAUDIBLE].

AUDIENCE: You're just dividing 4 by 2, right?

PROFESSOR: Yeah, exactly.

AUDIENCE: So wouldn't that make [INAUDIBLE]?

PROFESSOR: Oh, ha, OK.

AUDIENCE: You want to be able to [INAUDIBLE] by 4, so it'd be S12 1 2 equals 1/2 the difference to be equal to that.

PROFESSOR: OK, OK. Thank you. OK, let me then go back to what I started out to do. And I will write just one line of how we would go about expanding this just to indicate how intricate it is, and then I'll give a few examples for two point groups of how Young's modulus varies with direction.

OK. What we're going to say, then, is that one of these compliances, S1 1 1 1 prime is going to be equal to l 1/4 times S1 1 1 1 plus l 2/4 times S2 2 2 2 plus l 3/4 times S3 3 3 3 . And just now doing what we know, that any Sijkl prime is going to be Cii, Cj capital J, Ck capital K, Cl capital L-- these are direction cosines now, not stiffnesses-- times Sijkl. So it's a quadrupole summation over all of the non-zero tensor elements.

So what I'm doing now is taking these terms and expanding them to the full tensor form. So S1 1 1 1 prime, that's the first term in the transformed tensor. That's going to be these terms plus l1l2 squared times S1 1 2 2 plus S2 2 1 1 plus l1 squared l3 squared times S1 1 3 3 plus S3 3 1 1, and then still another term, l2 squared l3 squared times S2 2 3 3 plus S3 3 2 2. And then there will be terms of the form l2 squared l3 squared times, again, four terms, S3 2 3 2 plus S1 2 3 3 plus S2 3 3 2 plus S2 3 2 3, and then similarly, terms in the squares of l1 and l2, and these would involve four terms of the form S1 2 1 2 plus permutation 0, and then l1 squared l3 squared times, again, four terms, S1 3 1 3 and permutation 0 for four terms.

OK. So what do we have now? We have an expression for S1 1 1 1 prime. And this is, in fact, 1 over Young's modulus E. And we've done this summation over the supposed non-zero tensor elements in the matrix for an isotropic material.

And if I simplify this, and this will be just one more tedious and then we can see, that is for these equalities, we ought to, for l1, l2, l3, get the same value, S1 1. And that

is indeed what happens. So the summation, if I simplify, is l 1/4 times S1 1 plus l 2.4 times S2 2 plus l 3/4 times S3 3. And then a collection of terms plus l1 squared l2 squared, S1 2 plus S2 1. And then similar terms in l1 and l3, l1 squared l3 squared, S1 3 plus S 3 1 plus l2 squared l3 squared times S2 3 plus S3 2. And then some terms that stand by themselves, l2 squared l3 squared times S4 4 plus l1 squared l2 squared S6 6 plus l1 squared plus l3 squared times S5 5.

And consolidating this, this is going to be equal to l 1/4 plus l 2/4 plus l 3/4 times S1 1. And then combining terms in the second power of direction cosines, l1 squared l2 squared plus l1 squared l3 squared plus l2 squared l3 squared. And this is all times 2S1 2 plus 2S1 1 minus 2S1 2. So S1 2 drops out. And all this will be simply l 1/4 plus l 2/4 plus l 3/4. And then I'll add in the other terms here that involve products of squares of direction cosines.

And all this is times S1 1. But this turns out to be equal to simply l1 squared plus l2 squared plus l3 squared, the sum of the squares of the direction cosines of our rod, quantity squared times S1 1. And this term inside the parentheses is 1. So indeed, S1 1 prime is equal to S1 1. So the value of Young's modulus has not changed with direction if the form of the compliance tensor is like so.

And I think you would probably have taken my word for that, but it was intended primarily as an example of how, when S1 1 prime is the reciprocal of Young's modulus, how you would set up a transformation for S1 1 prime. And the form of this polynomial would be exactly the same, even for a triclinic crystal, if you included all the non-zero terms in this fashion.

All right, so let me wrap things up in another couple minutes for some cases that are real symmetries, where Young's modulus is anisotropic. And working in exactly the same way for the tensor that is the appropriate one for a cubic crystal, we would lift out the form of the stiffness matrix. The reciprocal of Young's modulus is S1 1 prime, so what we're saying is we have a long, skinny rod. This is x1. And what we're doing, if this is a single crystal, is examining how Young's modulus changes as we change the direction of the rod to a new orientation, x1 prime, that's

described by direction cosines l1, l2, L3, relative to x1.

The expression that results by exactly the process that we muddled through a moment ago is that for a cubic crystal, the form of S1 1 prime as a function of direction does indeed give anisotropy. Isaiah It's S1 1 minus 2 times S1 1 minus S1 2 minus 1/2 of S4 4. Remember, for a cubic crystal, 1 1, 2 2, and 1 4 are the only non-zero terms. And then this is times a polynomial l1 squared l2 squared plus l2 squared l3 squared plus l3 squared l1 squared. So this is, again the reciprocal of Young's modulus.

And it turns out to have a constant term, S1 1 , which is what we found for the isotropic material. But from that, as the direction cosines change, we subtract off a term that is a linear combination of 1 1, 1 2, and 4 4. So the question is, is this positive or negative? The direction cosines are all squared, so this term here is always going to be positive. So are we going to take the thing that we found for an isotropic material, which was a constant, Young's modulus, which was 1 over S1 1 prime, and that's equal to the Young's modulus E.

Are we going to add or subtract something to it? Well, it turns out that if you look at real materials, it can be either positive or negative. It's usually positive. So that says we are going to subtract off a term which goes as products of squares of direction cosines. And this is something that's going to be zero along the direction 1 0 0. So along the reference axes x1, x2, x3, which if you remember, where this form of the matrix came from was taking the axes along the edges of the cubic crystal. It turns out that this is going to be equal to 1/3 along the direction 1 1 1.

So if this term is plus, nothing gets added onto the surface in the directions that correspond to the four-fold axes or the twofold axes of a cubic crystal. Nothing gets added on if it's plus. But along the body diagonals, this takes on a value of 1 3, so a positive thing gets added on. And the best way I can describe this surface-- it's not a simple surface-- it looks like a cube with fuzzy edges. So we had a cube and started to dissolve it. So this is how the reciprocal of Young's modulus varies with direction.

If, on the other hand, this term is negative-- so this is what you get if it's positive. If

it's negative, again, you start with the basic isotropic variation of 1 over S1 1 1. If it is negative, and there's one metal, that's molybdenum-- molybdenum has a negative value of these compliances. It looks like a surface that has indentations along the 1 1 1 direction. So it looks like something with a dimple in it. There is a final case, which is not realized for any material that I know of, if that term is 0, then it's isotropic.

Let me give you one other variation that comes straight out of [INAUDIBLE]. And this is for a hexagonal crystal that might be a hexagonal close-packed crystal. For zinc specifically, which is a hexagonal close-packed metal, the form of the matrix is S1 1, S1 2, S1 3, 0, 0, 0, S1 1, S1 2, 0, 0, 0, S3 3, 0, 0, 0, S4 4, 0, 0, S4 4, 0, and S4 4. In the values of these specific compliances are 8.4 for S1 1, for S1 2 1.1, for S1 3, minus 7.8, for S3 3, 28.7, for S4 4, 26.4. And these are all in units of 10 to the minus 12 meters squared per Newton. That's good old MKS units. It turns out that there are 10 to the 2 meters squared per Newton per 1 centimeter squared per dyne, if you like CGS units.

The form of the reciprocal of Young's modulus, it turns out to be a surface of revolution. This is x3, which is the direction of C. And that's the surface of revolution and the value of S1 1 prime as a function of the angle theta. That's the only parameter we need examine the variation with since it's a surface of revolution. S1 1 prime is equal to S1 1 times the sine of theta to the fourth power plus S3 3 times the cosine of theta to the fourth power plus S4 4 plus 2S1 3 times sine squared theta, cosine squared theta.

So it's a fairly exotic surface, even as a surface of revolution. And what this looks like, it's something that peaks out at 3 times 10 to the minus 12 centimeters squared per dyne along the direction of C. It's something that comes down very sharply as you approach the normal to the C axis. And then there's a cute little wiggle just near the axis. So it looks something like that. Not a terribly isotropic surface, and a variation of Young's modulus of 3 to 1 in the direction parallel to C and perpendicular to C. So there are lots of exotic surfaces of this sort.

All right. I did run a little bit over. And I will absent myself and let you express yourself candidly on the questionnaire. Except for these numbers that I just put up on the blackboard, I haven't really said anything about, much about, numbers. So let me pass around some examples, not for metals, which we just looked at, but for oxides.

And this is interesting, because you'll remember there was an additional equality between the stiffnesses called the [? Kowshi ?] equality, which was supposed to hold for physical reasons, not for reasons of symmetry, if the forces were central, if the crystal was under a state of no stress, and if all of the atoms were situated at a center of symmetry. The first set of data that you have on the top sheet is for MGO, which is cubic. It's predominantly ionic compounds. And the atoms are all octahedrally coordinated. So all of the requirements for the [? Kowshi ?] equality should hold.

And you can see they're not really exact. And that's probably due to the covalent character. The two different sets of data here are for measurements by two separate observers. The second page is for aluminum oxide. And again, you see the order of magnitudes of the numbers on the order of 10 to the 12 dynes per centimeter squared in general, and variation from one to another for a lumen of a factor of about four.

I meant to bring the book in, but I left it behind in my rush to come off. Where do you get these numbers? And it's hard to really find. And the book from which I took these data are by two workers, Simmons and Wang. And it's called *Handbook of Elastic Constants*, I think that is. And this is a book that was published by MIT Press. Valuable repository of data, but it's all numbers. And there are many, many materials in here.

Simmons and Wang, interestingly, are geophysicists. Why should geophysicists care about stiffnesses? Why? Because these guys are going all over the face of the earth setting off little bits of explosive that send off seismic waves. And they probed the interior of the earth by the propagation of elastic waves. And the elastic waves

depend on the square root of stiffnesses over the density of the material. So they're very, very interested in the elastic properties of particularly rock-forming minerals.

Another interesting comment on this book is that there is no set of data for a single triclinic crystal. Too many stiffnesses that are independent, I guess, and too few materials that, fortunately, are anisotropic and triclinic.

OK, I am going to quit. And thank you for your attention. You've been a good class. And we've covered all sorts of exotic aspects of the behavior of crystalline materials. And you might think with some justification that you know all there is to know about symmetry and tensor properties of materials. But nevertheless, I would caution you that you don't know everything. So I have a final handout fill in the chinks, the cracks that we've not been able to fill. And this is a set of data that is titled "Think You Know Everything?" And this will fill in some things that you really don't know.

And it has a lot of very useful items on here. For example, there are 293 different ways to make change for $1. You didn't know that. 2/3 of the world's eggplant is grown in New Jersey. I'm a native of New Jersey, and even I didn't know that. The longest one-syllable word in the English language is "screeched." OK. And so it goes. In most advertisements-- this is one I never checked out-- in most advertisements the time displayed on a watch is 10 minutes after 10:00. Don't know why. How many ridges are around the edge of a dime? 118. And how many little dimples are there on a regulation golf ball? If you count them up, you'll find that there are 336.

So I'll end with one for the benefit of some of our visiting students. In England, the Speaker of the House is not allowed to speak. There is my oxymoron of the day. And with that I shall leave you, and I hope you enjoy these. It's one of these things that circulates around on the internet. So as the sheet finishes, now you do know everything.

So I'll leave you, and again I thank you for your faithful attendance. And you shall see me when you come to call for your quizzes. And I will, as most of you requested, send out an email to let you know when you can come and pick those

up. So again, thank you. Enjoy the mid-term break. And take advantage of some of the really interesting things that go on extracurricularly around the institute. So that's it. Thank you, and au revoir.

[APPLAUSE]