3.46 PHOTONIC MATERIALS AND DEVICES

Lecture 5: Waveguide Design—Optical Fiber and Planar Waveguides

Lecture

Fiber Optics

Optical fiber $\equiv$ core + cladding

guided if $n_2 > n_1$

power loss to cladding if $n_2 < n_1$

$$\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right)$$

each mode travels with β, v_g, $U(x,y)$, $\vec{P}$, $\vec{k}$

single mode (small core)

multi mode (large core)

modal dispersion: modes have different v_g

graded index fiber: gradual reduction of $n_2 \downarrow$

step index fiber: $n_2 \to n_1$ step change @ boundary

modal dispersion reduced for graded index: $v_g \uparrow$ as $n \downarrow$ i.e. large θ rays travel farther but faster.

Step Index Fiber

typically: $\frac{2a}{2b} = \frac{50\ \mu m}{125\ \mu m}$ multi-mode fiber

2a ~8-10 μm single-mode fiber

Δ = fractional index change

$= \frac{n_2 - n_1}{n_2} \ll 1$

Typical dopants to SiO_2: Ti, Ge, B

$n_2 : (1.44 - 1.46)$

$\Delta : (0.001 - 0.02)$

numerical aperture: NA = light gathering power

guiding of ray incident from air $\bar{\theta}_c = \theta_a$ for air/core interface

1. $\sin\ \theta_a = n_2 \sin\ \theta_c$

$$\sin\theta_a = n_2(1 - \cos^2\bar{\theta}_c)^{\frac{1}{2}}$$

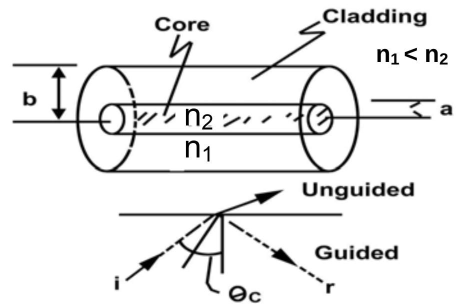
$$= n_2\left[1 - \left(\frac{n_1}{n_2}\right)^2\right]^{\frac{1}{2}}$$

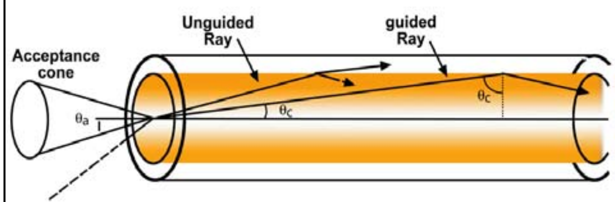
$$= \left(n_2^2 - n_1^2\right)^{\frac{1}{2}}$$

$$\theta_a = \sin^{-1}(\text{NA})$$

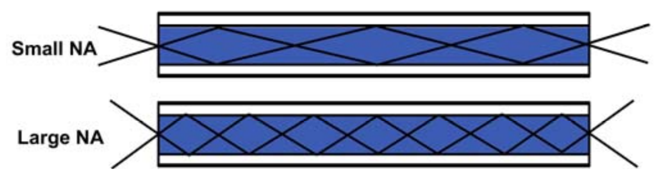
$$\text{NA} = (n_2^2 - n_1^2)^{\frac{1}{2}} \approx n_2(2\Delta)^{\frac{1}{2}}$$

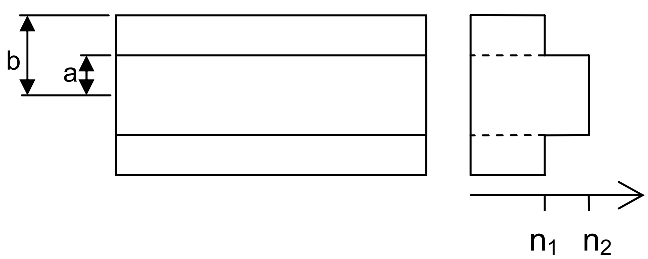
Notes





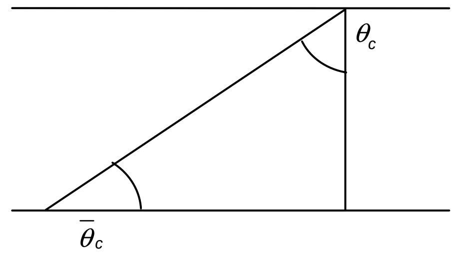
(a)





Lecture | Notes

$\theta_a =$ acceptance $\angle$ for fiber
$\quad \equiv$ exit angle for fiber
$\bar{\theta}_c =$ complementary critical $\angle$



e.g. SiO_2 <u>fiber</u>
$n_2 = 1.46,\ \Delta = 0.01$

$\therefore \bar{\theta}_c = \cos^{-1}\left(\frac{n_1}{n_2}\right) = 8.1°$

$\theta_a = 11.9°$
NA = 0.206

<u>Unclad fiber</u>
$n_2 = 1.46$, $n_1 = 1$, Δ=0.96
$\theta_c = 46.8°$, $\theta_a = 90°$
NA = 1
(all rays are guided)

Guided Waves

Helmholtz equation
$\nabla^2 u + n^2 k_0^2 u = 0$
$n = n_2$, $r < a$; $n = n_1$, $r > a$

$k_0 = \frac{2\pi}{\lambda_0}$

$u(r,\phi,z) = u(r)e^{-j\phi}e^{-j\beta z}$
$(b \to \infty)$

<u>Condition for guiding</u>
$n_1 k_0 < \beta < n_2 \ {}_0$

k_T = rate of change of u(r) in core

γ = rate of U(r) in cladding

rate of decay high $\Rightarrow$ low penetration

$k_T{}^2 = n_2{}^2 k_0{}^2 - \beta^2$

$\gamma^2 = \beta^2 - n_1^2 k_0^2$

$\boxed{k_T^2 + \gamma^2 \quad (n_2^2 - n_1^2)k_0^2 = (NA)^2 k_0^2}$

$k_T \uparrow, \gamma \downarrow \Rightarrow$ penetration into cladding

$k_T > NA \cdot \ {}_0 \Rightarrow \gamma$ imaginary, wave escape core

Lecture

Notes

V-parameter

Single mode fiber design

Define:

$X = k_T$ = normalized transverse <u>phase constant</u> in core

$Y = \gamma_T$ = normalized transverse <u>attenuation constant</u> in clad

$X^2 + Y^2 = V^2$

$$\boxed{V = 2\pi \frac{a}{\lambda_0} NA}$$

= normalized frequency

≤ 2.405 for single mode

<u>core radius</u> requirement for single mode

$$a < \frac{1.2\lambda_0}{\pi(n_2^2 - n_1^2)^{\frac{1}{2}}}$$

if $\Delta = 0.003$, $a = 8 - 10\ \mu m$

most single mode fiber designed @ V = 2.8 for better confinement of fundamental mode.

<u>Weakly</u> guiding fiber

$$\boxed{n_2 \simeq n_1,\ \Delta \ll 1}$$

guided waves are TEM

guided waves are paraxial

linear polarization (x, y) orthogonal

LP_{lm} = linear polarization mode

l = propogation constant

m = spatial distribution

$\vec{E} \perp z$

($\parallel$ fiber axis)

(X + Y polarization travel equally ω no coupling)

M, number of modes

V >> 1

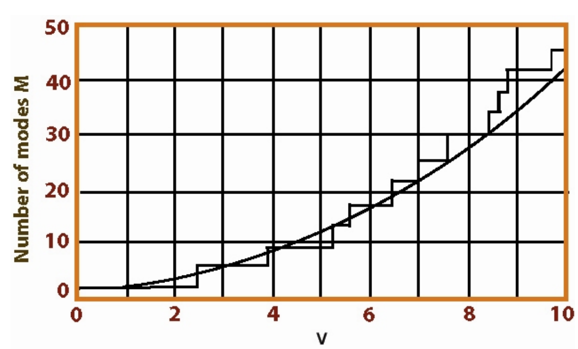
e.g. SiO_2 fiber

$n_2 = 1.452$, $\Delta = 0.01$, NA = 0.205

$\lambda_0 = 0.85\ \mu m$ (GaAs)

a (core) = 25 μm

$\Rightarrow$ V = 37.9, M = 585



<u>remove cladding</u> $\Rightarrow$ n_1 = 1, NA = 1

$\Rightarrow$ V = 184.8, $\boxed{M > 13{,}800}$

Lecture | Notes

Group Velocity $v_g >> 1$

$$\boxed{v_{lm} = \frac{d\omega}{d\beta_{lm}}}$$

$$v_{lm} \simeq c_2 \left[1 - \frac{(l+2m)^2}{M}\Delta\right]$$

$$z < l + 2m < \sqrt{M}$$

$$c_2 > v_{lm} > c_2\left(\frac{n_1}{n_2}\right)$$

phase velocity > v_{lm} > high order modes

(high order modes slower)

$\therefore$ fractional charge in $v_g \simeq \Delta$

large $\Delta \rightarrow$ large NA
$\rightarrow$ large M

high modal dispersion

Single Mode Fibers

small core diameter
small NA
long λ_0
u(r) ~ Gaussian

e.g. SiO_2
$n_2 = 1.447$, $\Delta = 0.01$, NA = 0.205
$\lambda_0 = 1.3\ \mu m$
single mode $\Rightarrow 2a < 4.86\ \mu m$

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if $\Delta = 0.0025$
single mode $\Rightarrow$ $2a < 9.72\ \mu m$

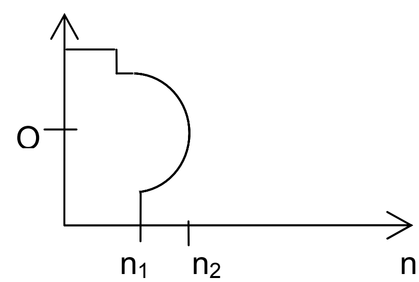
Graded Index Fiber

reduce modal dispersion
c_0 minimum @ center
$\rightarrow$shortest travel, slowest velocity
$\Rightarrow$ power low profile

$$n^2(r) = n_2^2\left[1 - 2\left(\frac{r}{a}\right)^p \Delta\right]$$

n $= n_2$ @ r = 0
$= n_1$ @ r = a

$$\boxed{\Delta = \frac{n_2^2 - n_1^2}{2n_2^2}}$$



$r \le a$
p = 1 $n^2(r)$ linear
p = 2 $n^2(r)$ quadratic
$p \rightarrow \infty$ $n^2(r)$ step function

Lecture | Notes

Number of Modes

$p \to \infty$ (step index)

$$\boxed{M \approx \frac{V^2}{2}}$$

$$v = 2\pi\left(\frac{a}{\lambda_0}\right)NA$$

Optimal profile

$p = 2$ $\Rightarrow v_g = c_2$

for all other modes (least modal dispersion for multimode)

$$\boxed{M = \frac{V^2}{4}}$$