

Fatigue Crack Growth

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Fatigue Crack Growth

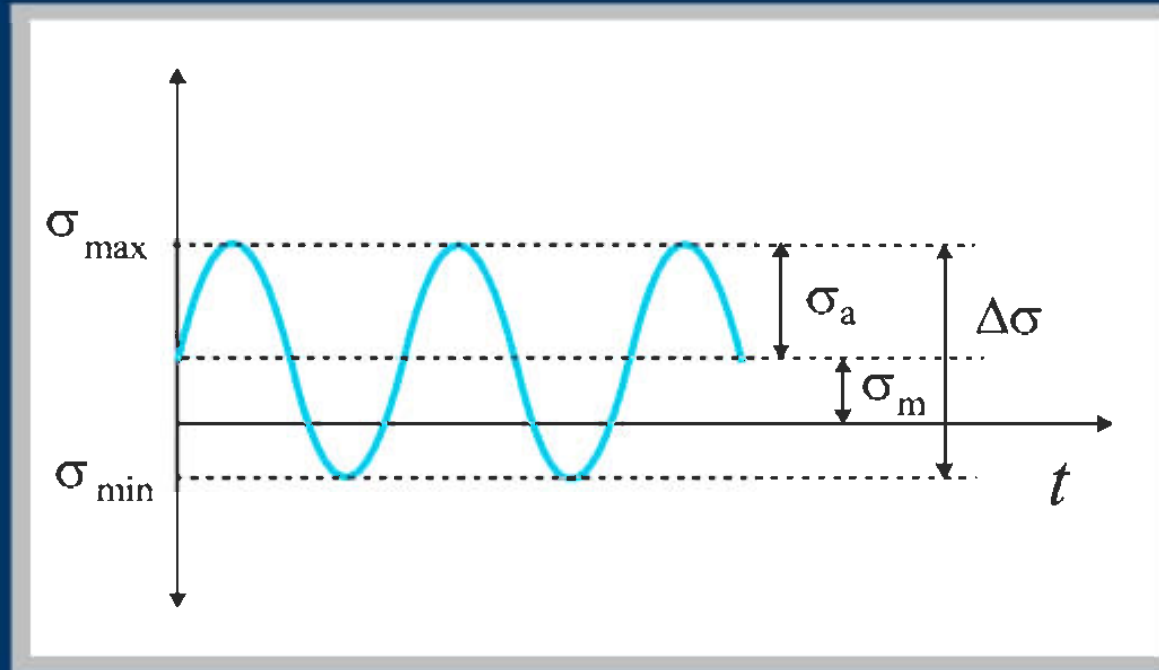
Importance of Fatigue

Key Idea: Fluctuating loads are more dangerous than monotonic loads.

Example: *Comet Airliner* (case study). The actual cabin pressure differential when the plane was in flight was ≈ 8.5 pounds per square inch (psi). The design pressure was ≈ 20 psi (a factor of safety greater than 2). Thought to be safe! However, crack growth due to *cyclic* loading caused catastrophic failure of the aircraft.

Cyclic Loading

A typical stress history during cyclic loading is depicted below:



Cyclic Loading

continued

What are the important parameters to characterize a given cyclic loading history?

- **Stress Range:** $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$
- **K-Range:** $\Delta K = K_{\max} - K_{\min}$
- **Stress amplitude:** $\sigma_a = \frac{1}{2} (\sigma_{\max} - \sigma_{\min})$
- **Mean stress:** $\sigma_m = \frac{1}{2} (\sigma_{\max} + \sigma_{\min})$
- **Load ratio:** $R = \frac{\sigma_{\min}}{\sigma_{\max}} = \frac{K_{\min}}{K_{\max}}$

Cyclic Loading

continued

- **Frequency:** ν or f in units of Hz. For rotating machinery at 3000 rpm, $f = 50$ Hz. In general only influences fatigue crack growth if there are environmental effects present, such as humidity or elevated temperatures.
- **Waveform:** Is the stress history a sine wave, square wave, or some other waveform? As with frequency, generally only influences fatigue crack growth if there are environmental effects.

Cyclic vs. Static Loading

Key difference between *static* and *cyclic* loading:

Static: Until applied K reaches K_c ($30 \text{ MPa}\sqrt{\text{m}}$ for example) the crack will not grow.

Cyclic: K applied can be well below K_c ($3 \text{ MPa}\sqrt{\text{m}}$ for example). Over time, the crack grows.

The design may be safe considering static loads, but any cyclic loads **must** also be considered.

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LEFM approach

continued

In 1961 the ideas of LEFM were applied to fatigue crack growth by Paris, Gomez and Anderson (Ref: A Rational Analytic Theory of Fatigue, *The Trend in Engineering*, Vol. 13, 9-14, 1961).

For a given cyclic loading, define ΔK as $K_{\max} - K_{\min}$ which can be found from $\Delta\sigma$ and the geometry of the cracked body.

Say that the crack grows an amount Δa during N cycles. Paris, Gomez and Anderson said that the rate of crack growth depends on ΔK in the following way:

$$\frac{\Delta a}{\Delta N} \rightarrow \frac{da}{dN} = C (\Delta K)^m$$

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LEFM approach

continued

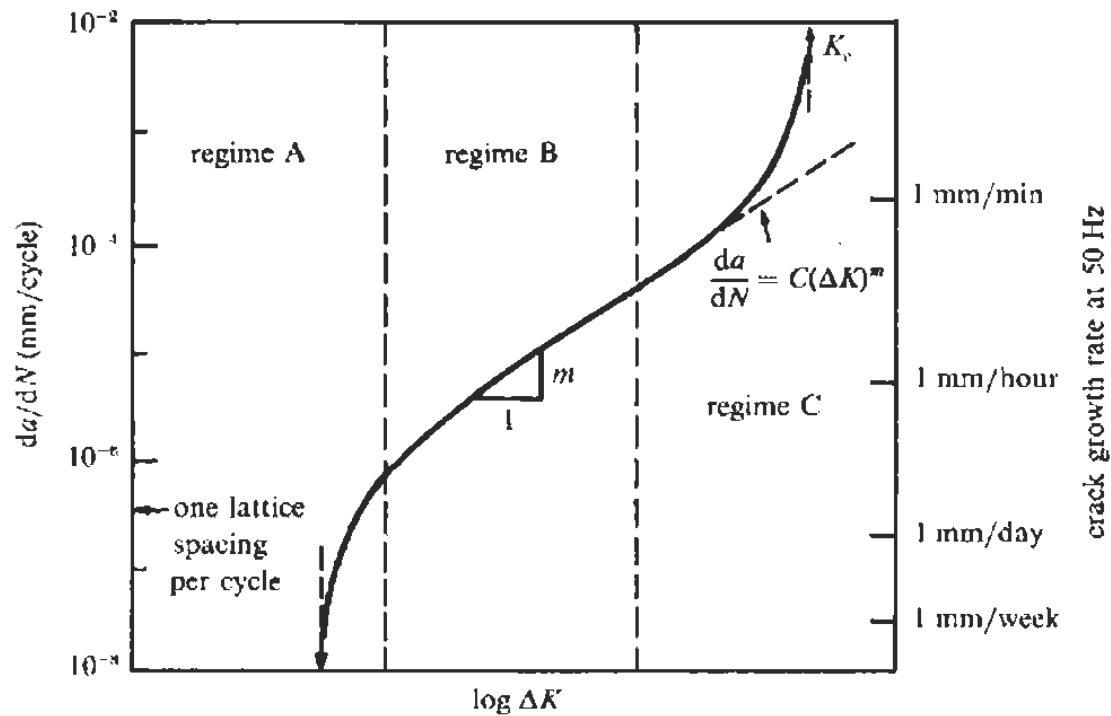
Thus a plot of $\log (da/dN)$ vs. $\log (\Delta K)$ should be a straight line with a slope of m .

The actual relationship between crack growth rate and ΔK is depicted on the following page. There are three different regimes of fatigue crack growth, A, B and C.

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LEFM approach

continued



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Three Regimes

Regime	A	B	C
Terminology	Slow-growth rate (near-threshold)	Mid-growth rate (Paris regime)	High-growth rate
Microscopic failure mode	Stage I, single shear	Stage II, (striations) duplex slip	Additional static modes
Fracture surface features	Faceted or serrated	Planar with ripples	Additional cleavage or microvoid coalescence
Crack closure levels	High	Low	—
Microstructural effects	Large	Small	Large
Load ratio effects	Large	Small	Large
Environmental effects	Large	*	Small
Stress state effects	—	Large	Large
Near-tip plasticity [†]	$r_c \leq d_g$	$r_c \geq d_g$	$r_c \gg d_g$

*large influence on crack growth for certain combinations of environment, load ratio and frequency.

[†] r_c and d_g refer to the cyclic plastic zone size and the grain size, respectively.

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Concept of the *threshold stress intensity factor* ΔK_{th} :

When ΔK is $\approx \Delta K_{th}$, where ΔK_{th} is the *threshold stress intensity factor*, the rate of crack growth is so slow that the crack is often assumed to be dormant or growing at an undetectable rate.

An operational definition for ΔK_{th} often used is that if the rate of crack growth is 10^{-8} mm/cycle or less the conditions are assumed to be at or below ΔK_{th} .

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Regime A

Continued

An important point is that these extremely slow crack growth rates represent an average crack advance of *less than one atomic spacing* per cycle. How is this possible? What actually occurs is that there are *many* cycles with no crack advance, then the crack advances by 1 atomic spacing in a single cycle, which is followed again by many cycles with no crack advance.

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When we are in regime B (*Paris regime*) the following calculation can be carried out to determine the number of cycles to failure. From the Paris Law:

$$\frac{da}{dN} = C (\Delta K)^m$$

ΔK can be expressed in terms of $\Delta\sigma$

$$\Delta K = Y \Delta\sigma \sqrt{\pi a}$$

Where Y depends the specific specimen geometry.

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Thus the Paris Law becomes:

$$\frac{da}{dN} = C (Y \Delta\sigma \sqrt{\pi a})^m$$

Assume that Y is a constant. Solve for da and integrate both sides:

$$\int_{a_0}^{a_f} \frac{da}{a^{m/2}} = CY^m (\Delta\sigma)^m \pi^{m/2} \int_0^{N_f} dN$$

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For $m > 2$:

$$N_f =$$

$$\frac{2}{(m-2) CY^m (\Delta\sigma)^m \pi^{m/2}} \left[\frac{1}{(a_0)^{(m-2)/2}} - \frac{1}{(a_f)^{(m-2)/2}} \right]$$

For $m = 2$:

$$N_f = \frac{1}{CY^2 (\Delta\sigma)^2 \pi} \ln \frac{a_f}{a_0}$$

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The constants C and m are material parameters that must be determined experimentally. Typically m is in the range 2-4 for metals, and 4-100 for ceramics and polymers.

For cases where Y depends on crack length, these integrations generally will be performed *numerically*.

Important: Note that in the Paris regime the rate of crack growth is weakly sensitive to the load ratio R . The key parameter governing crack growth is ΔK .

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Regime B

Continued

In these expressions, we need to determine the initial crack length a_0 and the final crack length a_f (sometimes called the critical crack length).

How do we determine the initial crack length a_0 ?

Cracks can be detected using a variety of techniques, ranging from simple visual inspection to more sophisticated techniques based on ultrasonics or x-rays. If no cracks are detectable by our inspection, we must assume that a crack just at the resolution of our detection system exists.

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Regime B

Continued

How do we determine the final crack length a_f ? We know that eventually the crack can grow to a length at which the material fails immediately, i.e.

$$K_{\max} \rightarrow K_c$$

or

$$Y \sigma_{\max} \sqrt{\pi a_f} \rightarrow K_c$$

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Regime B

Continued

Thus we may solve for a_f as follows:

$$a_f = \frac{1}{\pi Y^2 \sigma_{\max}^2} K_c^2$$

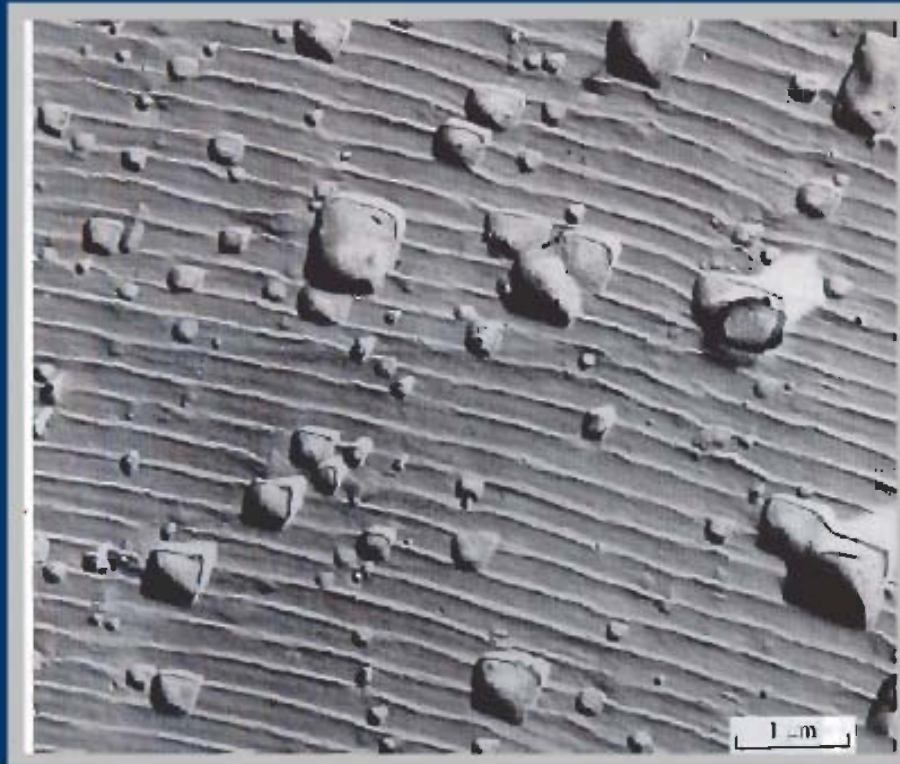
A very important idea that comes from this analysis is the following: even if a component has a detectable crack, it need not be removed from service! Using this framework, the remaining life can be assessed. The component can remain in service provided it is inspected periodically. This is the **crack-tolerant** or **damage tolerant** design approach.

Fatigue Striations

An advancing fatigue crack leaves characteristic markings called *striations* in its wake. These can provide evidence that a given failure was caused by fatigue. The striations on the fracture surface are produced as the crack advances over one cycle, i.e. each striation corresponds to da .

continued

Fatigue Striations



continued

Fatigue Striations

The previous slide showed fatigue striations on the etched fracture surface of a 2024-T3 aluminum alloy. Crack advance was from the lower left to the upper right.

Fatigue Striations

Models for Paris Regime

1. Geometric (or) CTOD models

$$\frac{da}{dN} \approx \Delta\delta_t = \beta \frac{(\Delta K)^2}{\sigma'_y E'}$$

\approx striation spacing

Important: This implies that the Paris Exponent m is equal to **2**.

Fatigue Striations

Advantages of CTOD-based models:

- Physically appealing
- Usefulness in multiaxial fatigue
- Link to microstructural size

2. *Damage Accumulation Models*

Basic Idea: Accumulation of local plastic strain over some microstructurally significant distance at crack tip leads to fracture.

$$\frac{da}{dN} = \frac{4\sigma_y}{u^*} \int_0^{r_c} \Delta u_y(x, 0) dx$$

Fatigue Striations

In the previous expression u^* represents the critical hysteresis energy, or the “work of fracture” and r_c represents some microstructurally significant distance.

$$\frac{da}{dN} \propto \frac{(\Delta K)^4}{\mu \sigma_y^2 u^*}$$

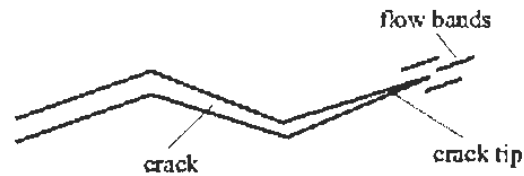
Note: This implies that the Paris Exponent m is equal to 4.

(Integration of local $\sigma - \epsilon$ hysteresis loops). Strain gradients at crack tip.

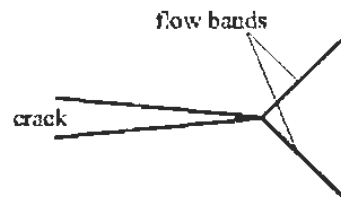
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Stage I and Stage II

- a) Near-Threshold: $r_y < d$
(Stage I, Modes II and I)



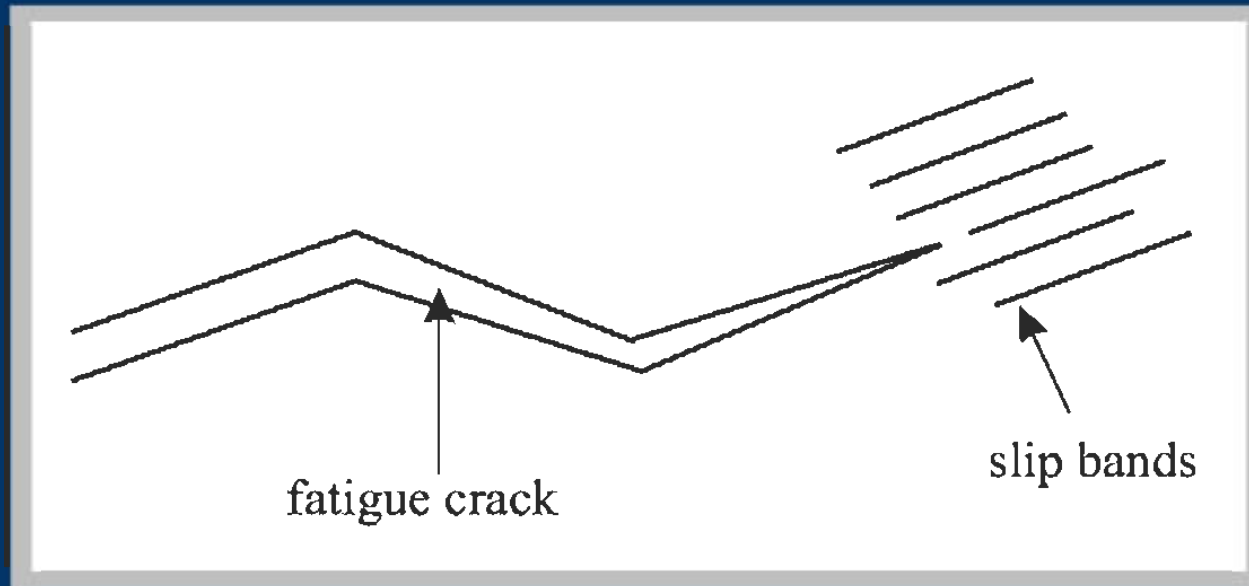
- b) Higher Growth Rates: $r_y > d$
(Stage II, Mode I)



(Forsyth, 1962)

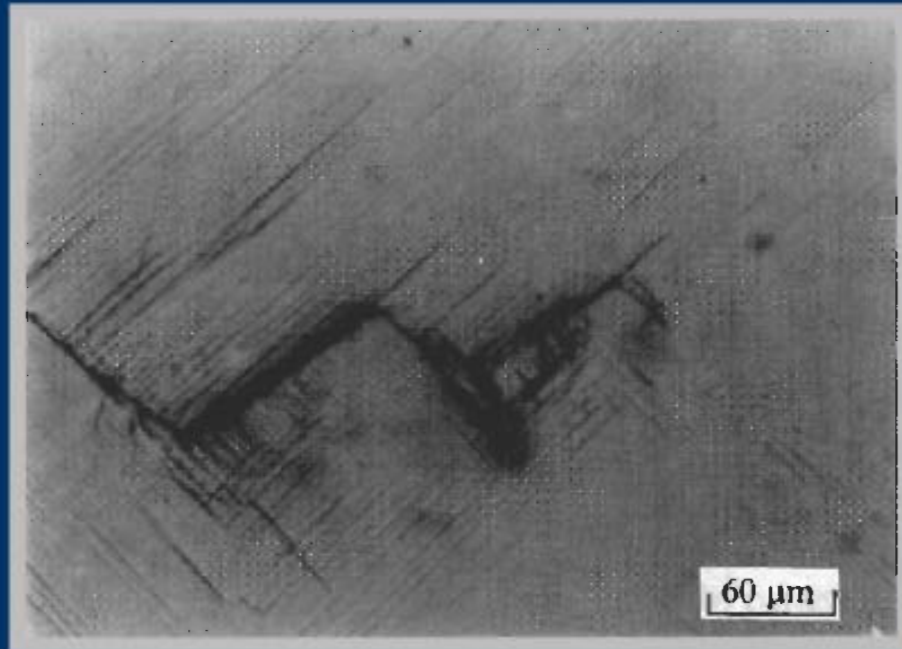
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Stage I



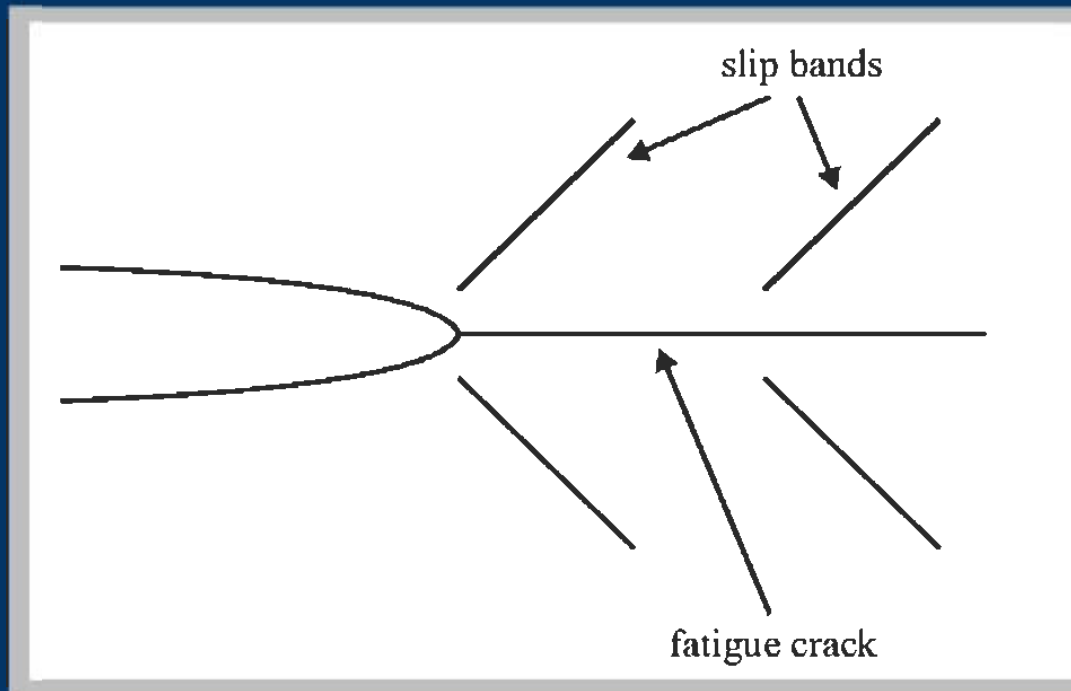
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Stage I



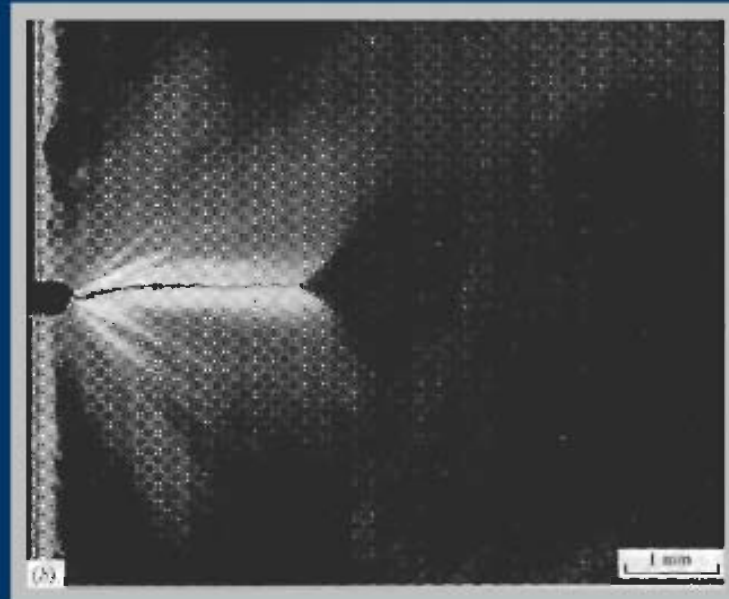
Stage II

Fatigue Striations



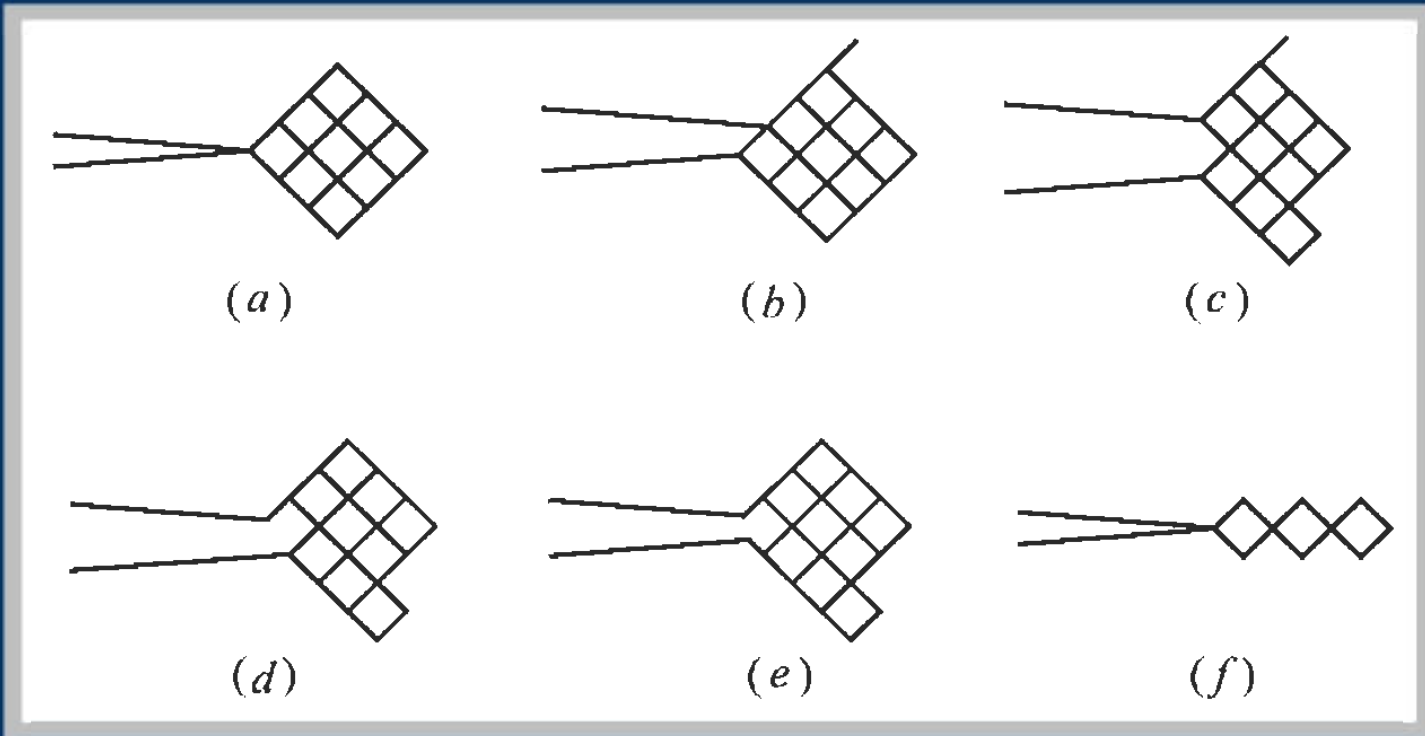
Stage II

Fatigue Striations



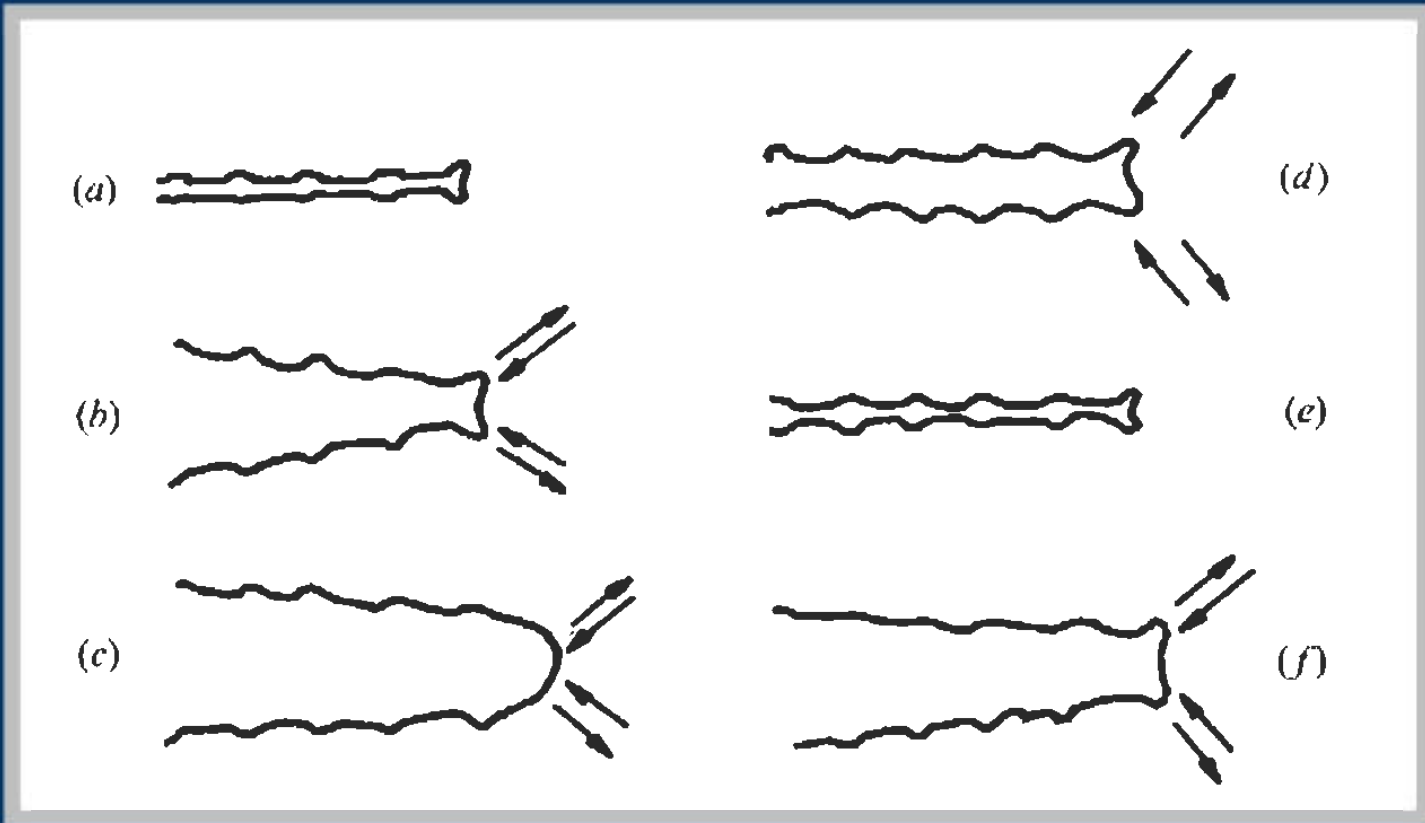
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Fatigue Striations



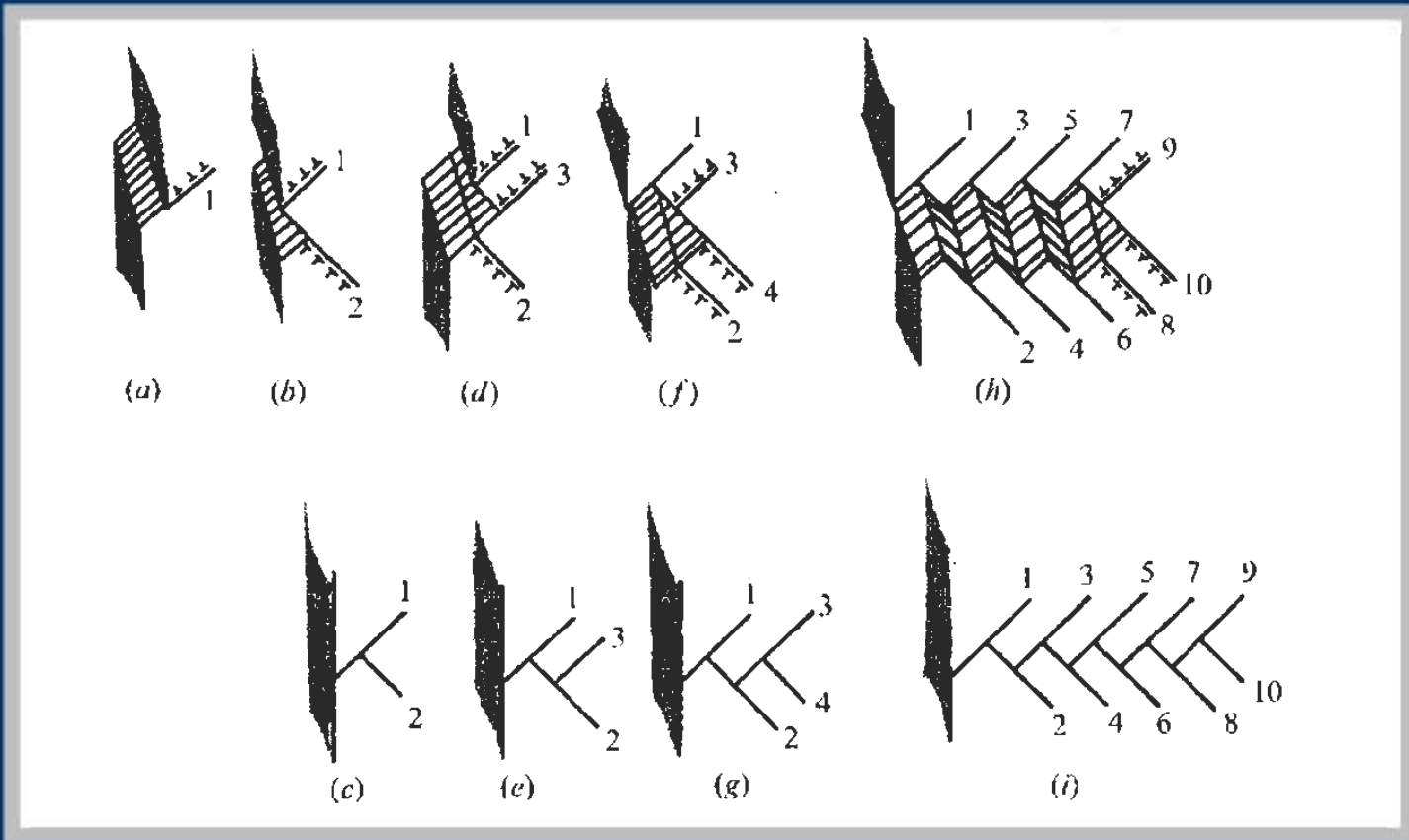
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Fatigue Striations



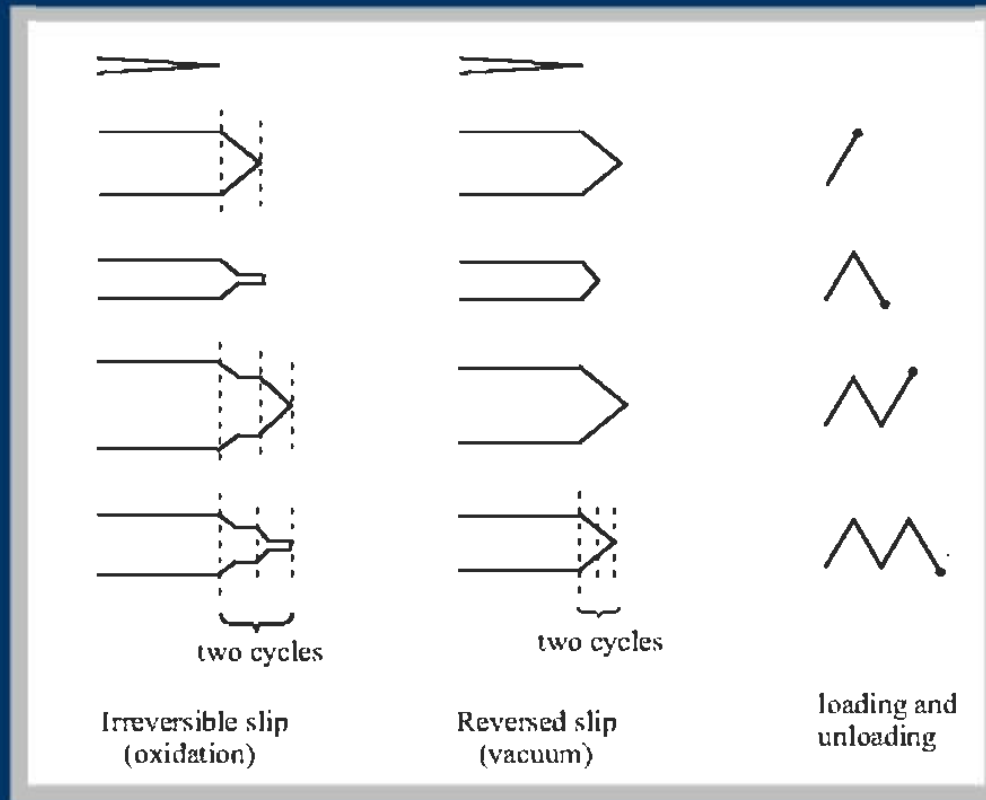
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Fatigue Striations



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Fatigue Striations



Fatigue Design

Safe-Life Approach

- Determine typical service spectra.
- Estimate useful fatigue life based on laboratory tests or analyses.
- Add factor of safety.
- At the end of the **expected** life, the component is retired from service, **even if** no failure has occurred and the component has considerable residual life.
- Emphasis on prevention of **crack initiation**.
- Approach is theoretical in nature.

Fatigue Design

- Even if an individual member of a component fails, there should be sufficient **structural integrity** to operate safely.
- Multiple load paths and **crack arresters**.
- Mandates periodic inspection.
- Accent on **crack growth** rather than **crack initiation**.

Retirement For Cause

F-100 gas turbine engines in F-15 and F-16 fighter aircraft for U.S. Airforce.

Old Approach:

- 1000 disks could be retired from service when, statistically, only one disk had a fatigue crack ($a \leq 0.75$ mm).

New, RFC Approach (since 1986):

- Retirement of a component occurs when the unique fatigue life of that particular component is expended.
- Retirement only when there is reason for removal (e.g. crack).

Retirement For Cause

Case Study

Continued

- Twin engine F-15 and single engine F-16.
- 3200 engines in the operational inventory of U.S. Air Force.
- 23 components of the engine are managed under RFC.
- 1986-2005 life cycle cost savings: \$1,000,000,000
- Additional labor and fuel costs: \$655,000,000