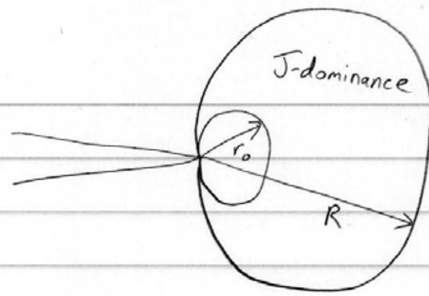
3.35 – Fracture and Fatigue
Problem Set 3 – Solutions
October 21, 2003

9.7



$r_o = 3\delta_t$

For plane strain and small-scale yielding conditions:

$$J = \frac{K_I^2}{E}(1-\nu^2)$$

$$r_p = \frac{1}{3\pi}\left(\frac{K_I}{\sigma_y}\right)^2 \quad \text{where } r_p \text{ is the monotonic plastic zone size}$$

J-dominance spans a distance R of up to $\frac{1}{4}r_p$

$$R = \frac{1}{12\pi}\left(\frac{K_I}{\sigma_y}\right)^2$$

CTOD: $\delta_t \approx \frac{0.6\,J}{\sigma_y}$ where $J = \frac{K_I^2}{E}(1-\nu^2)$

$$\delta_t = \frac{0.6\,K_I^2(1-\nu^2)}{E\sigma_y} \quad \text{We were given } \frac{E}{\sigma_y} = 500$$

$$\delta_t = \frac{0.6\,K_I^2(1-\nu^2)}{500\,\sigma_y^2} = \frac{0.6(1-0.33^2)}{500}\left(\frac{K_I}{\sigma_y}\right)^2$$

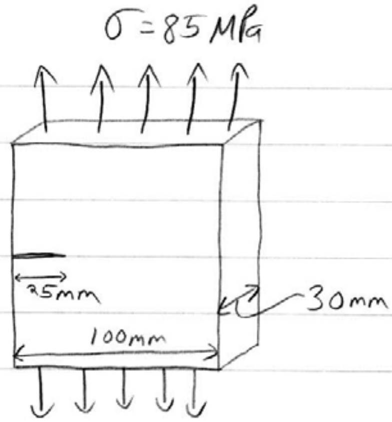
Comparing R to δ_t:

$$\frac{R}{\delta_t} = \frac{\frac{1}{12\pi}\left(\frac{K_I}{\sigma_y}\right)^2}{\frac{0.6(1-0.33^2)}{500}\left(\frac{K_I}{\sigma_y}\right)^2} = 24.8 \Rightarrow \boxed{R = 25\,\delta_t}$$

↳ J-dominance requirement satisfied.

9.8

(a)



From Appendix A.1

$$K_I = \sigma\sqrt{a}\, f\left(\frac{a}{\omega}\right) \rightarrow \frac{a}{\omega} = 0.25$$

$$f\left(\frac{a}{\omega}\right) = 2.665$$

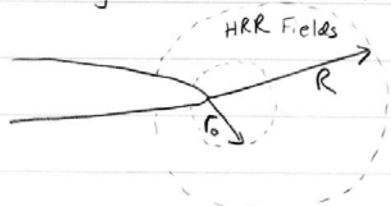
$$K_I = 85\,MPa\sqrt{0.025\,m}\,(2.665)$$

$$K_I = 35.82\,MPa\sqrt{m}$$

Assuming plane strain behavior:

$$r_p = \frac{1}{3\pi}\left(\frac{K_I}{\sigma_y}\right)^2 = 1.51\,mm$$

The region of J-dominance can be described as:



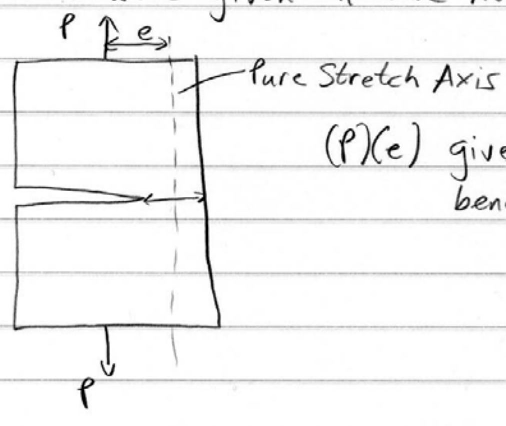
where $R \approx \frac{1}{4} r_p = 377.5\,\mu m$

Also, r_o should be greater than the size of the process zone (e.g. page 311 of the text: the grain size for transgranular cleavage or intergranular fracture, and the mean spacing of void-nucleating particles for ductile failure by void growth).

Therefore, the region of validity for the material with a grain size of $220\,\mu m$ is extremely small ($r_o \approx R$). Also, very few grains will fit into the above described region of validity (R). Therefore, crystal plasticity (rather than continuum) may dominate.

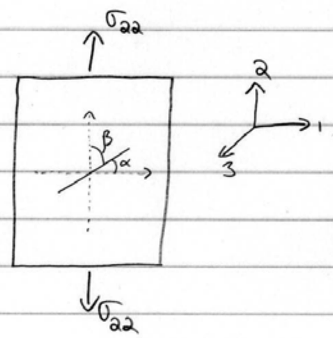
The region of validity of material 1, with a grain size of 10um, is much larger and continuum assumptions will hold.

(b) For elastic-plastic fracture toughness testing, the depth of the initial crack must be at least one-half the width of the specimen to ensure a large region of J-dominance. As discussed in class, pure tension with no eccentricity of loading leads to a very small region of J-dominance. With increasing amounts of eccentricity (and therefore bending), the region of J-dominance increases in size. Computational results to support this were given in the notes.

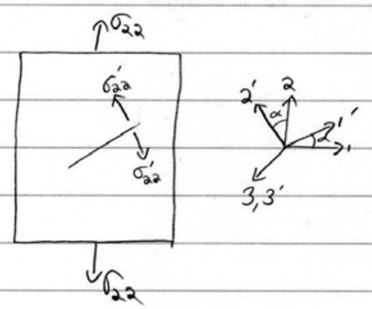


(P)(e) gives a measure of the bending moment.

9.9



Rotate the axes about the '3' direction by an angle $\alpha = 90 - \beta$



Transforming the stress tensor, using the direction cosine matrix, l_{ij}:

$$\sigma_{kl}' = l_{ik}\, l_{jl}\, \sigma_{ij} \quad \text{, where } l_{ij} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sigma_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

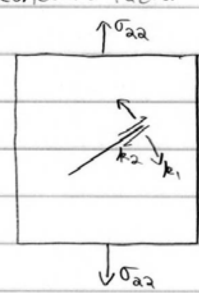
$$\sigma_{22}' = l_{22}\, l_{22}\, \sigma_{22} = (\cos^2\alpha)\sigma$$
$$= (\sin^2\beta)\sigma$$

$$\sigma_{12}' = l_{21}\, l_{22}\, \sigma_{22} = (\sin\alpha)(\cos\alpha)\sigma$$
$$= (\cos\beta)(\sin\beta)\sigma$$

Globally, $K_I = Y\sigma\sqrt{\pi a}$, where Y is a geometric factor

Locally, $k_1 = Y(\sin^2\beta)\sigma\sqrt{\pi a}$

$k_2 = Y(\sin\beta)(\cos\beta)\sigma\sqrt{\pi a}$



$$\boxed{\frac{k_1}{K_I} = \sin^2\beta \quad , \quad \frac{k_2}{K_I} = \sin\beta\cos\beta}$$

11.5 Fracture surface asperity height = 0.75 mm

We can assume linear elastic conditions for the ceramic:

$$J = \frac{K_I^2}{E}(1-\nu^2) \rightarrow \text{assume } \nu \approx 0.25$$

We also know that $\delta_t = d_n \frac{J}{\sigma_o}$

$d_n \approx 0.3$ since $n \approx 1$

Assume σ_o = the tensile rupture strength.

For a superimposed Mode I load, the maximum CTOD occurs at fracture, where $J \rightarrow J_{Ic}$:

$$J_{Ic} = \frac{K_{Ic}^2}{E}(1-\nu^2)$$

At fracture, $\delta_t = 0.3\left(\frac{(3\,MPa\sqrt{m})^2(1-0.25^2)}{(3.75\times10^5\,MPa)(250\,MPa)}\right)$

$$\delta_t = 2.7\times10^{-8}\,m$$

$$\boxed{\delta_t = 2.7\times10^{-5}\,mm \text{ @ fracture}}$$

↳ much lower than the fracture surface asperity height.

The apparent fracture resistance in mode III will always be higher than that in mode I because the crack faces are not able to separate, which leads to closure locking. In pure mode I loading, the surfaces of the crack will separate and fracture will occur at a lower applied load. This reinforces the fact that mode I loading is most damaging

Problem 5

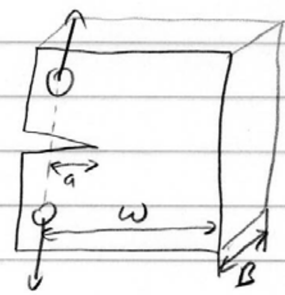
$\delta = 0.2mm$

$\sigma_y = 450 MPa$

$J_{IC} = 85 \times 10^3 J/m^2$

$E = 70 GPa$

$\nu = 0.3$



Valid K_{IC} test:

Plane Strain: $r_p = \frac{1}{3\pi}\left(\frac{K_{IC}}{\sigma_y}\right)^2$

Recall: $J_{IC} = \frac{K_{IC}^2}{E}(1-\nu^2) \rightarrow K_{IC} = \sqrt{\frac{J_{IC} E}{1-\nu^2}} = 80.8 MPa\sqrt{m}$

$r_p = \frac{1}{3\pi}\left(\frac{80.8 MPa\sqrt{m}}{450 MPa}\right)^2 = 3.43mm$

$\boxed{a, (W-a), B \geq 25 r_p = 86mm}$

Valid J_{IC} test:

$\boxed{a, (W-a), B \geq 25\left(\frac{J_{IC}}{\sigma_y}\right) = 4.7mm}$

↳ much less severe requirements