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3.23 Electrical, Optical, and Magnetic Properties of Materials  
Fall 2007

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**3.23 Fall 2007 – Lecture 1**  
**WAVES MECHANICS**



Courtesy of Jon Sullivan, <http://pdphoto.org>

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## The 3.23 Team

- Lectures
- Recitations

Nicola Marzari (Instructor)  
David Paul (I, Magnetic)  
Nicolas Poilvert (TA, Electronic)  
Nicephore Bonnet (TA, Optical/Magn)

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# Roadmap

- Sep 6. From particles to waves: the Schrödinger equation
- Sep 11. The mechanics of quantum mechanics: operators, expectation values
- Sep 13. Measurements and probabilities. The harmonic oscillator.
- Sep 18. The hydrogen atom and the periodic table
- Sep 20. Periodicity and phonons
- Sep 25. Electrons in a lattice: Bloch's theorem
- Sep 27. The nearly-free electron model
- Oct 2. The tight-binding model. Band structures
- Oct 4. Semiconductors and insulators
- Oct 11. Band structure engineering
- Oct 16. Transport of heat and electricity
- Oct 18. Inhomogeneous and hot carriers in semiconductors
- Oct 23. *Mid-term exam (during class, 1:30 hours)*
- Oct 25. The p-n diode

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# Roadmap

- Oct 30. Optical materials and refractive index
  - Nov 1. Electromagnetism in dielectric media
  - Nov 6. Classic propagation of waves
  - Nov 8. Interband absorption
  - Nov 13. Fundamental of ferromagnetic materials
  - Nov 15. Hysteresis loop and driving energies
  - Nov 20. Hard materials and permanent magnets
  - Nov 27. Soft materials: thin films and nanoparticles. Spintronics and GMR
  - Nov 29. Spin valves, spin switches, and spin tunneling
  - Dec 4. Excitons
  - Dec 6. Luminescence
  - Dec 11. Semiconductor quantum wells
- Dec 17 – Dec 21: *Final exam (3 hours, date will be fixed by Schedules' office)*  
**DO NOT BOOK YOUR FLIGHTS YET !**

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## Grading: Exams, Problem Sets

- **30% Problem Sets**
- **30% Mid-term Exam (Oct 23)**
- **40% Final Exam (Final's week – Dec 17-21)**
- **Exams are not “open book”, but you can bring one 2-sided, Letter-sized sheet of mnemonic aids**
- **For the exams, you'll probably need a very basic calculator**

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## Academic Integrity

### **Collaboration Policy for 3.23 - Fall Term 2007**

Before preparing your problem set, you are welcome to discuss it with your fellow students.

Data and figures may not be shared.

**All writing in in a problem set must be original:** do not copy any portion from reference material or the problem sets of other students, previous or current.

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# Textbooks

The class is based on these two **required** textbooks:

**John Singleton**

***Band Theory and Electronic Properties of Solids***

Paperback, Oxford University Press (2001)

ISBN-10: 0198506449, ISBN-13: 978-0198506447

**Mark Fox**

***Optical Properties of Solids***

Paperback, Oxford University Press (2001)

ISBN-10: 0198506120, ISBN-13: 978-0198506126

(Errata can be found at [www.mark-fox.staff.shef.ac.uk/ops\\_errata.html](http://www.mark-fox.staff.shef.ac.uk/ops_errata.html))

These can be found at any academic bookstore. They are also available from Oxford University Press ([www.oup.com](http://www.oup.com)). Last, [www.addall.com](http://www.addall.com) is a very good site to compare prices across

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# Other Textbooks

## Hayden Reserves

- **Stephen Blundell *Magnetism in Condensed Matter*, Oxford University Press**
- **Ashcroft and Mermin *Solid-state physics***
- **Charles Kittel *Introduction to solid-state physics* (Wiley)**

## Other

- **Bransden & Joachain *Quantum Mechanics (2<sup>nd</sup> ed)*, Prentice Hall (2000)**
- **Bransden & Joachain *Physics of Atoms and Molecules (2<sup>nd</sup> ed)***

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## Life at MIT (@ Prof Fink)

- Your experience should be wonderful and enjoyable (when averaged appropriately 😊)
- Finding an advisor (junior vs. senior, work style, group members, resources...)
- You can change the world ! (It might require some work)
- Are you stuck ? Unhappy ? Making progress ? Is it only you ?
- What if things don't work out initially ? (what are your options)
- Have a life (friends, home, gym, travel, music, museums...)

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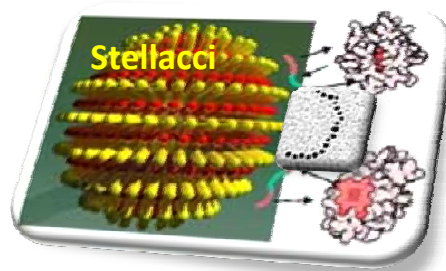
## Materials Breakthroughs (so 20<sup>th</sup> century...)

- Steel and cement - building and engines
- Aluminum alloys - air transportation
- Polymers - safe packaging, medical materials
- Silicon - computing
- Cobalt alloys - data storage
- Silica fibers - communications
- Transition-metal alloys – catalytic converters

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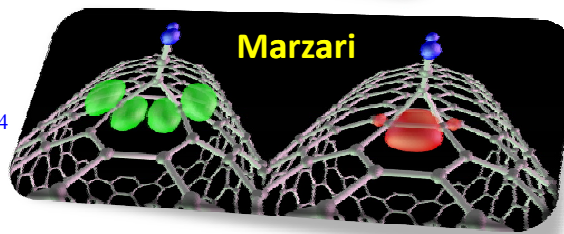
# Advanced Materials

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Please see <http://mit-pbg.mit.edu/img/NatureFiberWeb.jpg>.



Courtesy Francesco Stellacci.  
Used with permission.

Image removed due to copyright restrictions. Please see any image of the microstructure of nacre, such as <http://www.cas.org/ASSETS/E332CE654DC544398C837B46C102CA9D/abalone%20-%2020200.jpg>.



Courtesy Nicola Marzari and Young-Su Lee.  
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# Physical Origin of Material Properties



Courtesy flickr user [dymero](#).

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U. Landman @ Georgia Tech

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# From Classical to Quantum

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## Round Up the Usual Suspects

- Particles and electromagnetic fields
- Forces
- Dynamics

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# Particles and Fields

- Electrons
- Nuclei (protons, neutrons)

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Please see [http://www.cpepweb.org/images/chart\\_details/Structure.jpg](http://www.cpepweb.org/images/chart_details/Structure.jpg).

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# Particles and Fields

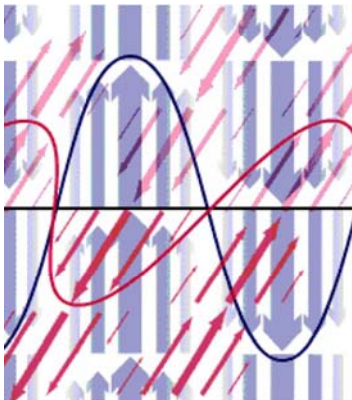


Image courtesy NASA.

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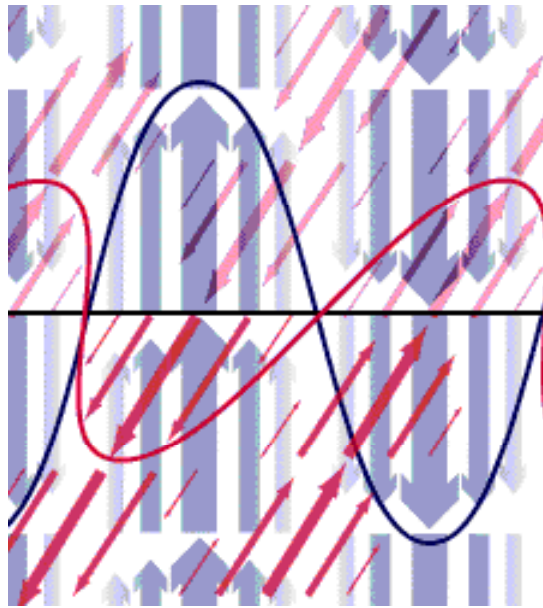


Image courtesy NASA.

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## Forces

- Electromagnetic interactions
- (Gravity, electroweak, strong)

## Dynamics of a Particle

$$m \frac{d^2 \vec{r}}{dt^2} = F(\vec{r}) \quad \longrightarrow \quad \begin{array}{l} \vec{r}(t) \\ \vec{v}(t) \end{array}$$

The sum of the kinetic and potential energy ( $E=T+V$ ) is conserved



Image from the Open Clip Art Library, <http://openclipart.org>

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# Electromagnetic Waves / Photons

$$E = h\nu = h \frac{c}{\lambda} = kT$$

$h$  is Planck's constant =  $6.626 \cdot 10^{-34}$  J s

$k$  is Boltzmann's constant =  $1.381 \cdot 10^{-23}$  J/K

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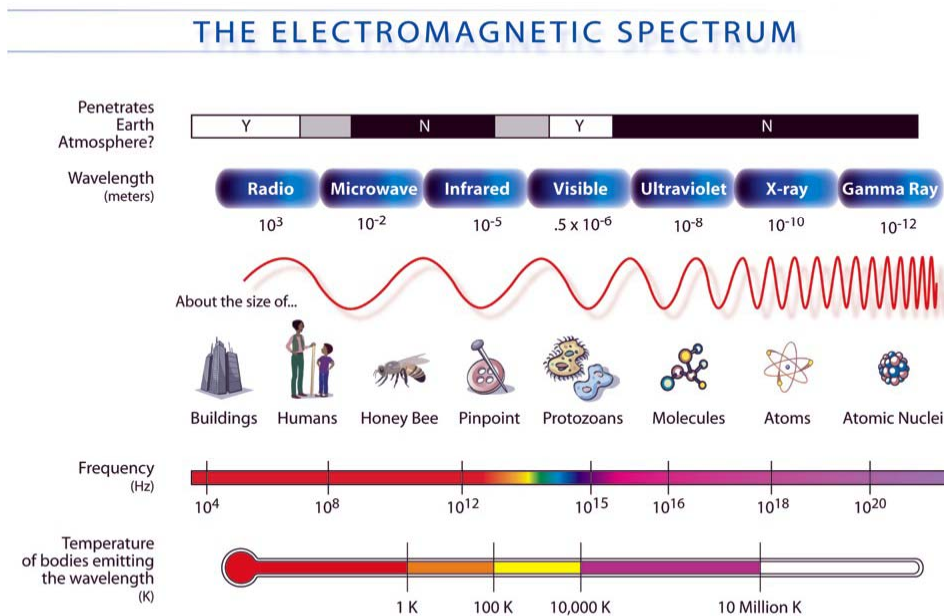


Image courtesy NASA.

Examples: <http://imagers.gsfc.nasa.gov/ems/ems.html>

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## Standard Model of Matter

- Atoms are made by **massive, point-like nuclei (protons+neutrons)**
- Surrounded by tightly bound, rigid shells of **core electrons**
- Bound together by a glue of **valence electrons** (gas vs. atomic orbitals)

Image removed due to copyright restrictions. Please see <http://static.howstuffworks.com/gif/atom-quantum.jpg>

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## Material Properties From First-Principles

- Energy at our living conditions (300 K): **0.04 eV** (kinetic energy of an atom in an ideal gas).
- Differences in bonding energies are within one order of magnitude of **0.29 eV** (hydrogen bond).
- Binding energy of an electron to a proton (hydrogen): **13.6058 eV = 0.5 atomic units (a.u)**
- Everything, from the muscles in our hands to the minerals in our bones is made of atomic nuclei and core electrons bonded together by valence electrons (**standard model** of matter)

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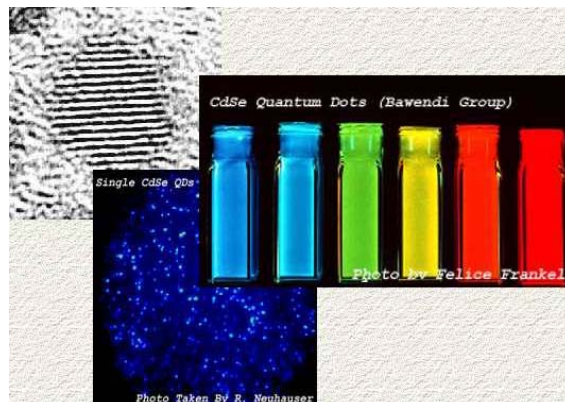
# Why do we need quantum mechanics ?

## Structural properties (fracture in silicon)

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"Directional Anisotropy in the Cleavage Fracture of Silicon." *Physical Review Letters* 84 (June 5, 2000): 5347-5350.

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## Electronic, optical, magnetic properties

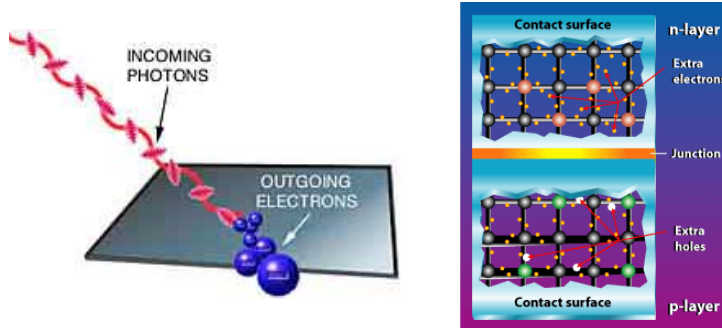


Courtesy of Prof. M. Bawendi and Felice Frankel. Used with permission.

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## Wave-particle Duality

- Waves have particle-like properties:
  - Photoelectric effect: quanta (photons) are exchanged discretely
  - Energy spectrum of an incandescent body looks like a gas of very hot particles



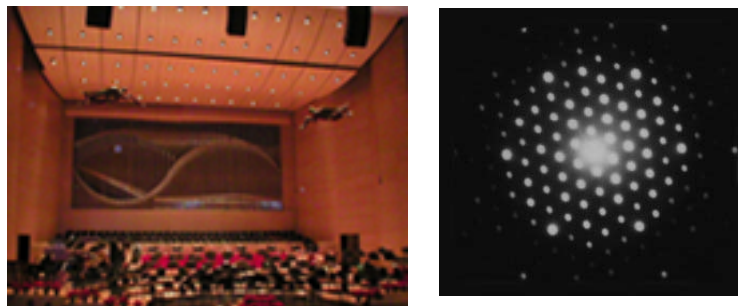
Courtesy Physics 2000, <http://www.colorado.edu/physics/2000/cover.html>.  
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Image courtesy US Dept. of Energy.

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## Wave-particle Duality

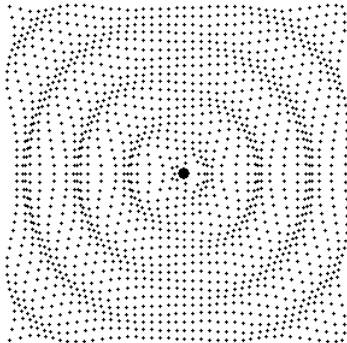
- Particles have wave-like properties:
  - Quantum mechanics: Electrons in atoms are standing waves – just like the harmonics of an organ pipe
  - Electrons beams can be diffracted, and we can see the fringes (Davisson and Germer, at Bell Labs in 1926...)



Courtesy of flickr user [holisticgeek](#).

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## Description of a Wave



The wave is an excitation (a vibration): We need to know the amplitude of the excitation at every point and at every instant

$$\Psi = \Psi(\vec{r}, t)$$

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# Principle of linear superposition

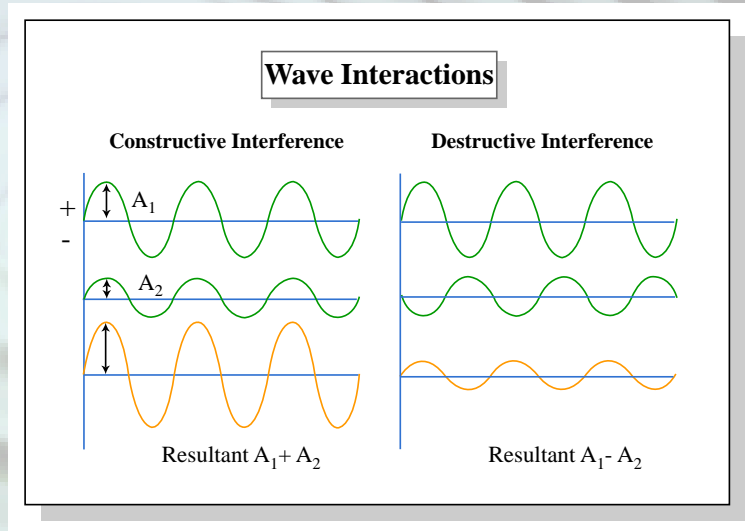


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## Interference in Action

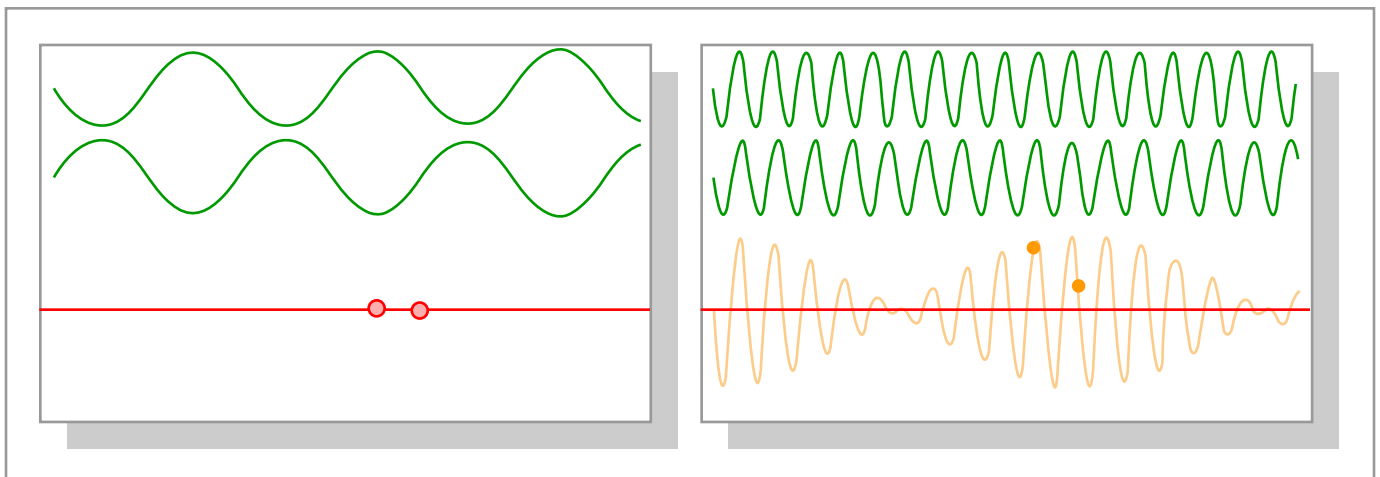


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# Interference in Action

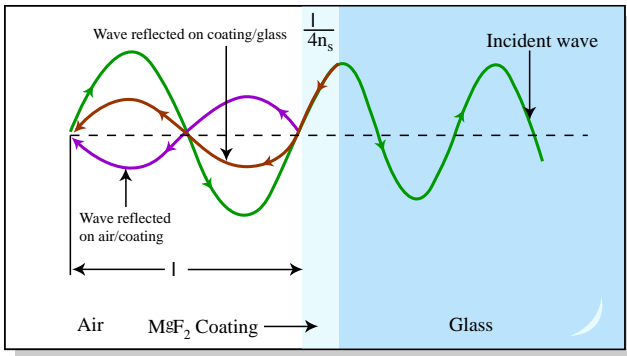


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# Harmonic Oscillator (I)

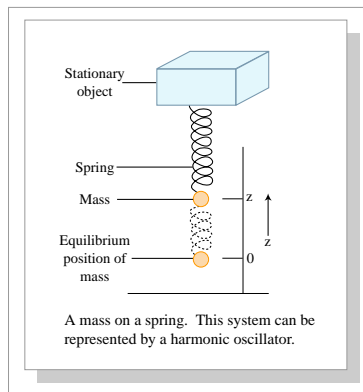


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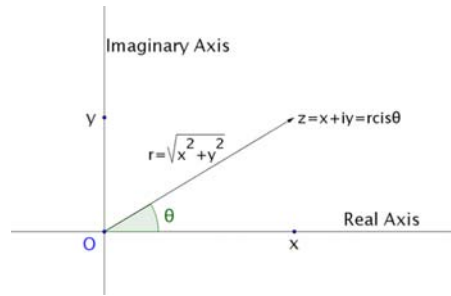
$$ma = F$$

$$m \frac{d^2 z(t)}{dt^2} = - \frac{d}{dz} V(z)$$

$$V(z) = \frac{1}{2} k z^2$$

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## Harmonic Oscillator (II)



$$\frac{d^2 z}{dt^2} = -\frac{k}{m} z$$

$$e^{i\alpha z}, e^{\alpha z}$$

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## Harmonic Oscillator (III)

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Please see any graph of harmonic oscillator position and velocity, such as <http://commons.wikimedia.org/wiki/File:HarmOsc2.png>.

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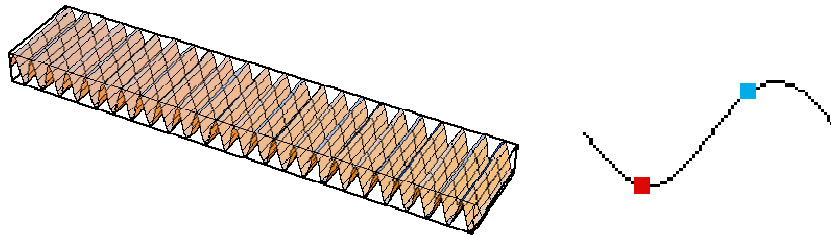
## The total energy of the system

- Kinetic energy  $K$
- Potential energy  $V$

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## A Traveling “Plane” Wave

$$\Psi(\vec{r}, t) = A \exp[i(\vec{k} \cdot \vec{r} - \omega t)]$$



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## When is a particle like a wave ?

Wavelength • momentum = Planck



Image of a double-slit experiment simulation removed due to copyright restrictions.  
Please see "[Double Slit Experiment](#)." in *Visual Quantum Mechanics*.

$$\lambda \cdot p = h$$

$$(h = 6.626 \times 10^{-34} \text{ J s} = 2\pi \text{ a.u.})$$

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## Time-dependent Schrödinger's equation (Newton's 2<sup>nd</sup> law for quantum objects)

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(\vec{r}, t) + V(\vec{r}, t) \Psi(\vec{r}, t) = i\hbar \frac{\partial \Psi(\vec{r}, t)}{\partial t}$$

1925-onwards: E. Schrödinger (wave equation), W. Heisenberg (matrix formulation), P.A.M. Dirac (relativistic)

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## Plane waves as free particles

Our free particle  $\Psi(\vec{r}, t) = A \exp[i(\vec{k} \cdot \vec{r} - \omega t)]$  satisfies the wave equation:

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(\vec{r}, t) = i\hbar \frac{\partial \Psi(\vec{r}, t)}{\partial t} \quad (\text{provided } E = \hbar\omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m})$$