3.225 Electronic and Mechanical Properties of Materials
Test 1: Mechanical Properties
October 21, 2004

1. (a) Bone is an orthotropic material, with the following elastic constants:

$E_1 = 12.8$ GPa
$E_2 = 15.6$ GPa
$E_3 = 20.1$ GPa
$G_{12} = 4.7$ GPa
$G_{13} = 5.7$ GPa
$G_{23} = 6.7$ GPa
$\nu_{12} = \nu_{13} = \nu_{23} = 0.28$

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Calculate the elastic strain resulting from an applied stress field:

$$\sigma = \begin{bmatrix} 10 & 0 & 3 \\ 0 & 5 & 4 \\ 3 & 4 & 20 \end{bmatrix} MPa.$$

(b) The Young's modulus of a material is related to changes in the internal energy per atom, U_a, and the entropy per atom, S_a, with strain, ε, according to:

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$$E = \frac{1}{\Omega}\left(\frac{d^2U_a}{d\varepsilon^2} - T\frac{d^2S_a}{d\varepsilon^2}\right)$$

where Ω is the atomic volume. Explain why the Young's modulus of crystalline materials is controlled by the first term, while that for rubbers is controlled by the second term. Derive an expression for the Young's modulus of crystalline materials in terms of the melting temperature and the atomic volume.

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(c) Explain the origin of the bounds on Poisson's ratio for an isotropic material: $-1 < \nu < 0.5$. Why to rubbers have values of Poisson's ratio approaching 0.5?

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2. (a) An aluminum alloy ($\sigma_y = 240$ MPa) component is subjected to the following stress state:

$$\sigma = \begin{bmatrix} 200 & 40 & 0 \\ 40 & 100 & 50 \\ 0 & 50 & 0 \end{bmatrix} \text{ MPa}$$

Does the component yield according to the von Mises criterion?

(b) Why are covalently bonded materials intrinsically hard while metals are intrinsically soft?

(c) Why does the increase in yield strength from precipitation hardening depend on the surface energy of the precipitate while that from dispersion hardening depends on the shear modulus of the primary metal?

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3. (a) What changes occur at the glass transition temperature that lead to viscoelastic behaviour in amorphous polymers?

(b) Describe two mechanisms of diffusion that give rise to diffusional flow creep behaviour in metals.

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1. (a) $E_1 = 12.8$ GPa $\Rightarrow$ $S_{11} = 1/E_1 = 7.81 \times 10^{-11}$ GPa

$E_2 = 15.6$ GPa $\Rightarrow$ $S_{22} = 1/E_2 = 6.41 \times 10^{-11}$ GPa

$E_3 = 20.1$ GPa $\Rightarrow$ $S_{33} = 1/E_3 = 4.98 \times 10^{-11}$ GPa

$G_{12} = 4.7$ GPa $\Rightarrow$ $S_{66} = 1/G_{12} = 2.13 \times 10^{-10}$ GPa

$G_{13} = 5.7$ GPa $\Rightarrow$ $S_{55} = 1/G_{13} = 1.75 \times 10^{-10}$ GPa

$G_{23} = 6.7$ GPa $\Rightarrow$ $S_{44} = 1/G_{23} = 1.49 \times 10^{-10}$ GPa

$\nu_{12} = \nu_{13} = \nu_{23} = 0.28 \Rightarrow \nu_{12} = -\dfrac{\epsilon_2}{\epsilon_1} = -\dfrac{S_{12}\cancel{\sigma_1}}{S_{11}\cancel{\sigma_1}} \Rightarrow S_{12} = \dfrac{-\nu_{12}}{E_1}$

$$S_{12} = -\frac{0.28}{12.8\text{ GPa}} = -2.19 \times 10^{-11}$$

$$S_{13} = \frac{-\nu_{13}}{E_1} = -\frac{0.28}{12.8} = -2.19 \times 10^{-11}$$

$$S_{23} = \frac{-\nu_{23}}{E_2} = \frac{-0.28}{15.6} = -1.79 \times 10^{-11}$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{bmatrix} = \begin{bmatrix} 7.81 & -2.19 & -2.19 & & & \\ -2.19 & 6.41 & -1.79 & & & \\ -2.19 & -1.79 & 4.98 & & & \\ & & & 14.9 & & \\ & & & & 17.5 & \\ & & & & & 21.3 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \\ 20 \\ 4 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.34 \times 10^{-4} \\ -2.57 \times 10^{-4} \\ 6.88 \times 10^{-4} \\ 5.96 \times 10^{-4} \\ 5.25 \times 10^{-4} \\ 0 \end{bmatrix}$$

②

1(a) $\epsilon_1 = 23.4 \times 10^{-5}$ $\epsilon_4 = 2\epsilon_{23} = 5.96 \times 10^{-4}$

$\epsilon_2 = -25.7 \times 10^{-5}$ $\epsilon_5 = 2\epsilon_{13} = 5.25 \times 10^{-4}$

10 $\epsilon_3 = 68.8 \times 10^{-5}$ $\epsilon_6 = 2\epsilon_{12} = 0.$

1(b) $E = \frac{1}{\Omega}\left(\frac{d^2 u_a}{d\epsilon^2} - T\frac{d^2 S_a}{d\epsilon^2}\right)$

crystalline materials - uniaxial def^n atoms pulled apart or pushed together - relative positions don't change much -
⇒ stretch bonds
3 change in entropy small : $d^2S_a/d\epsilon^2 \ll d^2u_a/d\epsilon^2$

rubbers - random C-C chains, occasionally cross-linked by covalent cross-links. Uniaxial def^n aligns chains, making them more ordered (chains can slide over one another
3 without stretching C-C bonds along the length of the chain).

7 $d^2S/d\epsilon \gg d^2u/d\epsilon^2$

crystalline materials $E = f(u_a, \Omega)$

dimensionless group: $\frac{E\Omega}{u_a}$ = constant

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$u_a \propto kT_m \Rightarrow \frac{E\Omega}{kT_m}$ = constant

$$\boxed{E = \frac{CkT_m}{\Omega}}$$

(c) bounds on ν isotropic material

$G = \frac{E}{2(1+\nu)} \Rightarrow \nu > -1$

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$K = \frac{E}{3(1-2\nu)} \Rightarrow \nu < 1/2$

rubbers ⇒ large K, small E

hyd. press. $K \propto \frac{d^2u_a}{d\epsilon^2}$ uniax stress $E \propto -T\frac{d^2S_a}{d\epsilon^2}$

large K, small E ⇒ $\nu \rightarrow \frac{1}{2}$

2(a) $\sigma_y = 240\,\text{MPa}$

$$\sigma = \begin{bmatrix} 200 & 40 & 0 \\ 40 & 100 & 50 \\ 0 & 50 & 0 \end{bmatrix}$$

Van Mises:

$$\sigma_e = \sqrt{\frac{1}{2}\left\{(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2\right\} + 3\sigma_{12}^2 + 3\sigma_{13}^2 + 3\sigma_{23}^2}$$

$$= \sqrt{\frac{1}{2}\left\{100^2 + 100^2 + 200^2\right\} + 3(40)^2 + 3(0)^2 + 3(50)^2}$$

$$= \sqrt{30{,}000 + 4800 + 7500}$$

$$= \sqrt{42{,}300}$$

$= 206\,\text{MPa} < \sigma_y \Rightarrow$ NO YIELD.

(b) coval. bond - intrins. hard

$U_{att} \propto \frac{1}{r^m}$ $m \approx 6 \Rightarrow U_{att}$ falls rapidly with r

$\Rightarrow$ only nearest neighbours contribute to bonding: z, no bonded neighbours per atom is small

$\Rightarrow$ energy per bond $= Q_b = U_a/(z/2) = 2U_a/z$

$\Rightarrow \frac{E\Omega}{U_a} =$ constant ($\doteq 3$)

$\Rightarrow Q_b = \frac{2}{3}\frac{E\Omega}{z}$

$\Rightarrow$ work done by disl. in breaking 1 bond: $\iota(\tau b)(b)(b) \approx \frac{\tau\Omega}{2}$

$\Rightarrow \frac{\tau\Omega}{2} = \frac{2}{3}\frac{E\Omega}{z} \Rightarrow \tau^\circ = \frac{4}{3}\frac{E}{z}$ z small τ° large $\Rightarrow$ intrins hard

2(b) metals - intrinsically soft

$\Rightarrow$ non-localized bonds, free electrons $\Rightarrow z$ v. large

$\tau^\circ = \frac{4}{3}\frac{E}{z}$ $\quad$ τ° small (eg. fcc $\tau^\circ/E = 10^{-6}$)

2(c) precipitation hardening

- dislocation cuts through ppt. - new surface formed - requires surface energy
- energy balance: $(\tau \cancel{b})(L)(\cancel{\lambda}) = \Gamma(\cancel{\lambda})(\cancel{b})(2)$

$$\boxed{\tau^\circ_{ppt} = 2\Gamma/L}$$

dispersion hardening

- dislocation pinned at hard dispersion obstacle
- can't cut through
- disl. line stretches - line tension $= Gb^2/2$ = change in strain energy when disl. line extends unit length
- strain en $\propto G$, shear mod. of primary metal

force balance $\quad 2T = \tau^\circ_d \, b \, L$

$$\cancel{2}\frac{Gb^{\cancel{2}}}{\cancel{2}} = \tau^\circ_d \cancel{b} L$$

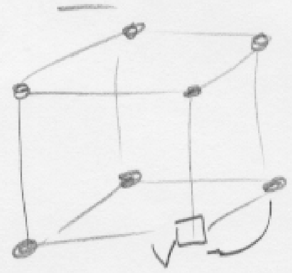
$$\boxed{\tau^\circ_d = \frac{Gb}{L}}$$

3 (a) T_g

- sec. crosslink bonds betn chains melt
- free vol. increases with temp more rapidly
- motion of chains sliding across one another easier.

(b) diffusional flow mechanisms

- vacancy diffusion



- grain boundary diffusion
  - grain boundary is a region of disorder
  - more open structure
  - high conductivity channel for diffusion
  - lower Q than for bulk/vacancy diffusion