I wanted to add a few comments to what Raymundo has mentioned. The
point of this problem is in part to see what the variables are and
what role they play.

If you work in energy representation:

    du = TdS - pdV : V and S are the extensive variables
              T and (-p) are the intensive conjugates

    When Legendre transforming U, you subtract the intensive x
extensive
    couple, which appropriately so, has units of energy (otherwise
    you could not subtract it from U).

    OK, you say, I already knew this. Good.
    Try to do formally the same thing for entropy, but acknowledging
    the different role each variable plays.

Working in entropy representation:

    dS = 1/T dU + (p/T)dV

    The extensive variables in this function are now U and V. Its
    intensive conjugates are (1/T) and (p/T). (Note: I think one
    of the "errors" you make in your derivation is to treat "T" rather
    than "1/T" as the relevant variable.)
    Note that the product: intensive x extensive has units of entropy
now.

    A Legendre transform of a function is: FUNCTION - intensive x
extensive

    Simply apply that to S, thereby one for one replacing extensive
    by intensive.

    e.g. S - (1/T)U will give you a new function with variables (1/T)
and V

In each case, if you substitute what S (or U) is from the Euler
relation, you will find that the Legendre transforms of S (the
Massieu functions) are very much related to Legendre transforms of U.
In many cases differing only by a factor.

    You can do this now substituting other variables away.

I hope this helps. Let me know if it does not.

gerd ceder