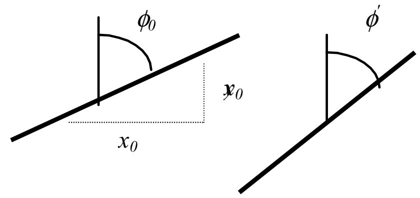
Prob. 2.22

`Line rotation from uniaxial extension:`

Consider a line inclined at an angle p from the vertical, with a slope y_0/x_0. After stretching by an amount $\lambda_y = \lambda$, we have:



$$\lambda_y = \lambda \Rightarrow \lambda_x = \lambda_z = \frac{1}{\sqrt{\lambda}}$$

$$\tan\phi' = \frac{x}{y} = \frac{\lambda_x x_0}{\lambda_y y_0} = \frac{\left(\frac{1}{\sqrt{\lambda}}\right)x_0}{\lambda y_0} = \frac{1}{\lambda^{3/2}}\tan\phi_0$$

Prob. 2.23

`Orientation function:`

Fraction of sigments in range *d(ϕ):*

f(phi):=2*Pi*r^2*sin(phi)/(2*Pi*r^2);

$$f(\phi) := \sin(\phi)$$

Segment orientation after stretching (from previous problem);

phi_2:=arctan((1/lambda^(3/2))*tan(phi));

$$phi\_2 := \arctan\left(\frac{\tan(\phi)}{\lambda^{3/2}}\right)$$

Integrate over sphere to obtain mean square orientation angle:

phi_avg:= simplify(int(cos(phi_2)^2*f(phi),phi=0..Pi/2));

$$phi\_avg := \frac{\left(\sqrt{\lambda^3-1} - \arctan\left(\frac{1}{\sqrt{\lambda^3-1}}\right) - \arctan\left(\frac{1}{2}\frac{\lambda^3-2}{\sqrt{\lambda^3-1}}\right)\right)\lambda^3}{\left(\lambda^3-1\right)^{3/2}}$$

Evaluate for λ=3:

Digits:=4;evalf(subs(lambda=3,phi_avg));

$$Digits := 4$$

$$.7581$$

Denote the Herrman orientation parameter as ff:

ff:= (1/2)*(3*phi_avg-1) ;

$$ff := \frac{3}{2}\,\frac{\left(\sqrt{\lambda^3-1} - \arctan\left(\frac{1}{\sqrt{\lambda^3-1}}\right) - \arctan\left(\frac{1}{2}\,\frac{\lambda^3-2}{\sqrt{\lambda^3-1}}\right)\right)\lambda^3}{\left(\lambda^3-1\right)^{3/2}} - \frac{1}{2}$$

Evaluate for λ=3:

evalf(subs(lambda=3,ff));

$$\boxed{.6370}$$

Plot orientation function versus extension ratio:

plot(ff, lambda=.1..5);