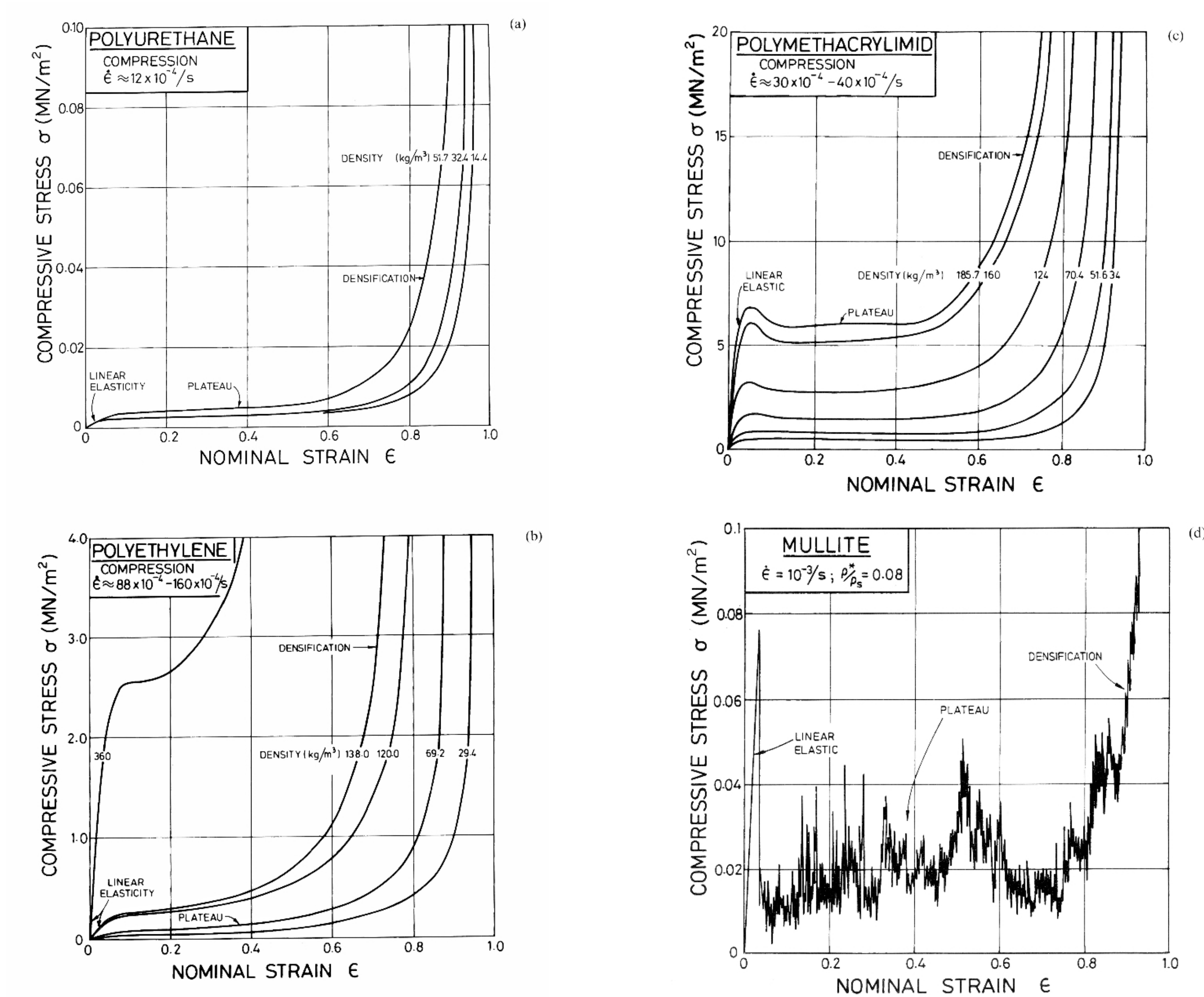
Open-cell foams

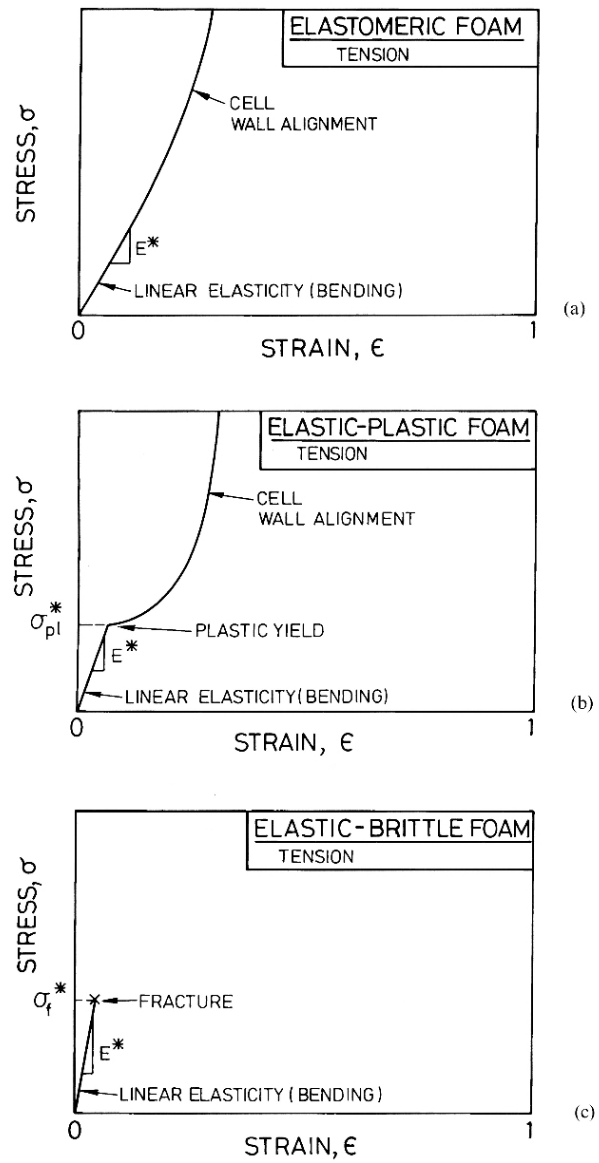
- stress-strain curves: deformation + failure mechanisms
- compression - 3 regimes - linear elastic - bending
  - stress plateau - cell collapse by buckling / yielding / crushing
  - densification
- tension - no buckling
  - yielding can occur
  - brittle fracture

Linear elastic behaviour

- initial linear elasticity - bending of cell edges (small t/l)
- as t/l ↑, axial deformation becomes more significant
- consider dimensional argument, which models mechanism of deformation + failure, but not cell geometry
- consider cubic cell, square cross-section members of area t^2, length, l



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

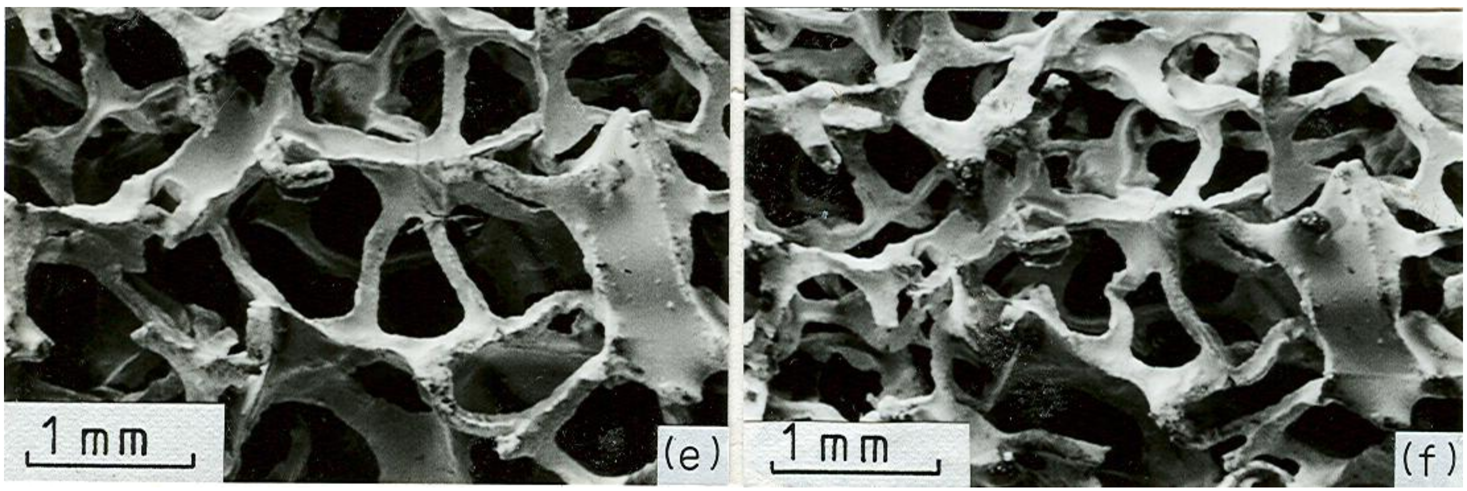


Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Foams: Bending, Buckling

Figure removed due to copyright restrictions. See Fig. 3: Gibson, L. J., and M. F. Ashby. "The Mechancis of Three-Dimensional Cellular Materials." *Proceedings of The Royal Society of London A* 382 (1982): 43-59.

Foams: Plastic Hinges



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Foams: Cell Wall Fracture

- regardless of specific cell geometry chosen:

$$\rho^*/\rho_s \propto (t/l)^2 \qquad I \propto t^4$$

$$\sigma \propto F/l^2 \qquad \epsilon \propto \delta/l \qquad \delta \propto Fl^3/E_s I$$

$$E^* \propto \frac{\sigma}{\epsilon} \propto \frac{F}{l^2}\frac{l}{\delta} \propto \frac{F}{l}\frac{E_s t^4}{F l^3} \propto E_s\left(\frac{t}{l}\right)^4 \propto E_s\left(\frac{\rho^*}{\rho_s}\right)^2$$

$$\boxed{E^* = C_1 E_s (\rho^*/\rho_s)^2}$$

C_1 includes all geometrical constants.

Data: $C_1 \approx 1$

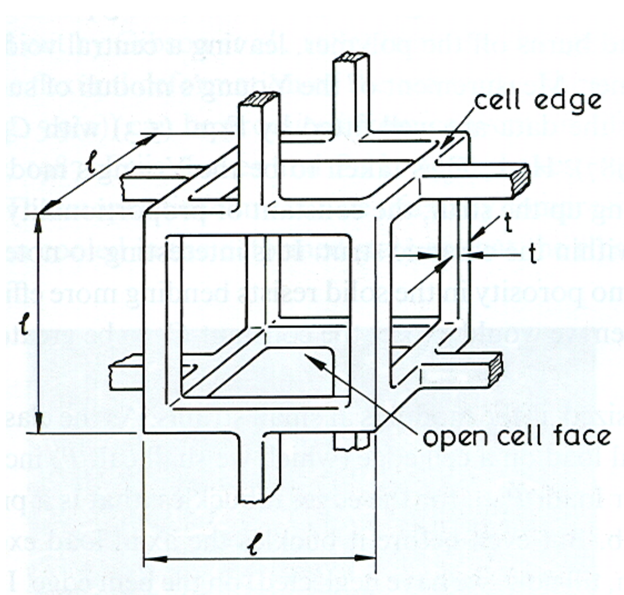
- data suggest $C_1 = 1$
- analysis of open cell tetrakaidecahedral cells with Plateau borders gives $C_1 = 0.98$
- shear modulus $\boxed{G^* = C_2 E_s (\rho^*/\rho_s)^2}$ $\quad C_2 \sim 3/8$ if foam isotropic

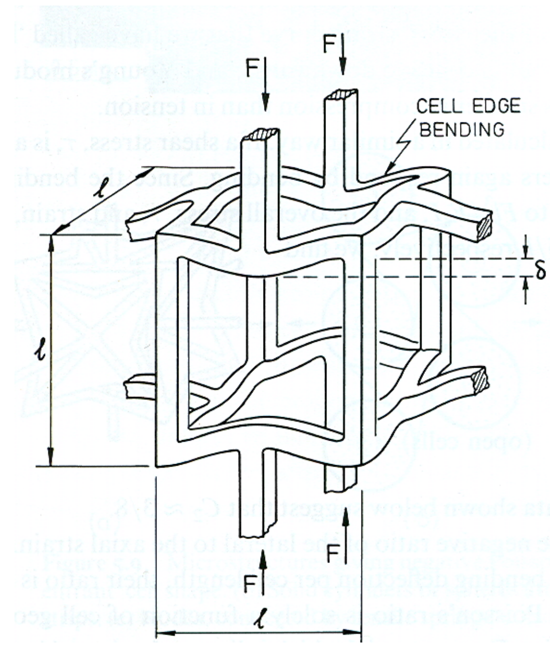
isotropy: $G = \dfrac{E}{2(1+\nu)}$

- Poisson's ratio $\nu^* = \dfrac{E}{2G} - 1 = \dfrac{C_1}{2C_2} - 1$ = constant, independent of E_s, t/l

$\boxed{\nu^* = C_3}$ (analogous to honeycombs in-plane)

Foam: Edge Bending

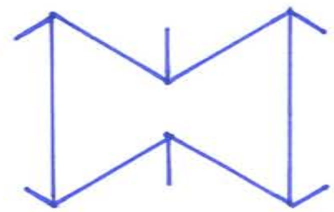




Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Poisson's ratio

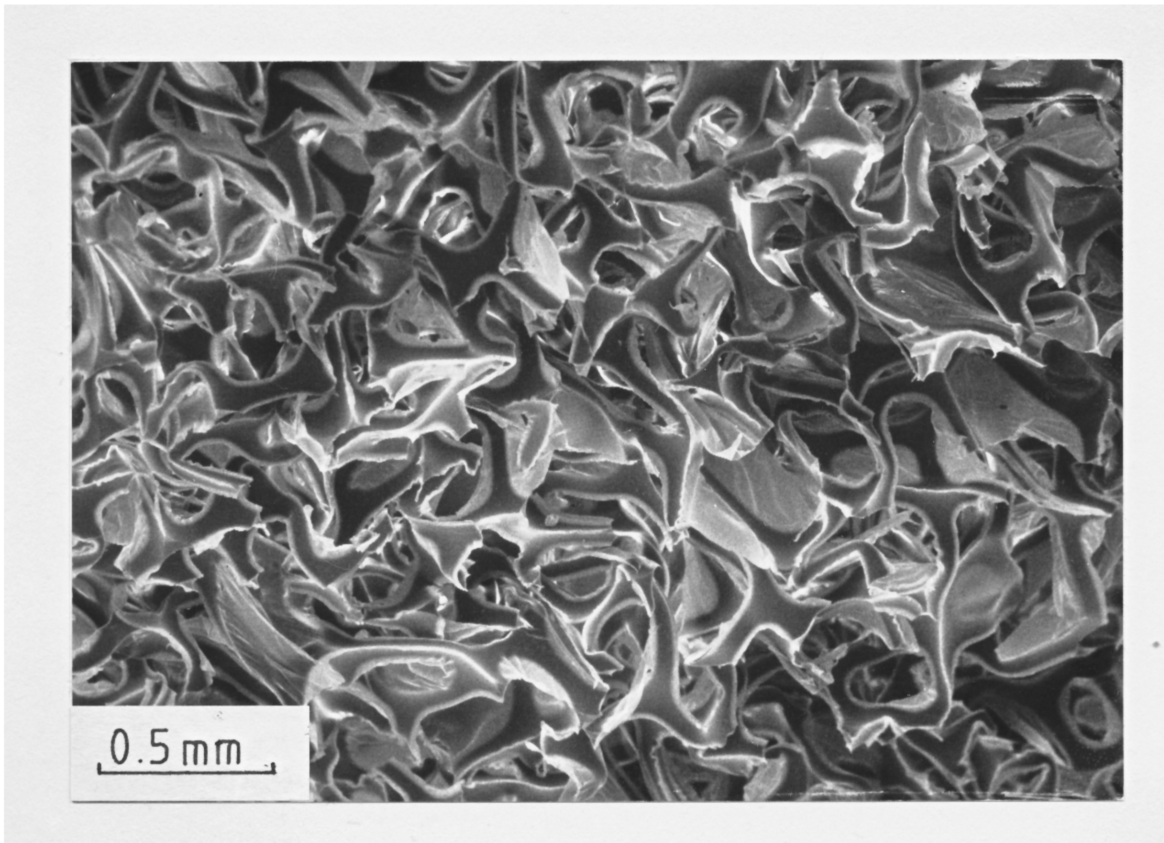
- can make negative Poisson's ratio foams
- invert cell angles (analogous to honeycomb)
- eg. thermoplastic foams - load hydrostatically + heat to $T > T_g$, then cool + release load so that edges of cell permanently point inward.



Closed-cell foams

- edge bending as for open-cell foams
- also: face stretching + gas compression
- polymer foams: surface tension draws material to edges during processing
  - define t_e, t_f in figure
- apply F to the cubic structure

Negative Poisson's Ratio



- external work done $\propto F\delta$
- internal work bending edges $\propto \frac{F_e}{\delta_e}\delta_e^2 \propto \frac{E_s I}{l^3}\delta^2$
- internal work stretching faces $\propto \sigma_f \epsilon_f V_f \propto E_s \epsilon_f^2 V_f \propto E_s (\delta/l)^2 t_f l^2$

$$\therefore \quad F\delta = \alpha \frac{E_s t_e^4}{l^3}\delta^2 + \beta E_s\left(\frac{\delta}{l}\right)^2 t_f l^2$$

$$E^* \propto \frac{F}{l^2}\frac{l}{\delta} \Rightarrow F \propto E^* \delta l$$

$$\therefore \quad E^* \delta^2 l = \alpha \frac{E_s t_e^4}{l^3}\delta^2 + \beta E_s\left(\frac{\delta}{l}\right)^2 t_f l^2$$

$$E^* = \alpha E_s (t_e/l)^4 + \beta E_s (t_f/l)$$

Note: open cells, uniform t : $\rho^*/\rho_s \propto (t/l)^2$

closed cells, uniform t: $\rho^*/\rho_s \propto (t/l)$

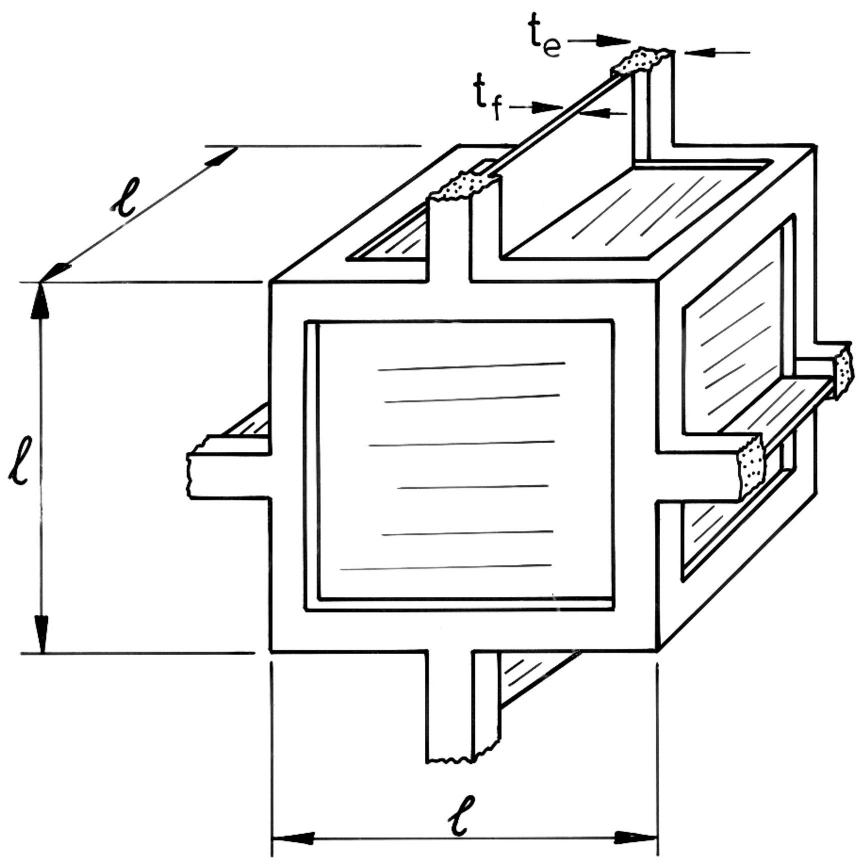
if ϕ = volume fraction of solid in cell edges:

$$t_e/l = C\phi^{1/2}(\rho^*/\rho_s)^{1/2}$$

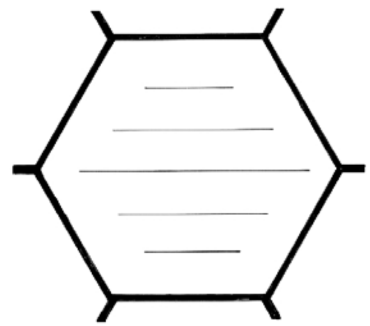
$$t_f/l = C'(1-\phi)(\rho^*/\rho_s)$$

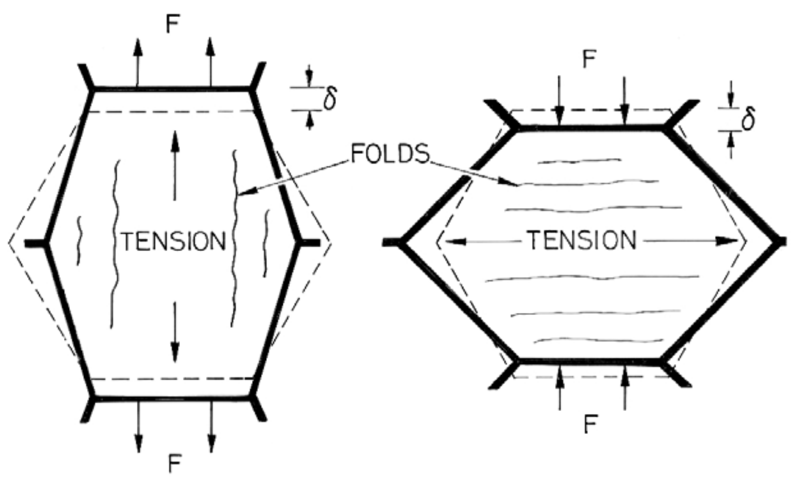
$$\boxed{\frac{E^*}{E_s} = C_1\phi^2\left(\frac{\rho^*}{\rho_s}\right)^2 + C_1'(1-\phi)\frac{\rho^*}{\rho_s}}$$

Closed-Cell Foam



Cell Membrane Stretching





Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Closed cell foams – gas within cell may also contribute to E^*

- cubic element of foam of volume V_o
- under uniaxial stress, axial strain in direction of stress is ϵ
- deformed volume V is:

$$\frac{V}{V_o} = 1 - \epsilon(1-2\nu^*)$$

taking compressive strain as positive
neglecting ϵ^2, ϵ^3 terms

$$\frac{V_g}{V_g^o} = \frac{1-\epsilon(1-2\nu^*) - \rho^*/\rho_s}{1-\rho^*/\rho_s}$$

V_g = volume gas
V_g^o = " " initially

- Boyle's law: $p V_g = p_o V_g^o$

p = pressure after strain ϵ
p_o = pressure initially

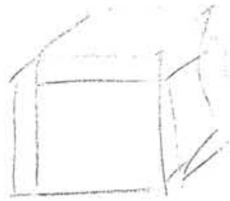
- pressure that must be overcome is $p' = p - p_o$

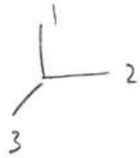
$$p' = \frac{p_o\,\epsilon\,(1-2\nu^*)}{1-\epsilon(1-2\nu^*) - \rho^*/\rho_s}$$

- contribution of gas compression to the modulus, E^*:

$$E_g^* = \frac{dp'}{d\epsilon} = \frac{p_o(1-2\nu^*)}{1-\rho^*/\rho_s}$$

ϵ I





$V_0 = l_0^{\,3}$

$V = l_1 l_2 l_3$

$\epsilon_1 = \dfrac{l_1 - l_0}{l_0} \Rightarrow l_1 = l_0 + \epsilon_1 l_0 = l_0(1+\epsilon_1)$

$\epsilon_2 = \dfrac{l_2 - l_0}{l_0} \Rightarrow l_2 = l_0 + \epsilon_2 l_0$

$= l_0 - \nu\epsilon_1 l_0$

$= l_0 (1 - \nu\epsilon_1)$

$\nu = -\dfrac{\epsilon_2}{\epsilon_1}$

$\epsilon_2 = -\nu\epsilon_1$

$\epsilon_3 = l_0 (1 - \nu\epsilon_1)$.

$V = l_1 l_2 l_3 = l_0 (1+\epsilon_1) l_0 (1 - \nu\epsilon_1) l_0 (1 - \nu\epsilon) = l_0^{\,3} (1+\epsilon_1)(1-\nu\epsilon)^2$

$\dfrac{V}{V_0} = \dfrac{l_0^3 (1+\epsilon)(1-\nu\epsilon)^2}{l_0^3} = (1+\epsilon)(1 - 2\nu\epsilon + \nu^2\epsilon^2)$

$= (1 - 2\nu\epsilon + \nu^2\epsilon^2) + \epsilon - 2\nu\epsilon^2 + \nu^2\epsilon^3$

$= 1 - \epsilon + 2\nu\epsilon$ (taking comp. as +)

$= 1 - \epsilon(1 - 2\nu)$

$$p' = p - p_0$$

$$p = \frac{p_0 V_g^0}{V_g}$$

$$p' = p - p_0 = \frac{p_0 V_g^0}{V_g} - p_0 = p_0\left(\frac{V_g^0}{V_g} - 1\right) \quad \cancel{= p_0\left(\frac{V_g^0 - V_g}{V_g}\right)}$$

$$= p_0\left[\frac{1 - \rho/\rho_s}{1 - \epsilon(1-2\nu^*) - \rho^*/\rho_s} - 1\right]$$

$$= p_0\left[\frac{\cancel{1} - \cancel{\rho/\rho_s} - \left(\cancel{1} - \epsilon(1-2\nu^*) - \cancel{\rho^*/\rho_s}\right)}{1 - \epsilon(1-2\nu^*) - \rho^*/\rho_s}\right]$$

$$= p_0\left[\frac{\epsilon(1-2\nu^*)}{1 - \epsilon(1-2\nu^*) - \rho^*/\rho_s}\right] \checkmark$$

Closed cell foam

$$\frac{E^*}{E_s} = \phi^2\left(\frac{\rho^*}{\rho_s}\right)^2 + (1-\phi)\left(\frac{\rho^*}{\rho_s}\right) + \frac{p_o\,(1-2\nu^*)}{E_s\,(1-\rho^*/\rho_s)}$$

↓ edge bending ↓ face stretching ↓ gas compression

- note: if $p_o = p_{atm} = 0.1\,MPa$, gas compression term negligible, except for closed-cell elastomeric foams
- gas comp. can be significant if $p_o \gg p_{atm}$; also modifies shape of stress plateau in elastomeric closed cell foams

---

Shear modulus: edge bending, face stretching; shear $\Delta V = 0$ gas contrib. $= 0$

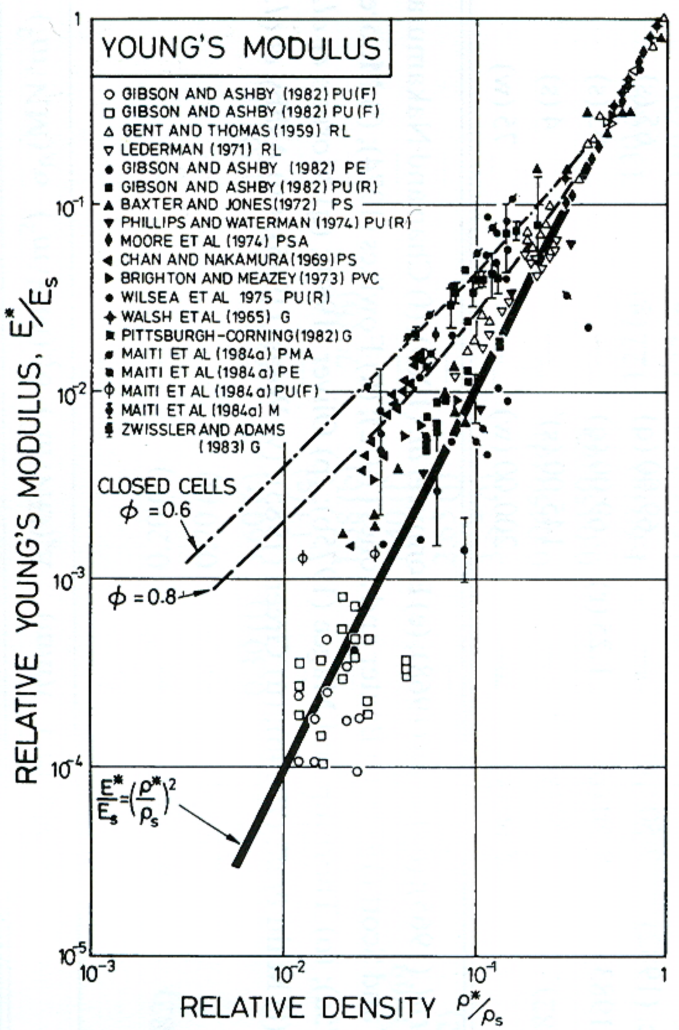
$$\frac{G^*}{E_s} = \frac{3}{8}\left[\phi^2\left(\frac{\rho^*}{\rho_s}\right)^2 + (1-\phi)\left(\frac{\rho^*}{\rho_s}\right)\right] \quad \text{(isotropic foam)}$$

Poisson's ratio = f (cell geometry only) $\nu^* \approx 1/3$

Comparison with data

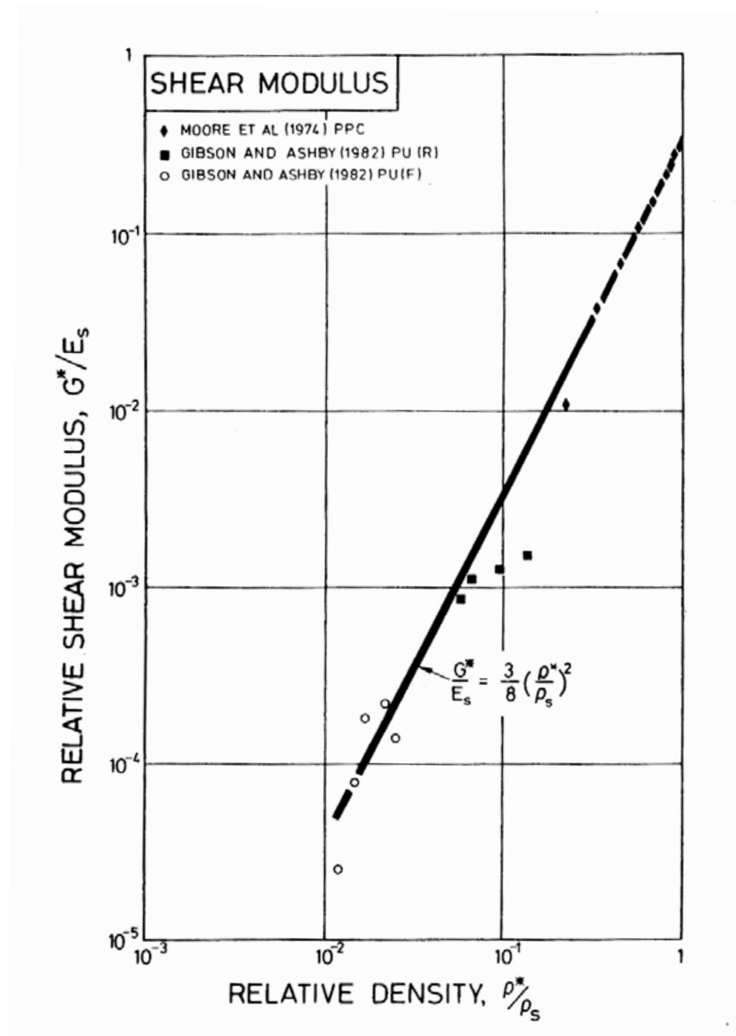
- data for polymers, glasses, elastomers
- E_s, ρ_s Table 5.1 in book
- open cells - open symbols
- closed cells - filled symbols

Young's Modulus



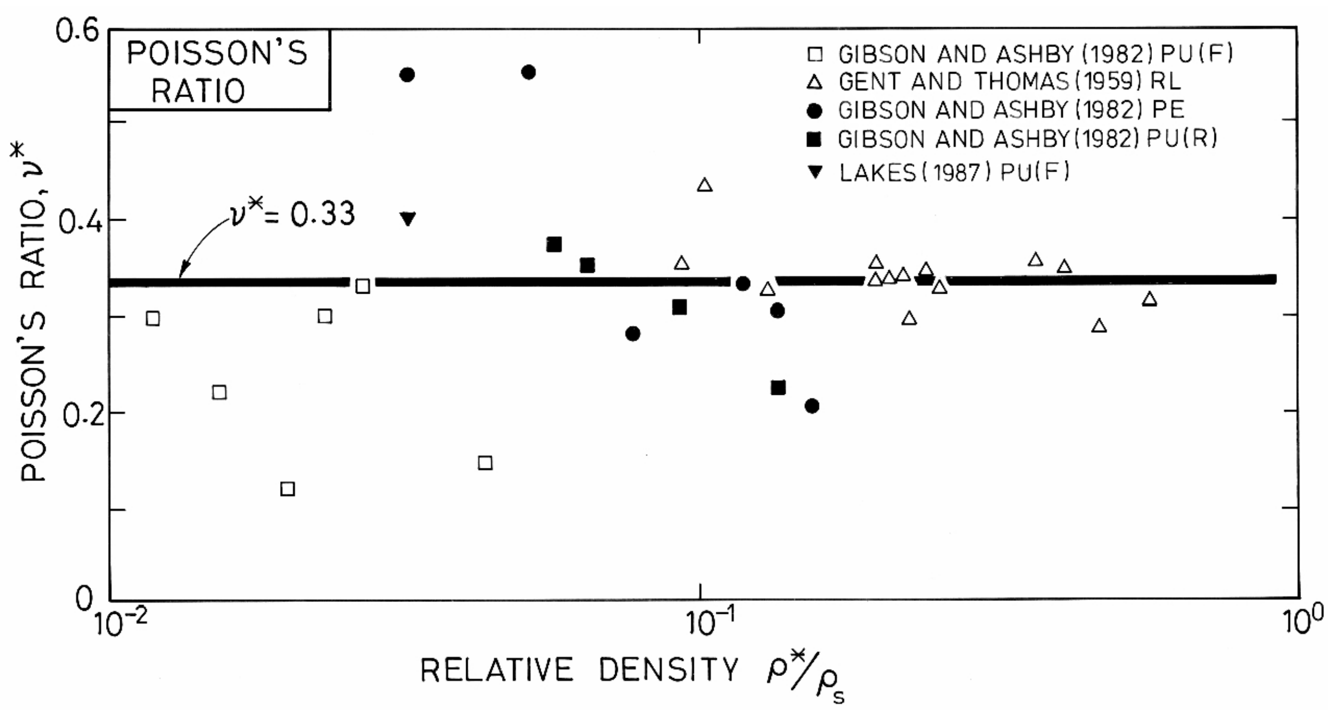
Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Shear Modulus



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Poisson's Ratio



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Non-linear elasticity

Open cells:

$$P_{cr} = \frac{n^2 \pi^2 E_s I}{l^2}$$

$$\sigma^*_{el} \propto \frac{P_{cr}}{l^2} \propto E_s \left(\frac{t}{l}\right)^4$$

$$\boxed{\sigma^*_{el} = C_4 E_s (\rho^*/\rho_s)^2}$$

Data: $C_4 \approx 0.05$, corresponds to strain when buckling initiates, since $E^* = E_s (\rho^*/\rho_s)^2$

Closed cells

- t_f often small compared to t_e (surface tension in processing) - contrib. small
- if $p_o \gg p_{atm}$, cell walls pretensioned; buckling stress has to overcome this

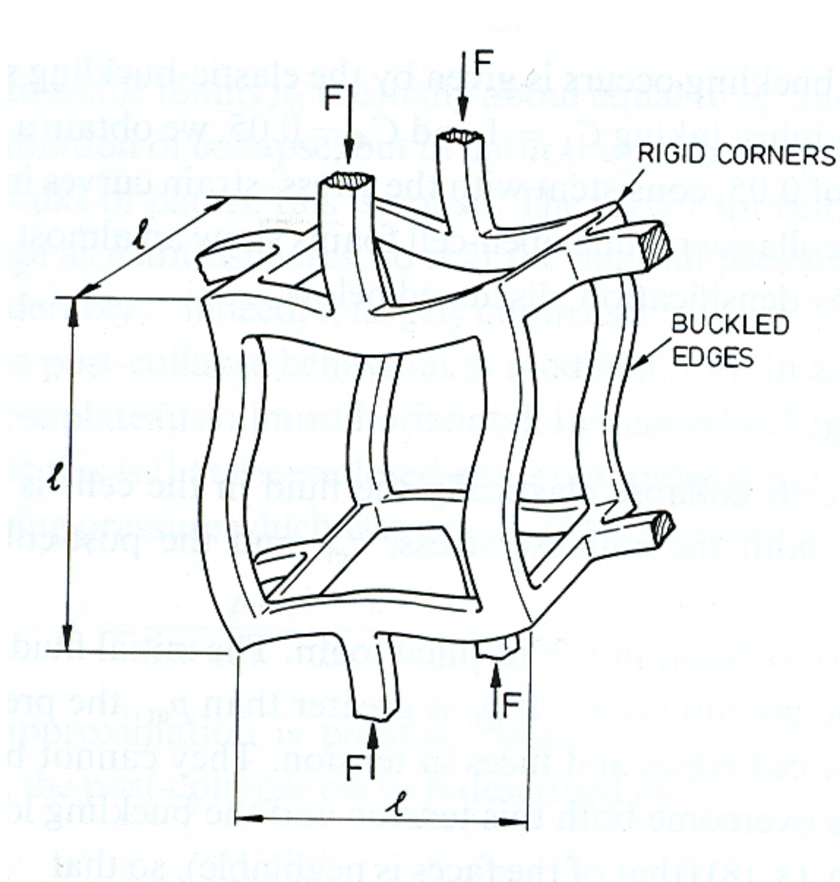
$$\sigma^*_{el} = 0.05 E_s (\rho^*/\rho_s)^2 + p_o - p_{atm}$$

- post-collapse behaviour - stress plateau rises due to gas compression (if faces don't rupture) $\nu^* = 0$ in post-collapse regime

$$p' = \frac{p_o \epsilon (1-2\nu^*)}{1 - \epsilon(1-2\nu^*) - \rho^*/\rho_s} = \frac{p_o \epsilon}{1 - \epsilon - \rho^*/\rho_s}$$

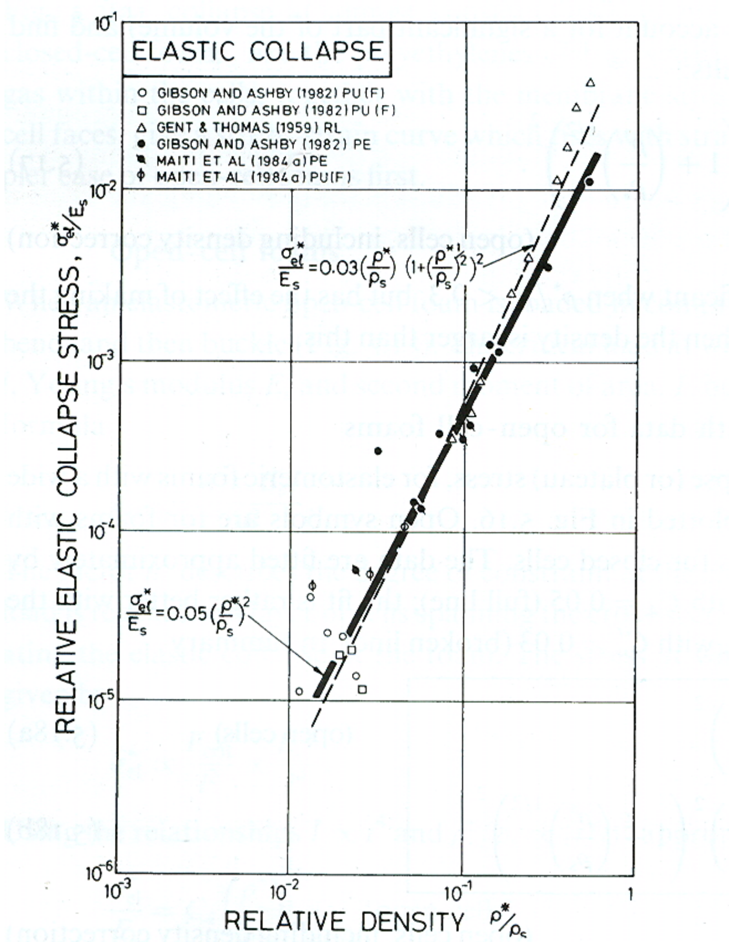
$$\boxed{\sigma^*_{post\text{-}collapse} = 0.05 E_s \left(\frac{\rho^*}{\rho_s}\right)^2 + \frac{p_o \epsilon}{1 - \epsilon - \rho^*/\rho_s}}$$

Elastic Collapse Stress



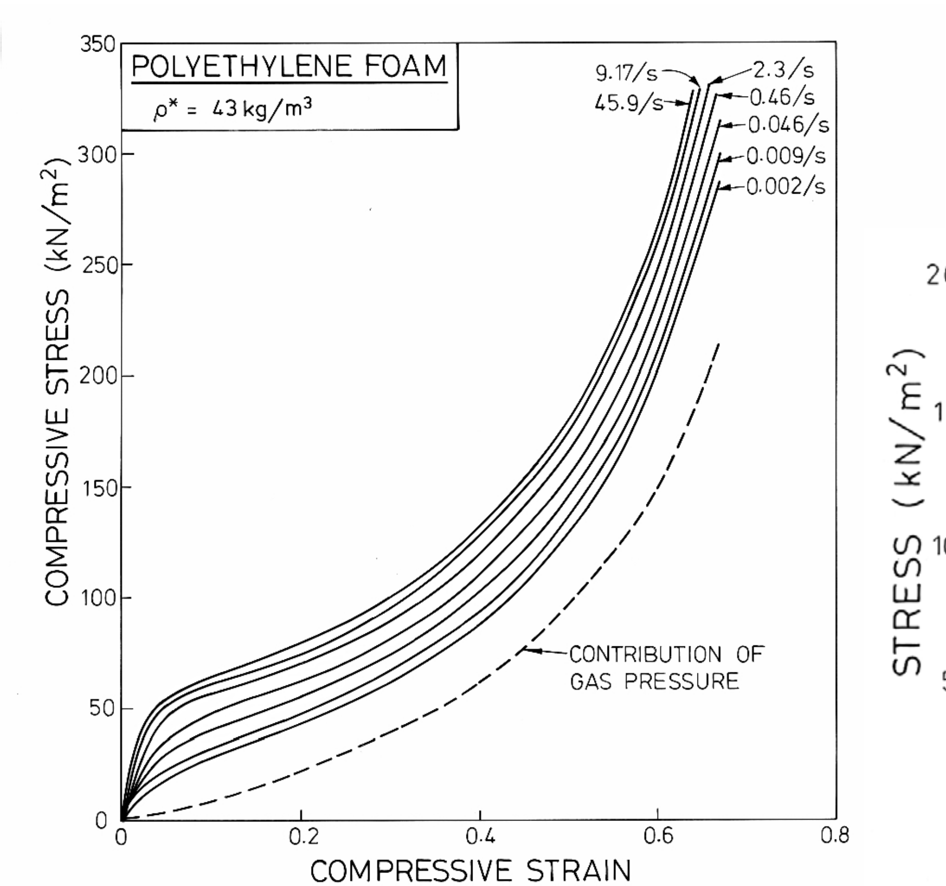
Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

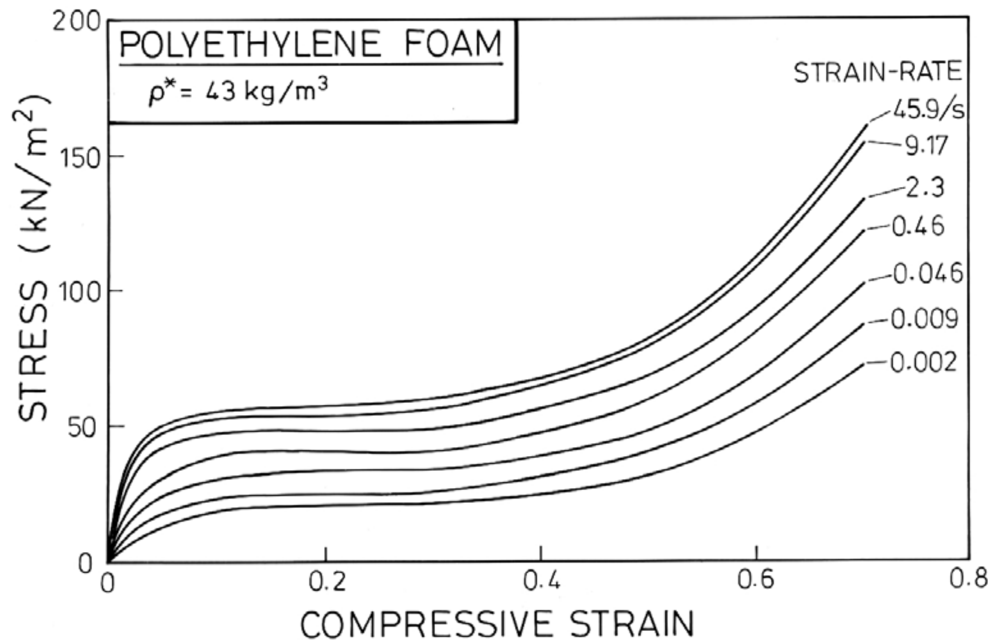
Elastic Collapse Stress



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Post-collapse stress strain curve





Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Plastic collapse

Open cells

- failure when $M = M_p$
- $M_p \propto \sigma_{ys} t^3$ $\quad M \propto \sigma^*_{pl} l^3$

$$\boxed{\sigma^*_{pl} = C_5 \, \sigma_{ys} \, (\rho^*/\rho_s)^{3/2}}$$

$C_5 \sim 0.3$, from data.

- elastic collapse precedes plastic collapse if $\sigma^*_{el} < \sigma^*_{pl}$

- $0.05 \, E_s (\rho^*/\rho_s)^2 \leq 0.3 \, \sigma_{ys} (\rho^*/\rho_s)^{3/2}$

$$(\rho/\rho_s)_{critical} \leq 36 \, (\sigma_{ys}/E_s)^2$$

rigid polymers $(\rho^*/\rho_s)_{cr} < 0.04$ $\left(\frac{\sigma_{ys}}{E_s} \sim \frac{1}{30}\right)$

metals $(\rho^*/\rho_s)_{cr} < 10^{-5}$ $\left(\frac{\sigma_{ys}}{E_s} \sim \frac{1}{1000}\right)$

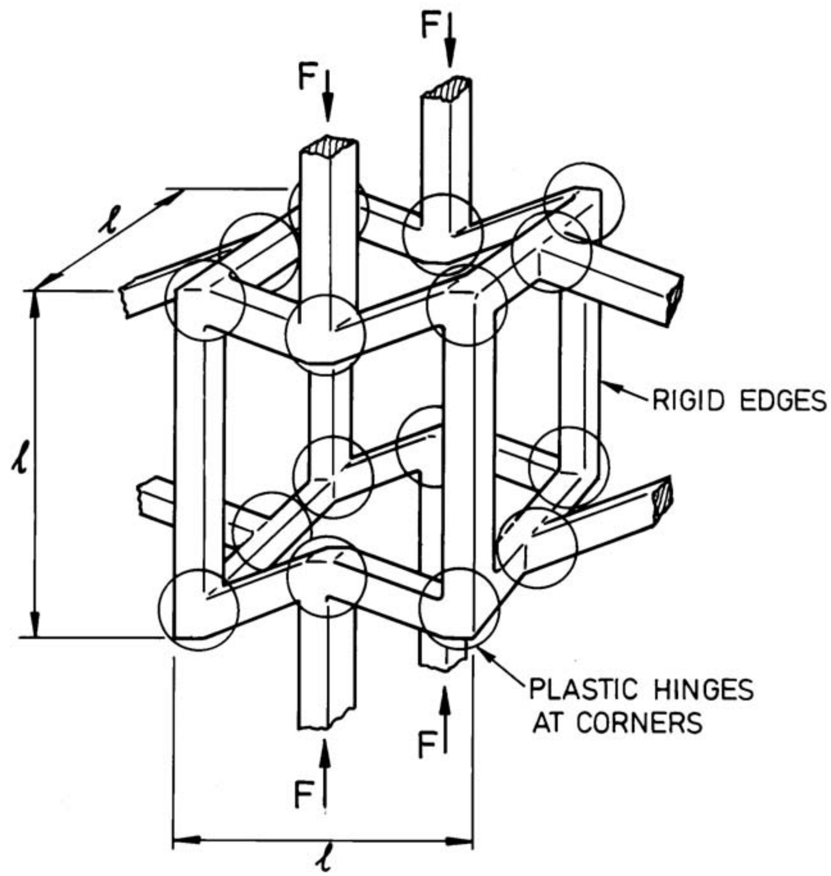
Closed cells

- including all terms $\sigma^*_{pl} = C_5 \, \sigma_{ys} (\phi \, \rho^*/\rho_s)^{3/2} + C_5' \, \sigma_{ys} (1-\phi)(\rho^*/\rho_s) + p_i - p_{atm}$

  ↑ edge bending; ↑ face stretching; ↑ gas

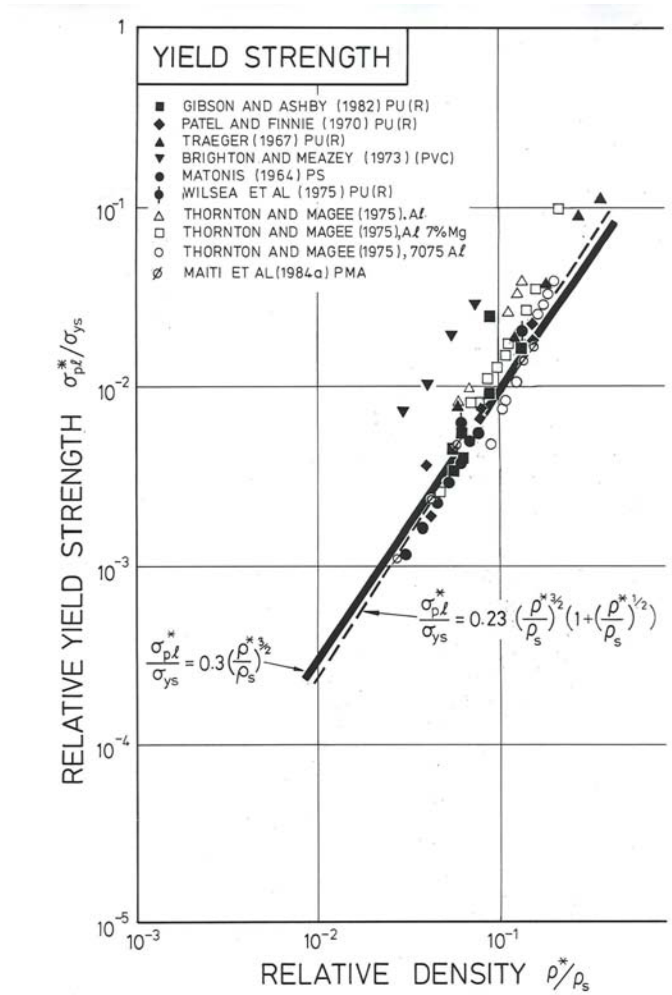
- but in practice, faces often rupture around σ^*_{pl} - often $\sigma^*_{pl} = 0.3 (\rho^*/\rho_s)^{3/2} \sigma_{ys}$

Plastic Collapse Stress



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Plastic Collapse Stress



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Brittle crushing strength

Open cells

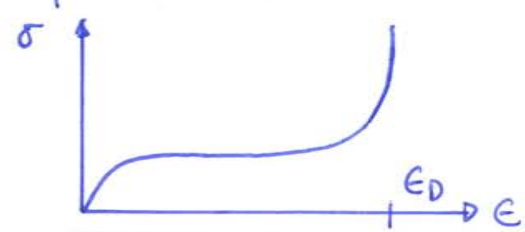
- failure when $M = M_f$ $M \propto \sigma^*_{cr}\, l^3$ $M_f \propto \sigma_{fs}\, t^3$

$$\boxed{\sigma^*_{cr} = C_6\, \sigma_{fs} \left(\rho^*/\rho_s\right)^{3/2}}$$ $C_6 \approx 0.2$

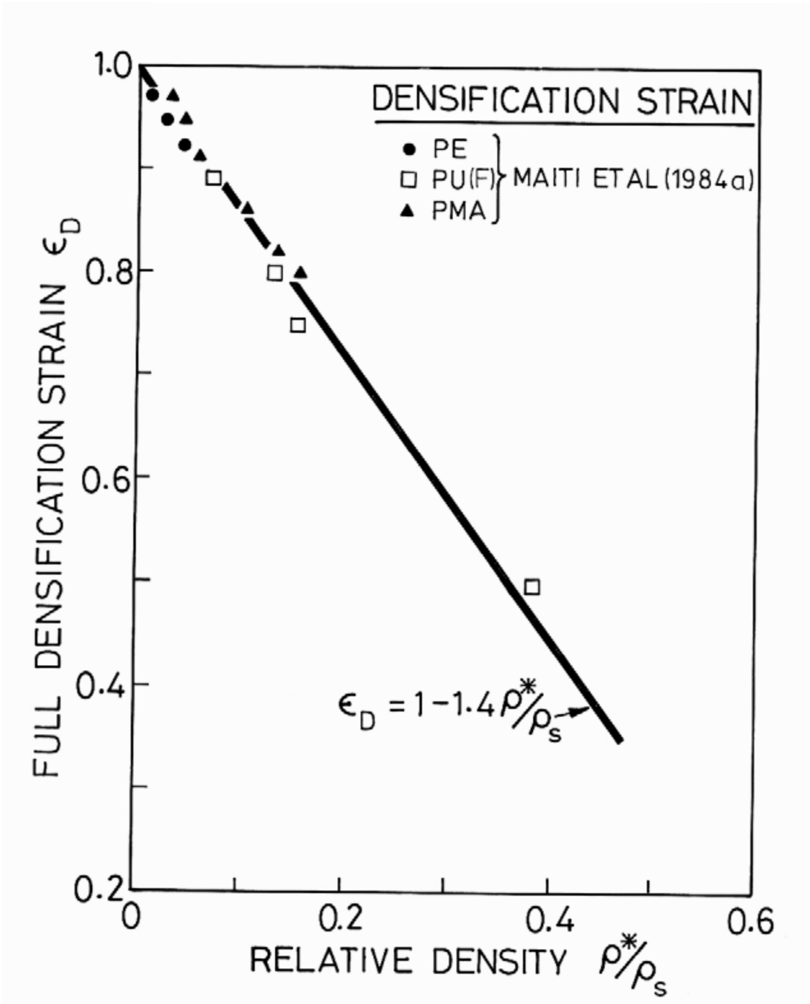
Densification strain, ϵ_D

- at large comp. strain, cell walls begin to touch, σ-ϵ rises steeply
- $E^* \rightarrow E_s$; σ-ϵ curve looks vertical, at limiting strain
- Might expect $\epsilon_D = 1 - \rho^*/\rho_s$
- walls jam together at slightly smaller strain than this:

$$\boxed{\epsilon_D = 1 - 1.4\, \rho^*/\rho_s}$$



Densification Strain



Fracture toughness

Open cells : crack length $2a$, local stress σ_ℓ, remote stress σ^∞

$$\sigma_\ell = \frac{C\,\sigma^\infty \sqrt{\pi a}}{\sqrt{2\pi r}}$$ a distance r from crack tip

- next unbroken cell wall a distance $r \approx \ell/2$ ahead of crack tip subject to a force (integrating stress over next cell)

$$F \propto \sigma_\ell\, \ell^2 \propto \sigma^\infty \sqrt{\frac{a}{\ell}}\, \ell^2$$

- edges fail when applied moment, M = fracture moment, M_f

$$M_f \propto \sigma_{fs}\, t^3$$

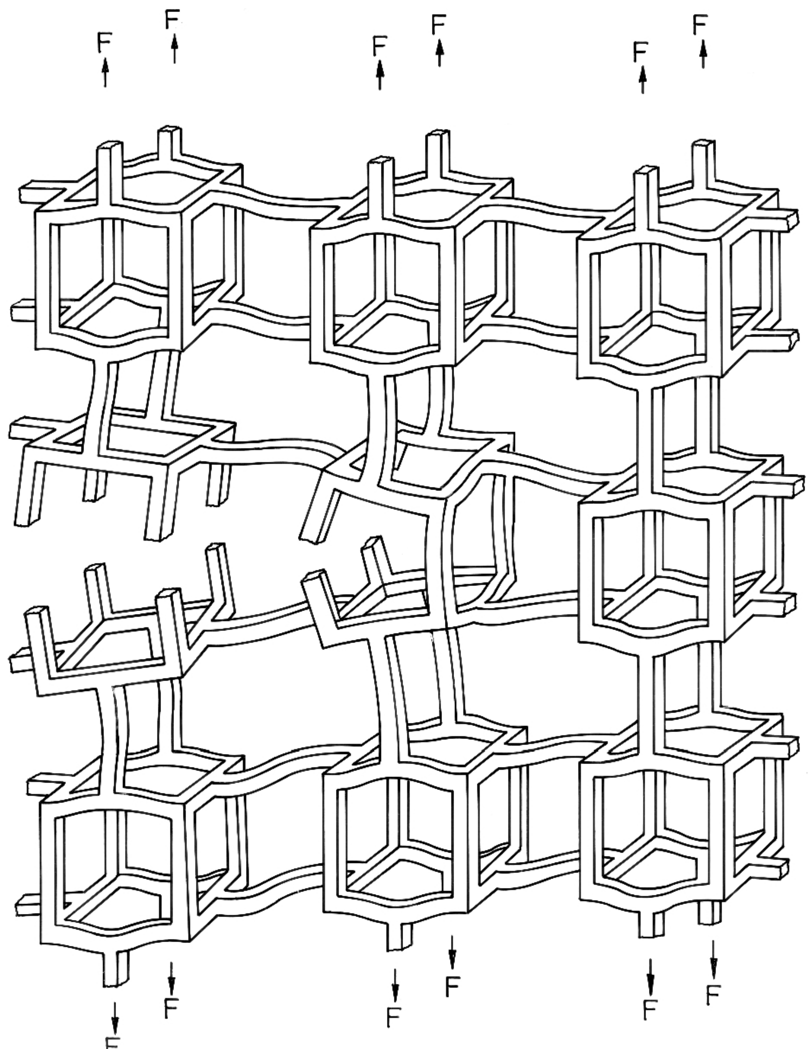
$$M \propto F\ell \propto \sigma^\infty \left(\frac{a}{\ell}\right)^{1/2} \ell^3 \qquad M = M_f \Rightarrow \sigma^\infty \left(\frac{a}{\ell}\right)^{1/2} \ell^3 \propto \sigma_{fs}\, t^3$$

$$\sigma^\infty \propto \sigma_{fs} \left(\frac{\ell}{a}\right)^{1/2} \left(\frac{t}{\ell}\right)^3$$

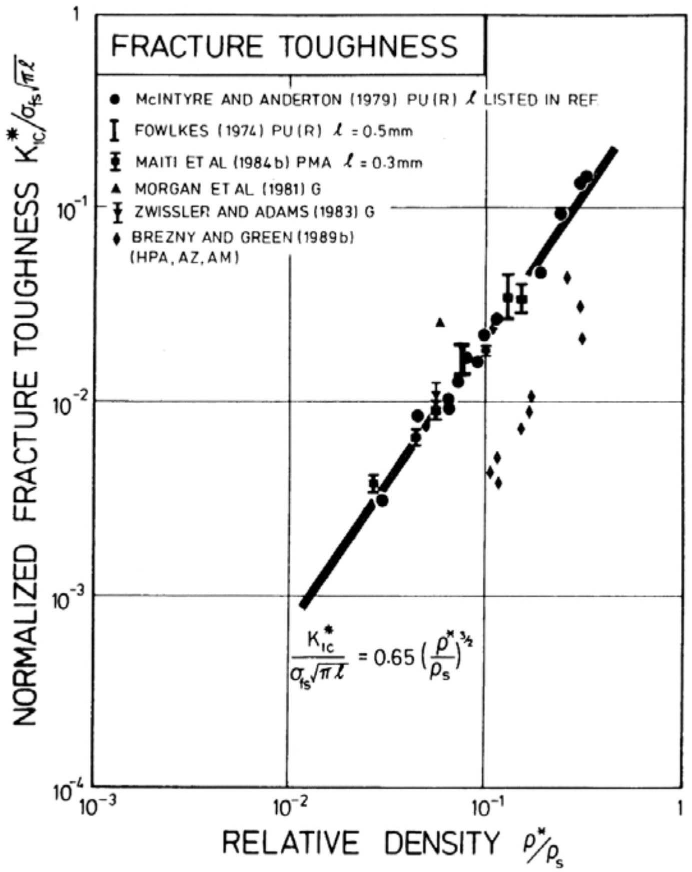
$$\boxed{K_{Ic}^* = \sigma^\infty \sqrt{\pi a} = C_8\, \sigma_{fs} \sqrt{\pi \ell}\, (\rho^*/\rho_s)^{3/2}}$$

Data: $C_8 \sim 0.65$

Fracture Toughness



Fracture Toughness



3.054 / 3.36 Cellular Solids: Structure, Properties and Applications
Spring 2015