Honeycombs: Out-of-plane behavior

- honeycombs used as cores in sandwich structures
  - carry shear load in $x_1 - x_3$ & $x_2 - x_3$ planes
- honeycombs sometimes used to absorb energy from impact - loaded in x_3 direction
- require out-of-plane properties
- cell walls extend or contract, rather than bend
- honeycomb much stiffer + stronger.

Linear-elastic deformation

- honeycomb has 9 independent elastic constants
  - 4 in-plane
  - 5 out-of-plane

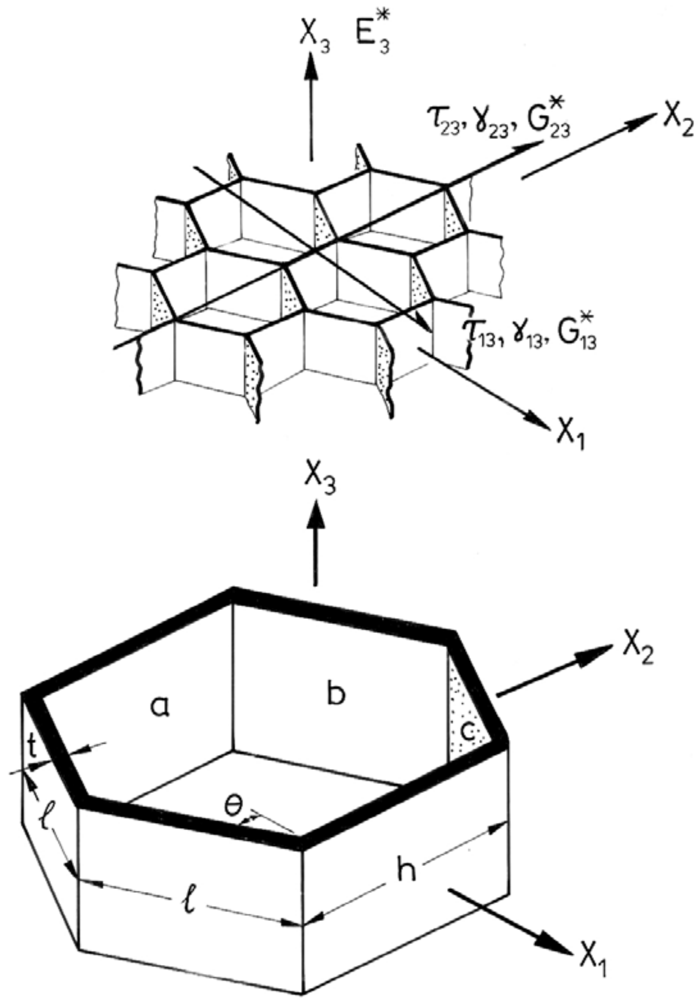
Young's modulus, E_3^*

- cell walls contract or extend axially
- E_3^* scales as area fraction of solid in plane $\perp$ to x_3

$$\boxed{E_3^* = E_s (\rho^*/\rho_s) = E_s \left(\frac{t}{l}\right) \frac{h/l + 2}{2(h/l + \sin\theta)\cos\theta}}$$

Notice: $E_3^* \propto t/l$ & $E_1^*, E_2^* \propto (t/l)^3$ $\Rightarrow$ large anisotropy

Out-of-Plane Properties



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Poisson ratios

- for loading in x_3 direction, cell walls strain by $\nu_s \epsilon_3$ in x_1, x_2 directions

$$\boxed{\nu^*_{31} = \nu^*_{32} = \nu_s}$$

(recall $\nu_{ij} = -\epsilon_j/\epsilon_i$)

- ν^*_{13} & ν^*_{23} can be found from reciprocal relation:

$$\boxed{\frac{\nu^*_{13}}{E^*_1} = \frac{\nu^*_{31}}{E^*_3} \quad \text{and} \quad \frac{\nu^*_{23}}{E^*_2} = \frac{\nu^*_{32}}{E^*_3}}$$

$$\therefore \nu^*_{13} = \frac{E^*_1 \, \nu^*_{31}}{E^*_3} = \frac{C_1 (t/l)^3 E_s \, \nu_s}{C_2 (t/l) E_s} \approx 0 \quad \text{for small } (t/l)$$

similarly, $\underline{\nu^*_{23} \approx 0}$

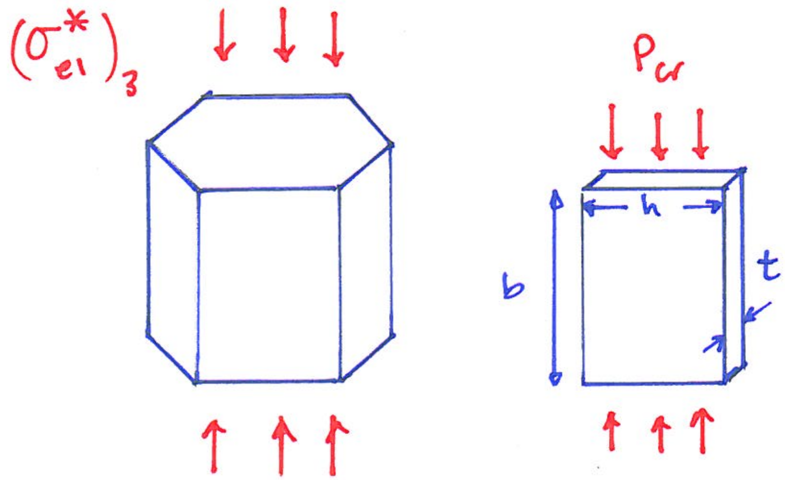
Shear moduli

- cell walls loaded in shear
- but constraint of neighbouring cell walls gives non-uniform strain in cell walls
- exact solution requires numerical methods
- can estimate as:

$$\boxed{G_{13}^* = G_s \left(\frac{t}{l}\right) \frac{\cos\theta}{h/l + \sin\theta} = \frac{1}{\sqrt{3}} G_s \frac{t}{l}}$$ for regular hexagons ($= G_{23}^*$).

- note linear dependence on (t/l)

Compressive strength : elastic buckling



- plate buckling

$$P_{cr} = \frac{K E_s t^3}{(1-\nu_s^2)\, h}$$ ← also, for l

- K end constraint factor

depends on stiffness of adjacent walls

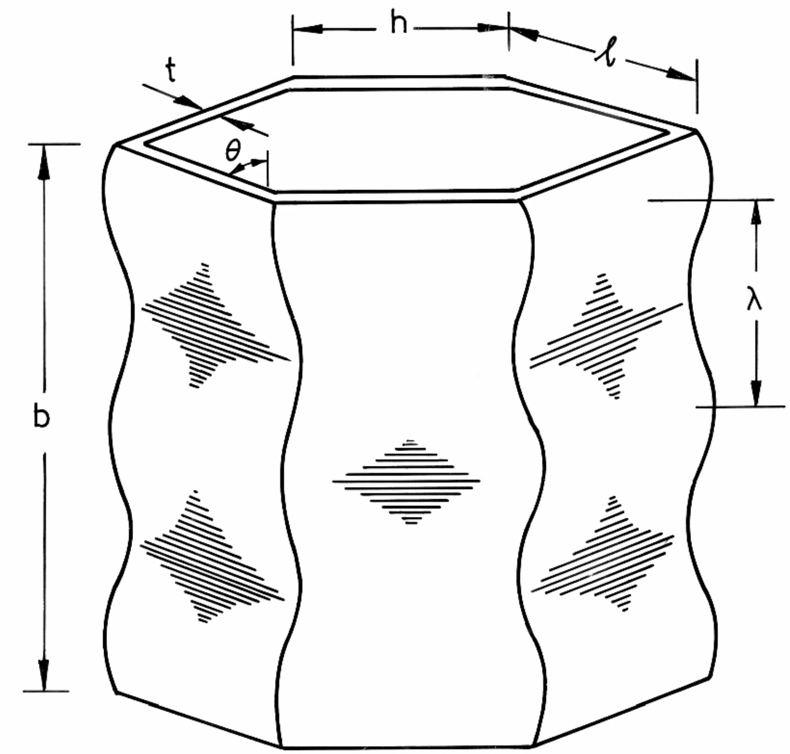
- if vertical edges simply supported (free to rotate) & $b > 3l$: $K = 2.0$
- " " " clamped + fixed : $K = 6.2$
- approximate $K \approx 4$

$P_{total} = \Sigma P_{cr}$ for each wall ($2l + h$ for each cell)

$$\boxed{(\sigma^*_{el})_3 \approx \frac{E_s}{1-\nu_s^2}\left(\frac{t}{l}\right)^3 \frac{2\,(l/h + 2)}{(h/l + \sin\theta)\cos\theta}}$$

- regular hexagons $(\sigma^*_{el})_3 = 5.2\, E_s (t/l)^3$
- same form as $(\sigma^*_{el})_2$ but ~ 20 x larger.

Out-of-Plane: Elastic Buckling



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Compressive strength: plastic collapse

- failure by uniaxial yield $(\sigma^*_{pl})_3 = \sigma_{ys}\,(\rho^*/\rho_s)$
- but, in compression, plastic buckling usually precedes this
- consider approximate calculation, simplified geometry $\Rightarrow$ isolated cell wall
- rotation of cell wall by π at plastic hinge
- plastic moment $M_p = \dfrac{\sigma_{ys} t^2}{4}(2l+h)$ (note $2l+h$ instead of b as before) for loading in x_1 or x_2
- internal plastic work $= \pi M_p$

- external work done $= \dfrac{P\lambda}{2}$ λ = wavelength of plastic buckling $\approx l$
  $P = \sigma_3\,(h + l\sin\theta)(2l\cos\theta)$

$$\therefore \frac{P\lambda}{2} = \pi M_p$$

$$\sigma_3\,(h+l\sin\theta)\,(2l\cos\theta)\,\frac{l}{2} = \pi\,\frac{\sigma_{ys} t^2}{4}\,(2l+h)$$

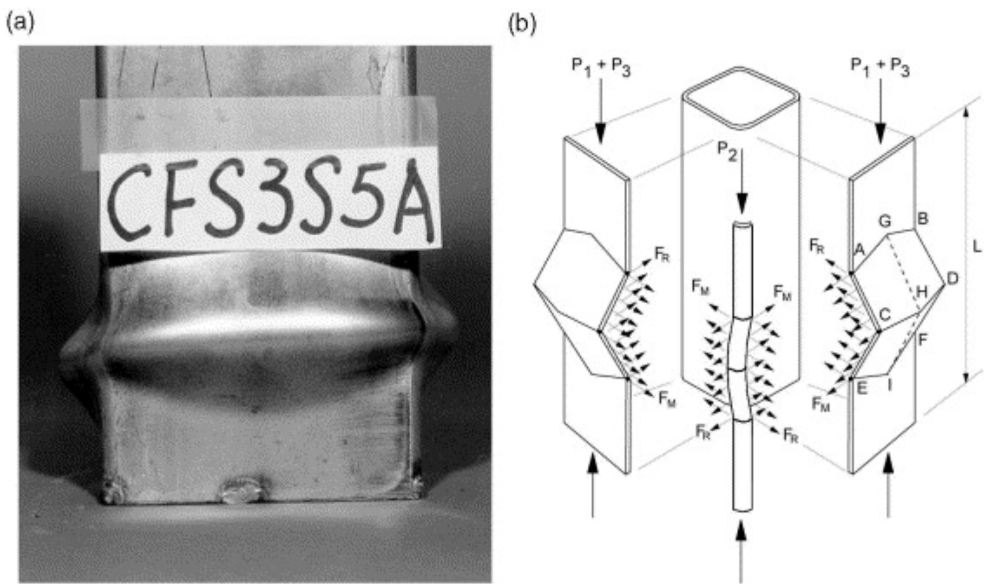
$$(\sigma^*_{pl})_3 \approx \frac{\pi}{4}\,\sigma_{ys}\left(\frac{t}{l}\right)^2 \frac{(h/l + 2)}{(h/l + \sin\theta)\cos\theta}$$

[note: misprint in book eqn before 4.115].

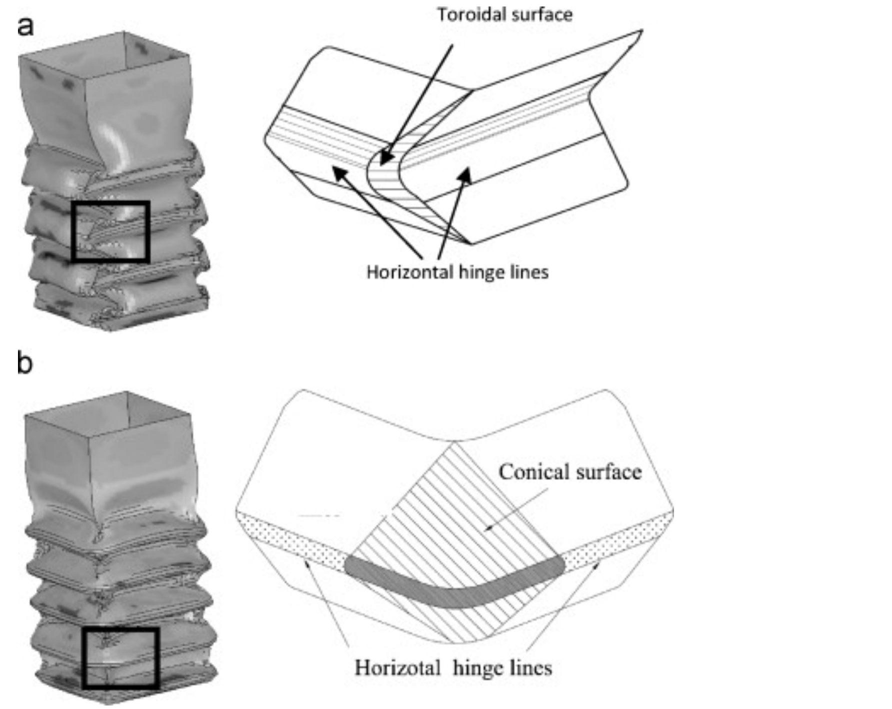
regular hexagons: $(\sigma^*_{pl})_3 \approx 2\,\sigma_{ys}\left(\dfrac{t}{l}\right)^2$

exact calculation regular hexagons $(\sigma^*_{pl})_3 = 5.6\,\sigma_{ys}\left(\dfrac{t}{l}\right)^{5/3}$

Out-of-Plane: Plastic Collapse



Source: Zhao X. L., B. Han, et al. "Plastic Mechanism Analysis of Concrete-Filled Double-skin (SHS Inner and SHS Outer) Stub Columns." *Thin-Walled Structures* 40 (2002): 815-33. Courtesy of Elsevier. Used with permission.

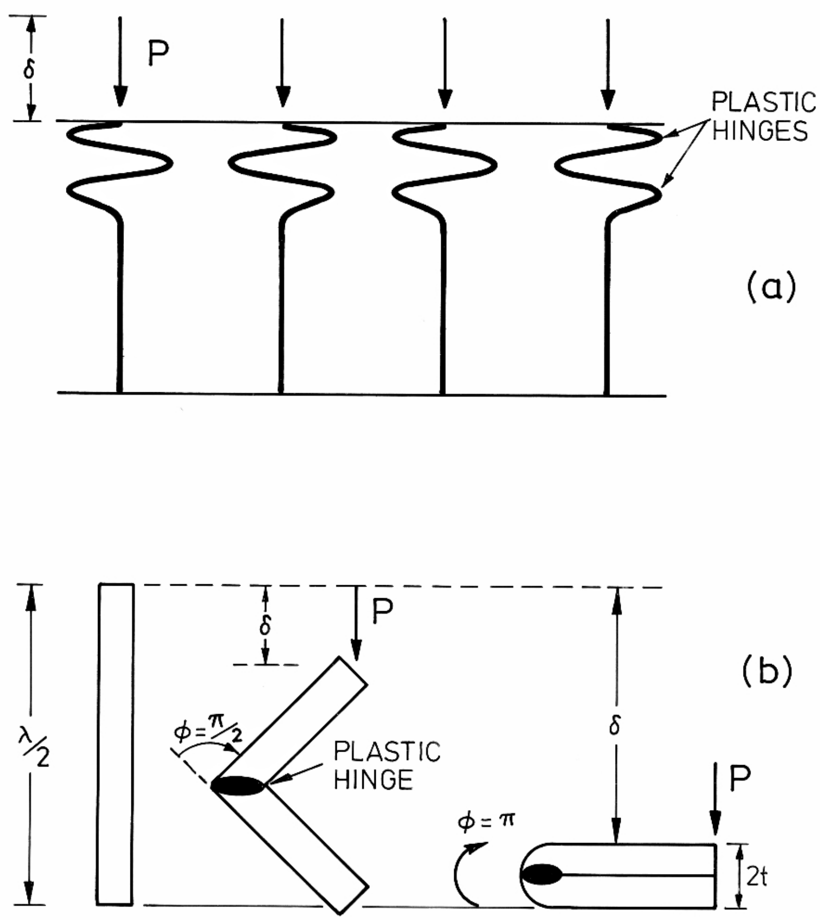


Source: Najafi, A., and M. Rais-Rohani. "Mechanics of Axial Plastic Collapse in Multi-cell, Multi-corner Crush Tubes." *Thin-Walled Structures* 49 (2011): 1-12. Courtesy of Elsevier. Used with permission.

Zhao XL, Han B and Grzebieta RH (2002) Thi-Walled Structures **40**, 815-533

Najafi A and Rais-Rohani M (2011) Thin-Walled Structures **49**, 1-12

Out-of-Plane: Plastic Collapse



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Out-of-plane, brittle fracture (tensile failure)

- defect free sample, walls see uniaxial tension

$$(\sigma_f^*)_3 = (\rho^*/\rho_s)\,\sigma_{fs} = \frac{h/l + 2}{2(h/l + \sin\theta)\cos\theta}\left(\frac{t}{l}\right)\sigma_{fs}$$

- if cell walls cracked ($a \gg l$) & crack propagates in plane normal to x_3

toughness, $G_c^* = (\rho^*/\rho_s)\,G_s$

fracture toughness, $K_{Ic}^* = \sqrt{E^* G_c^*} = \sqrt{(\rho^*/\rho_s)\,E_s\,(\rho^*/\rho_s)\,G_{cs}} = (\rho^*/\rho_s)\,K_{Ics}$

Out-of-plane: brittle crushing

σ_{cs} = compressive strength of cell wall

$$(\sigma_{cr}^*)_3 = (\rho^*/\rho_s)\,\sigma_{cs}$$

brittle materials $\sigma_{cs} \approx 12\,\sigma_{fs}$

$$\approx 12\,(\rho^*/\rho_s)\,\sigma_{fs}$$