Honeycombs - In-plane behavior

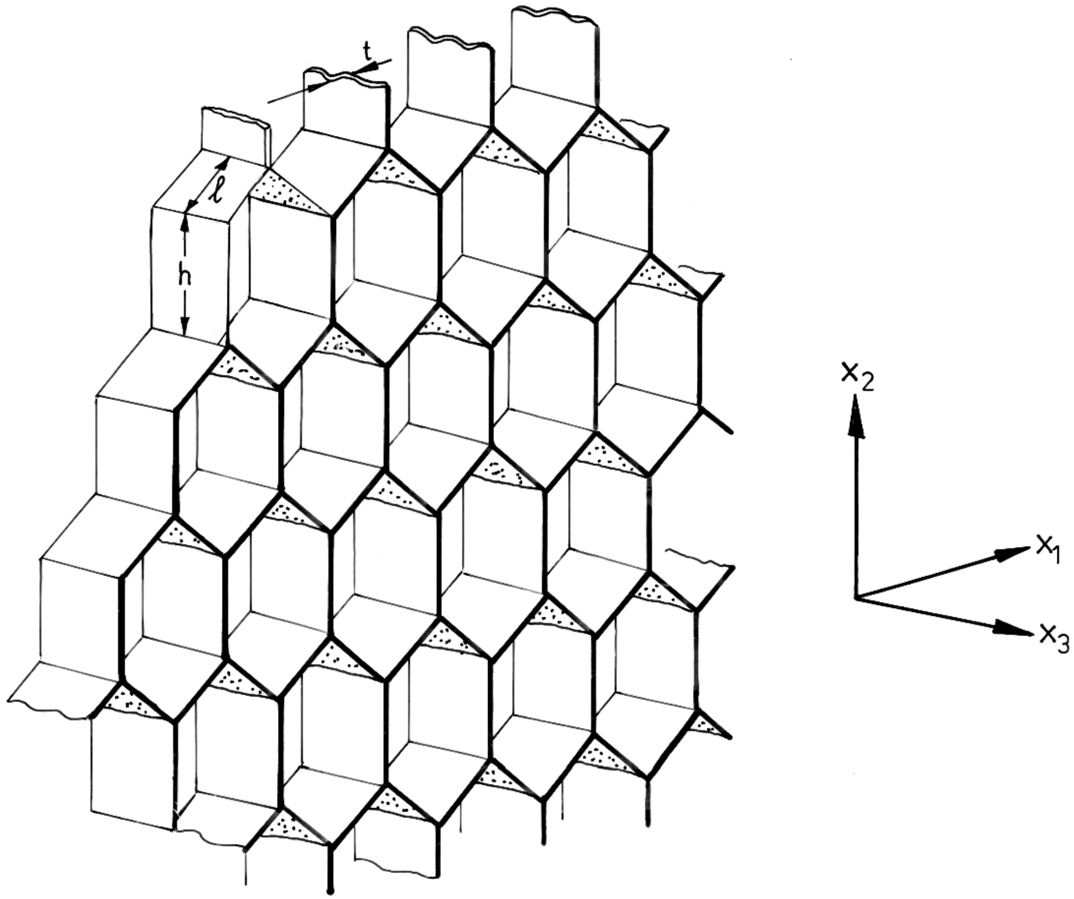
- prismatic cells
- polymer, metal, ceramic honeycombs widely available
- used for sandwich structure cores, energy absorption, carriers for catalysts
- some natural materials (eg. wood, cork) can be idealized as honeycombs
- mechanisms of deformation + failure in hexagonal honeycombs parallel those in foams
  - simpler geometry (unit cell) - easier to analyze
- mechanisms of deformation in triangular honeycombs parallels those in 3D trusses (lattice materials)

Stress - Strain curves + Deformation behaviour : In-Plane

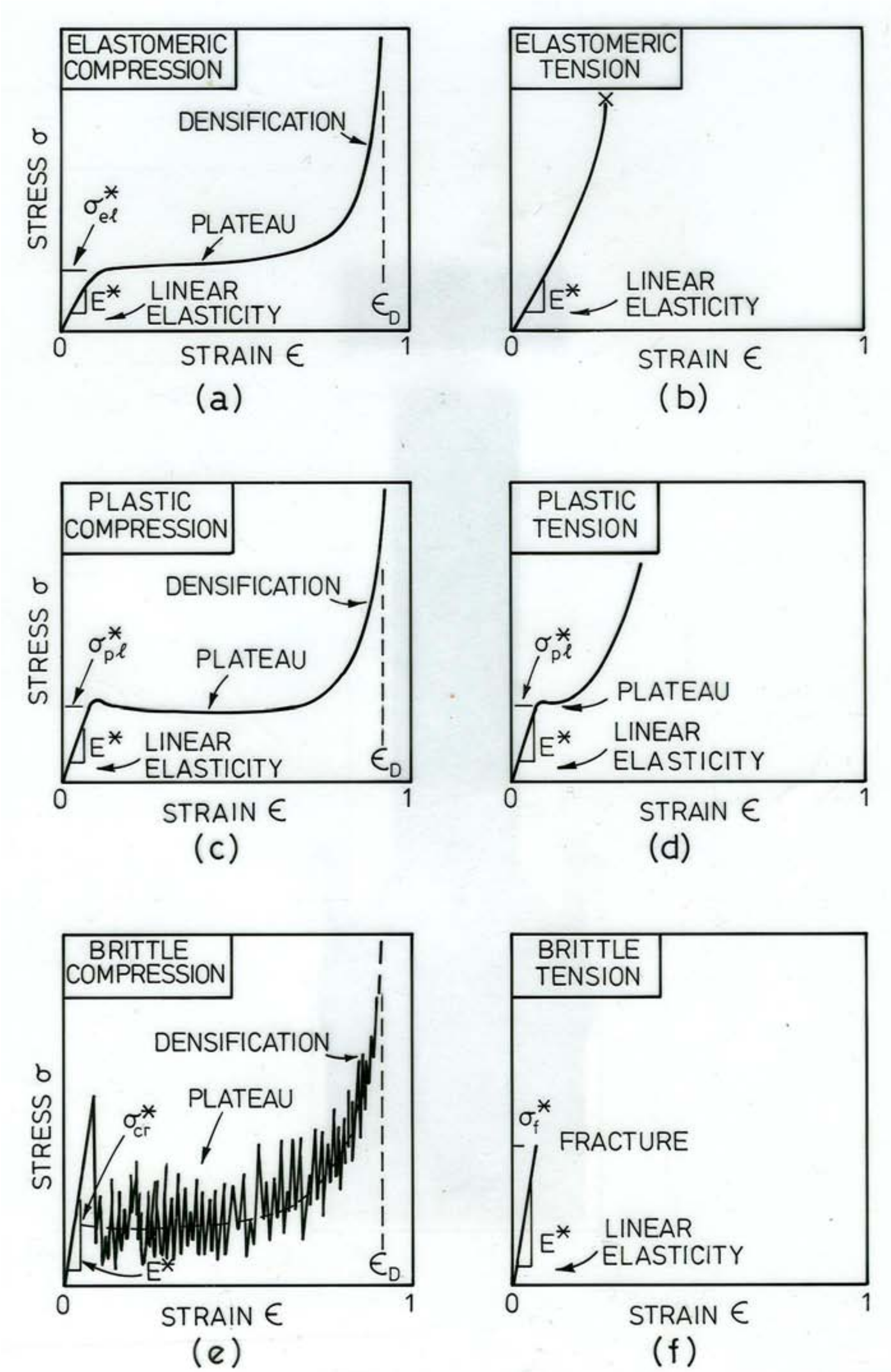
Compression

- 3 regimes
  - linear elastic - bending
  - stress plateau
    - buckling
    - yielding
    - brittle crushing
  - densification - cell walls touch
- increasing $t/l \Rightarrow E^* \uparrow \quad \sigma^* \uparrow \quad \epsilon_D \downarrow$

Honeycomb Geometry



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

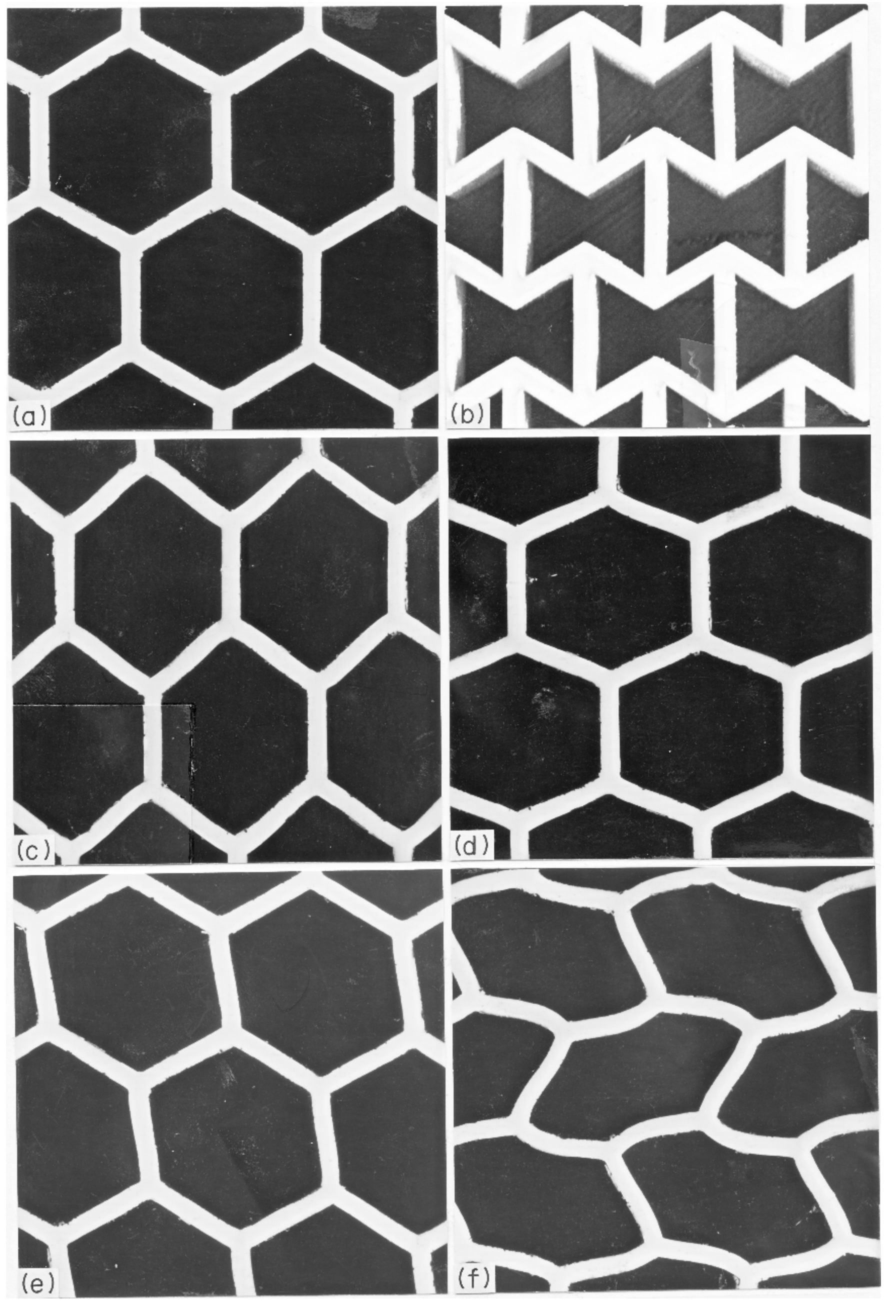
Deformation mechanisms

Bending
X_1 Loading

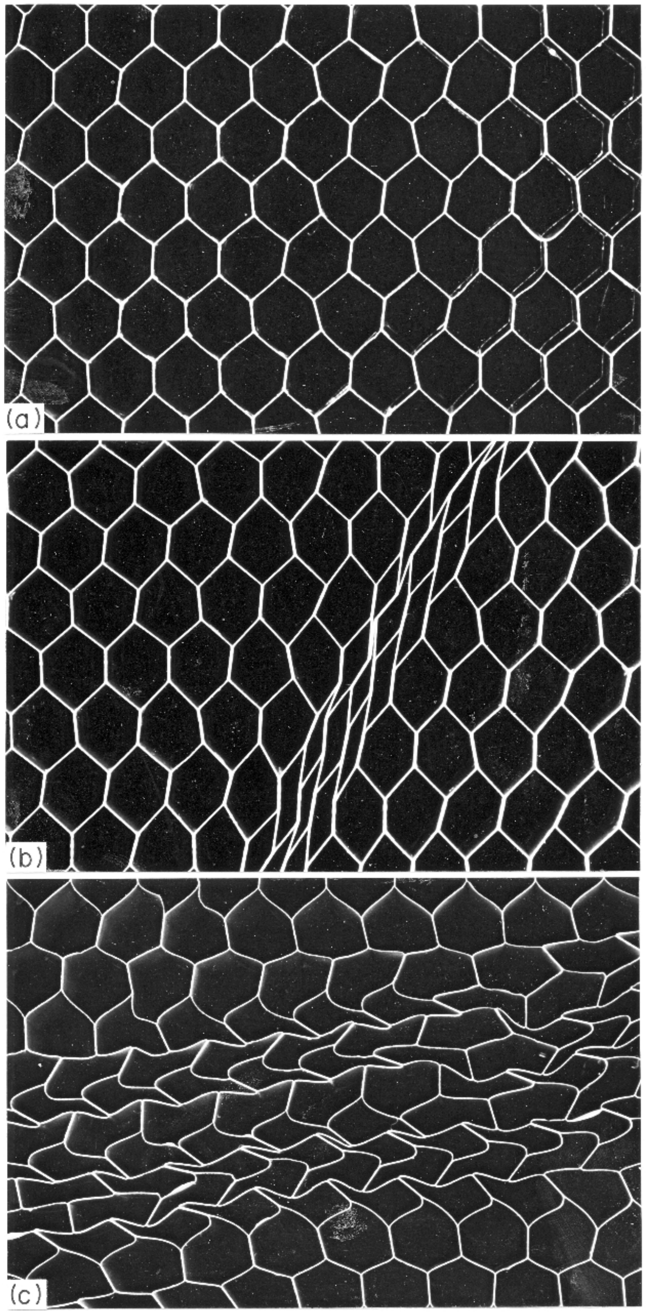
Bending
X_2 Loading

Bending
Shear

Buckling



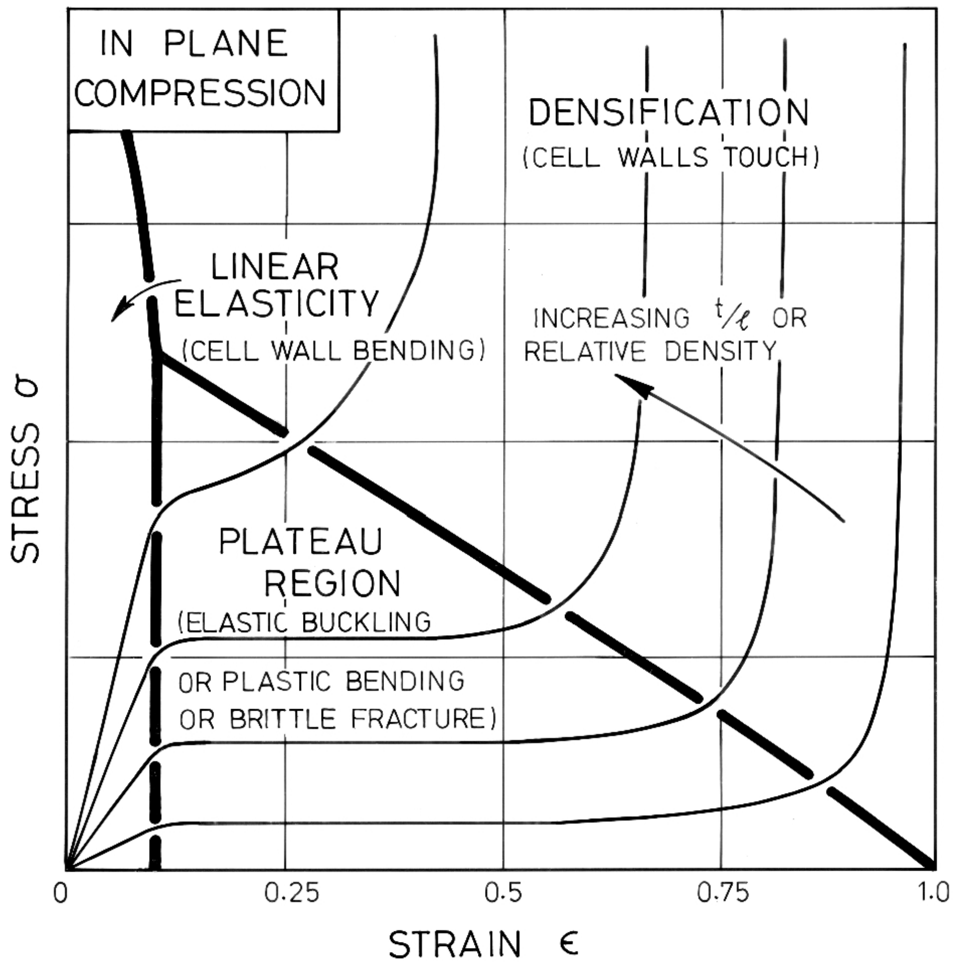
Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.



Plastic collapse in an aluminum honeycomb

Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Stress-Strain Curve



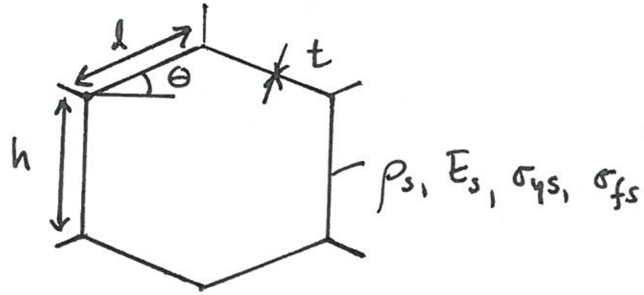
Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

tension

- linear elastic - bending
- stress plateau - exists only if cell walls yield
  - no buckling in tension
  - brittle honeycombs fracture in tension

Variables affecting honeycomb properties

- relative density $\frac{\rho^*}{\rho_s} = \frac{t/l\,(h/l+2)}{2\cos\theta\,(h/l+\sin\theta)} = \frac{2}{\sqrt{3}}\frac{t}{l}$ regular hexagons
- solid cell wall properties: ρ_s, E_s, σ_{ys}, σ_{fs}
- cell geometry: h/l, θ



In-plane properties

assumptions:

- t/l small (ρ^*/ρ_s small) - neglect axial + shear contribution to def'm
- deformations small - neglect changes in geometry
- cell wall - linear elastic, isotropic

symmetry

- honeycombs are orthotropic - rotate 180° about each of three mutually perpendicular axes & structure is the same

---

Linear elastic deformation

$$
\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{bmatrix}
=
\begin{bmatrix}
1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0 \\
-\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0 \\
-\nu_{13}/E_1 & -\nu_{23}/E_2 & -1/E_3 & 0 & 0 & 0 \\
 & & & 1/G_{23} & 0 & 0 \\
 & (\text{symmetric}) & & & 1/G_{13} & 0 \\
 & & & & & 1/G_{12}
\end{bmatrix}
\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix}
$$

- Matrix notation: $\epsilon_1 = \epsilon_{11}$ $\epsilon_2 = \epsilon_{22}$ $\epsilon_3 = \epsilon_{33}$ $\epsilon_4 = \gamma_{23}$ $\epsilon_5 = \gamma_{13}$ $\epsilon_6 = \gamma_{12}$ $\sigma_1 = \sigma_{11}$ $\sigma_2 = \sigma_{22}$ $\sigma_3 = \sigma_{33}$ $\sigma_4 = \sigma_{23}$ $\sigma_5 = \sigma_{13}$ $\sigma_6 = \sigma_{12}$

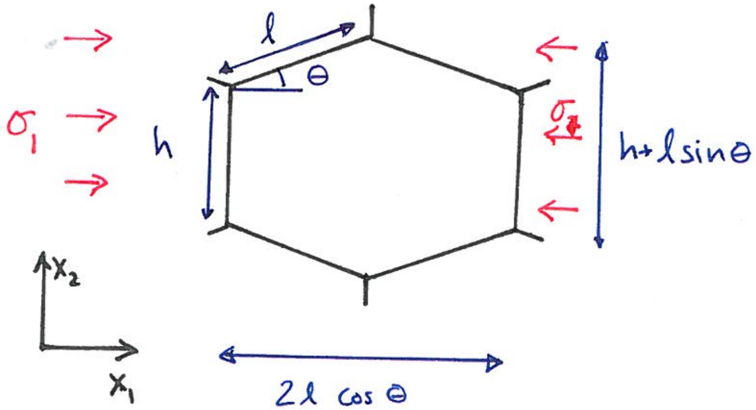
- in-plane $(x_1 - x_2)$: 4 independent elastic constants:

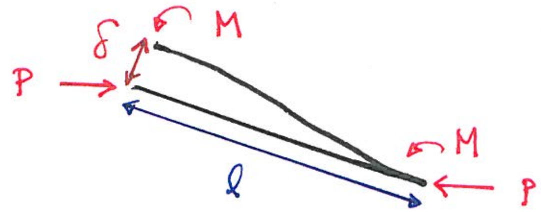
$$E_1 \quad E_2 \quad \nu_{12} \quad G_{12}$$

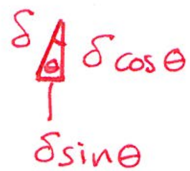
and compliance matrix symmetric $-\dfrac{\nu_{12}}{E_1} = -\dfrac{\nu_{21}}{E_2}$ (reciprocal relation)

[notation for Poisson's ratio: $\nu_{ij} = -\dfrac{\epsilon_j}{\epsilon_i}$]

Young's modulus in x_1 direction







$$\sigma_1 = \frac{P}{(h + \ell \sin\theta) b}$$

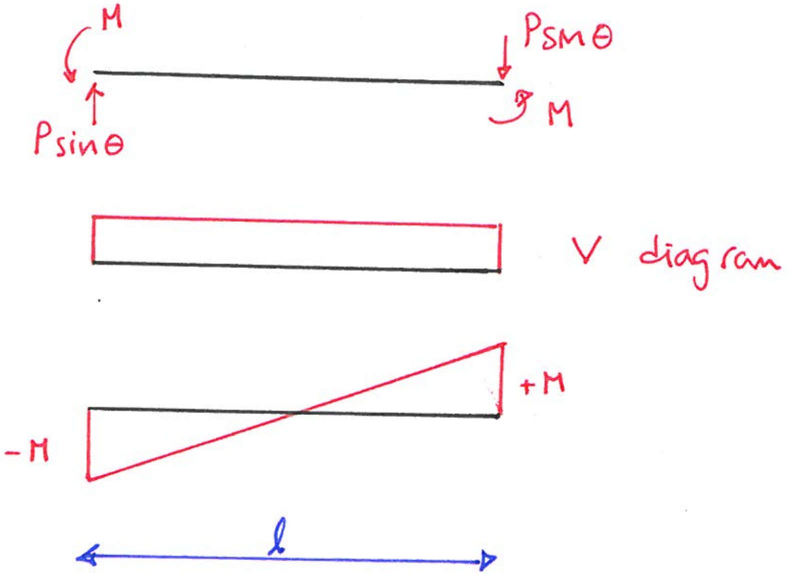
$$\epsilon_1 = \frac{\delta \sin\theta}{\ell \cos\theta}$$

unit cell in x_1 direction: $2\ell \cos\theta$

" " " [illegible]

In-Plane Deformation: Linear Elasticity

Figure removed due to copyright restrictions. See Figure 5: L. J. Gibson, M. F. Ashby, et al. "The Mechanics of Two-Dimensional Cellular Materials."



M diagram: 2 cantilevers of length $l/2$

$$\delta = 2 \cdot \frac{P \sin\theta \, (l/2)^3}{3 E_s I}$$

$$= \frac{2 \, P l^3 \sin\theta}{24 \, E_s I}$$

$$\delta = \frac{P l^3 \sin\theta}{12 \, E_s I} \qquad I = \frac{b t^3}{12}$$

Combining: $$E_1^* = \frac{\sigma_1}{\epsilon_1} = \frac{P}{(h + l\sin\theta)\, b} \; \frac{l \cos\theta}{\delta \sin\theta}$$

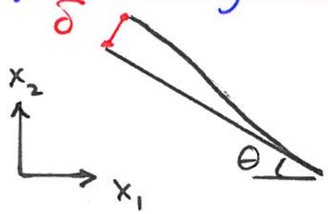
$$= \frac{\cancel{P}}{(h + l\sin\theta)\,\cancel{b}} \; \frac{l\cos\theta}{\cancel{P}\, l^3 \sin^2\theta} \; \cancel{12}\, E_s \, \frac{\cancel{b}\, t^3}{\cancel{12}}$$

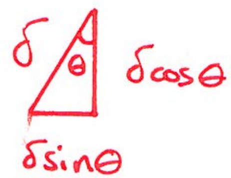
$$\boxed{E_1^* = E_s \left(\frac{t}{l}\right)^3 \frac{\cos\theta}{(h/l + \sin\theta)\, \sin^2\theta}} \qquad = \frac{4}{\sqrt{3}} \left(\frac{t}{l}\right)^3 E_s \quad \text{regular hexagons } h/l = 1 \;\; \theta = 30°$$

solid property, relative density, cell geometry

Poisson's ratio for loading in x_1 direction

$$\nu_{12}^* = -\frac{\epsilon_2}{\epsilon_1}$$





$$\epsilon_1 = -\frac{\delta \sin\theta}{l\cos\theta} \quad \text{(shortens)} \qquad \epsilon_2 = \frac{\delta\cos\theta}{h + l\sin\theta} \quad \text{(lengthens)}$$

$$\nu_{12}^* = -\frac{\delta\cos\theta}{h + l\sin\theta}\left(\frac{l\cos\theta}{-\delta\sin\theta}\right) = \frac{\cos^2\theta}{(h/l + \sin\theta)\sin\theta}$$

- ν_{12}^* depends ONLY on cell geometry (h/l, θ), not on E_s, t/l
- regular hexagonal cells: $\nu_{12}^* = 1$
- ν can be negative for $\theta < 0$

  eg. $h/l = 2$ $\quad \theta = -30°$ $\quad \nu_{12}^* = \dfrac{3/4}{(3/2)(-1/2)} = -1$

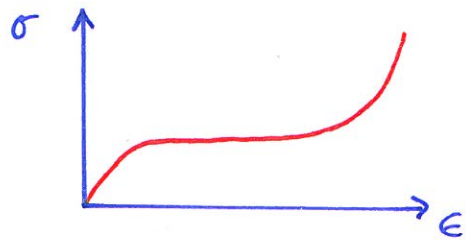
E_2^* ν_{21}^* G_{12}^*

- can be found in similar way; results in book.

Compressive strength (plateau stress)

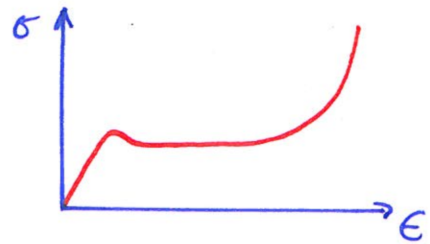
- cell collapse by:

(1) elastic buckling



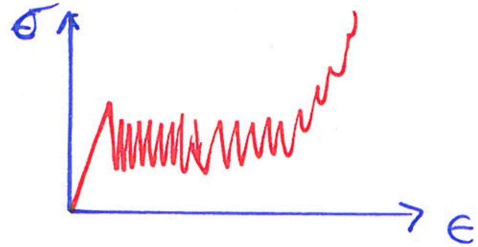
- buckling of vertical struts throughout honeycomb

(2) plastic yielding



- localization of yield
- as def'm progresses, propagation of failure band

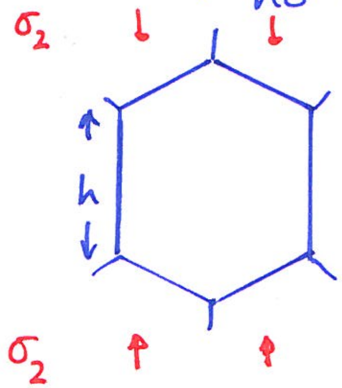
(3) brittle crushing



- peaks + valleys correspond to fracture of individual cell walls

Plateau stress: elastic buckling, σ^*_{el}

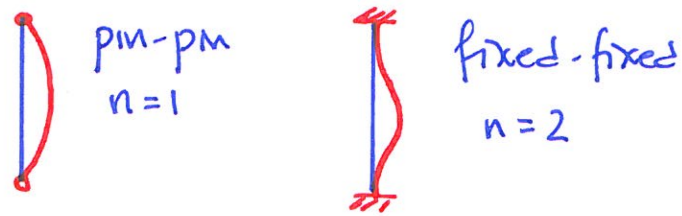
- elastomeric honeycombs - cell collapse by elastic buckling of walls of length h when loaded in x_2 direction
- no buckling for σ_1; bending of inclined walls goes to densification.



Euler buckling load

$$P_{cr} = \frac{n^2 \pi^2 E_s I}{h^2}$$

n = end constraint factor



Elastic Buckling

- here, constraint n depends on stiffness of adjacent inclined members
- can find by elastic line analysis (see appendix if interested)
- rotational stiffness at ends of column, h, matched to rotational stiffness of inclined members
- find $h/l = 1 \quad 1.5 \quad 2$

$n = 0.686 \quad 0.760 \quad 0.860$

and $$(\sigma^*_{el})_2 = \frac{P_{cr}}{2l\cos\theta\, b} = \frac{n^2\pi^2 E_s}{h^2\, 2l\cos\theta\, b}\frac{bt^3}{12}$$

$$\boxed{(\sigma^*_{el})_2 = \frac{n^2\pi^2}{24} E_s \frac{(t/l)^3}{(h/l)^2\cos\theta}}$$

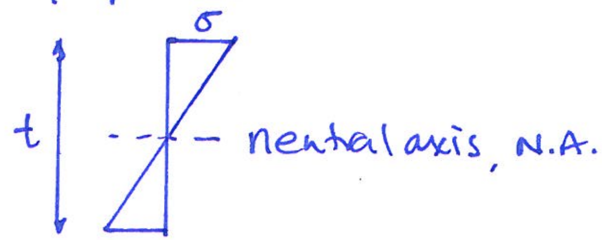
regular hexagons: $(\sigma^*_{el})_2 = 0.22\, E_s (t/l)^3$

& since $E^*_2 = 4/\sqrt{3}\, E_s (t/l)^3 = 2.31\, E_s (t/l)^3$

strain at buckling $(\epsilon^*_{el})_2 = 0.10$, for regular hexagons independent of E_s, t/l

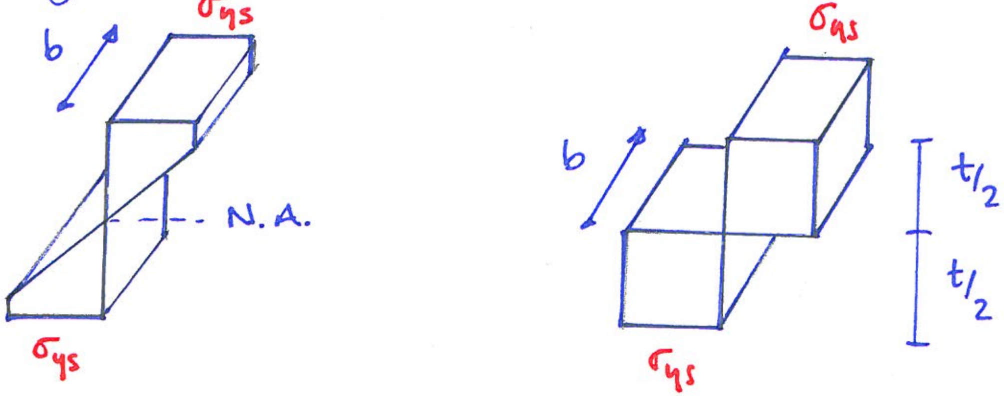
Plateau stress: plastic yielding, σ^*_{pl}

- failure by yielding in cell walls
- yield strength of cell walls $= \sigma_{ys}$
- plastic hinge forms when cross-section fully yielded
- Beam theory - linear elastic $\sigma = \frac{My}{I}$



- once stress at outer fiber $= \sigma_{ys}$, yielding begins & then progresses through the section, as the load increases

as P ↑



- when section fully yielded (right fig.), form plastic "hinge"
- section rotates, like a pin

Plastic Collapse

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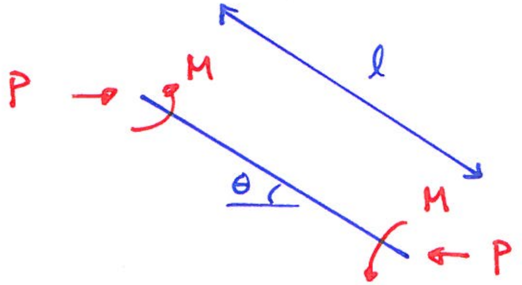
- moment at formation of plastic hinge (plastic moment, M_p):

$$M_p = \left(\sigma_{ys}\, b\, \frac{t}{2}\right)\left(\frac{t}{2}\right) = \frac{\sigma_{ys}\, b t^2}{4}$$

- applied moment, from applied stress



$$2 M_{app} - P l \sin\theta = 0$$

$$M_{app} = \frac{P l \sin\theta}{2}$$

$$\sigma_1 = \frac{P}{(h + l\sin\theta)\, b} \qquad M_{app} = \sigma_1 (h + l\sin\theta)\, b\, \frac{l\sin\theta}{2}$$

- plastic collapse of honeycomb at $(\sigma^*_{pl})_1$ when $M_{app} = M_p$

$$(\sigma^*_{pl})_1 (h + l\sin\theta)\, \cancel{b}\, \frac{l \sin\theta}{\cancel{2}} = \sigma_{ys} \frac{\cancel{b} t^2}{\cancel{4}\, 2}$$

$$\boxed{(\sigma^*_{pl})_1 = \sigma_{ys}\left(\frac{t}{l}\right)^2 \frac{1}{2(h/l + \sin\theta)\sin\theta}}$$

regular hexagons: $(\sigma^*_{pl})_1 = \frac{2}{3}\sigma_{ys}\left(\frac{t}{l}\right)^2$

similarly, $(\sigma^*_{pl})_2 = \sigma_{ys}\left(\frac{t}{l}\right)^2 \frac{1}{2\cos^2\theta}$

- can do similar

- for thin-walled honeycombs, elastic buckling can precede plastic collapse (for σ_2)
- elastic buckling stress = plastic collapse stress $(\sigma^*_{el})_2 = (\sigma^*_{pl})_2$

$$\frac{n^2\pi^2}{24} E_s \frac{(t/l)^3}{(h/l)^2 \cos\theta} = \frac{\sigma_{ys}(t/l)^2}{2\cos^2\theta}$$

$$(t/l)_{critical} = \frac{12\,(h/l)^2}{n^2\pi^2\cos\theta}\left(\frac{\sigma_{ys}}{E_s}\right)$$

regular hexagons: $(t/l)_{critical} = 3\,\dfrac{\sigma_{ys}}{E_s}$.

- e.g. metals $\sigma_{ys}/E_s \sim .002$ $(t/l)_{crit} \sim 0.6\%$
  - most metal honeycomb denser than this

  polymers $\sigma_{ys}/E_s \sim 3\text{-}5\%$ $(t/l)_{crit} \sim 10\text{-}15\%$
  - low density polymers with yield point may buckle before yield.

<u>Plateau stress: brittle crushing, $(\sigma_{cr}^*)_1$</u>

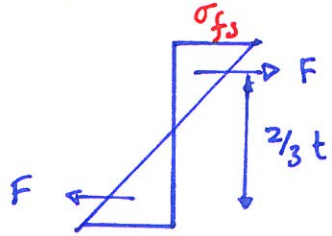
- ceramic honeycombs - fail in brittle manner
- cell wall bending - stress reaches modulus of rupture - wall fracture

loading in x_1 direction:

$$P = \sigma_1 (h + l\sin\theta) b \qquad \sigma_{fs} = \text{Modulus of rupture of cell wall}$$

$$M_{\substack{max \\ applied}} = \frac{P l \sin\theta}{2} = \frac{\sigma_1 (h + l\sin\theta) b l \sin\theta}{2}$$

Moment at fracture, M_f

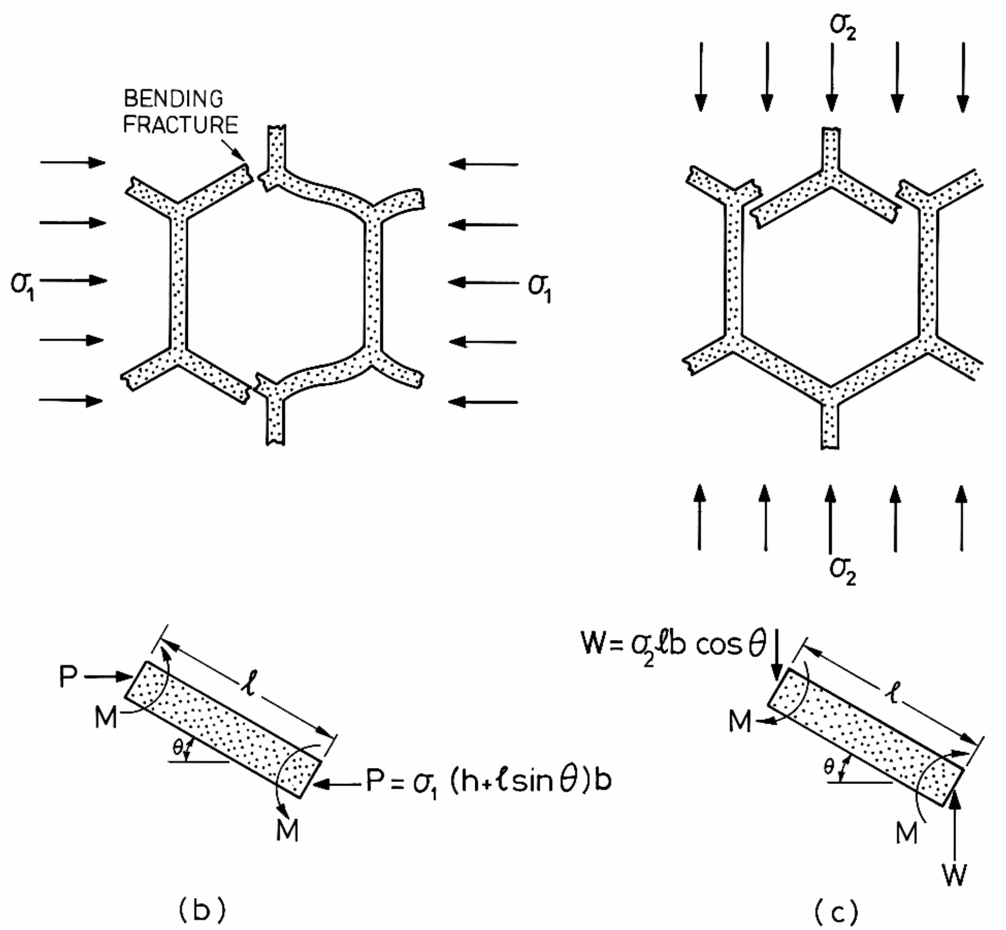


$$M_f = \left(\frac{1}{2}\sigma_{fs}\, b \frac{t}{2}\right)\left(\frac{2}{3}t\right) = \frac{\sigma_{fs}\, b t^2}{6}$$

$$\boxed{(\sigma_{cr}^*)_1 = \sigma_{fs}\left(\frac{t}{l}\right)^2 \frac{1}{3\,(h/l + \sin\theta)\sin\theta}}$$

regular hexagons: $(\sigma_{cr}^*)_1 = \frac{4}{9}\sigma_{fs}\left(\frac{t}{l}\right)^2$

Brittle Crushing



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Tension

- no elastic buckling
- plastic plateau stress approx same in tension + compression (small geometric difference due to deformation)
- brittle honeycombs: fast fracture

Fracture toughness

assume:
- crack length large relative to cell size (continuum assumption)
- axial forces can be neglected
- cell wall material has constant modulus of rupture, σ_{fs}

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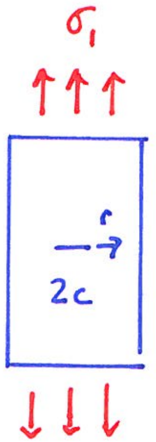
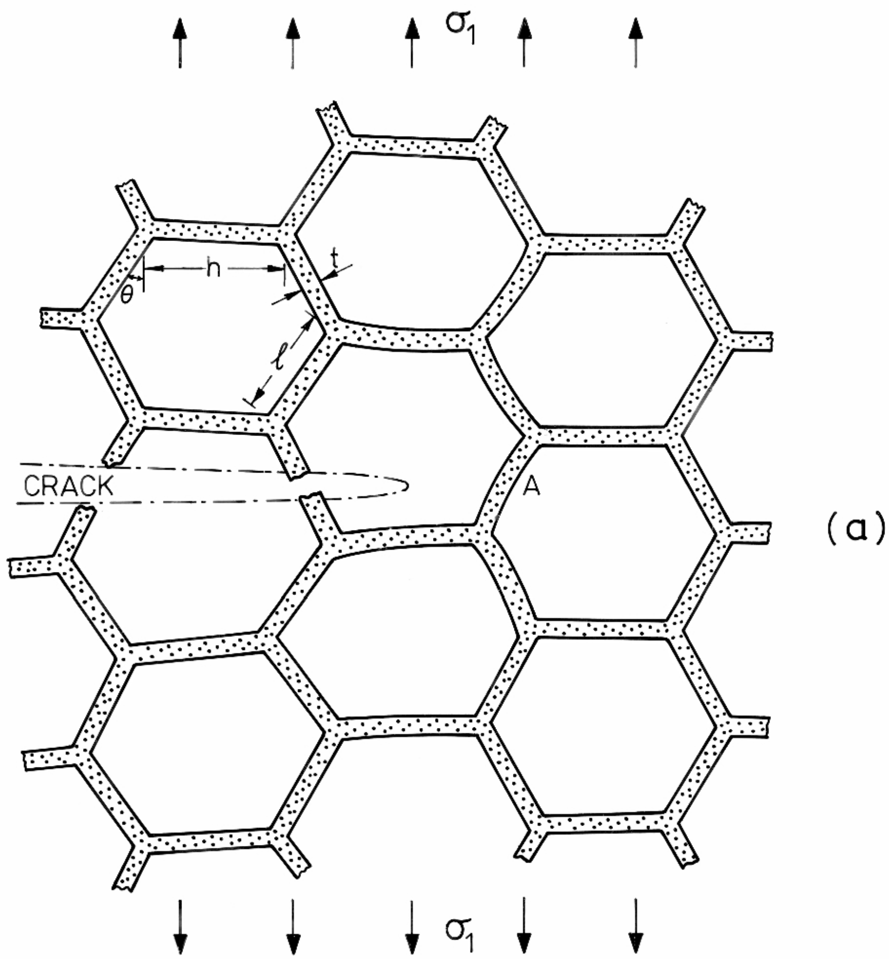
continuum: crack of length $2c$ in a linear elastic solid material normal to a remote tensile stress σ_1 creates a local stress field at the crack tip



$$\sigma_{local} = \sigma_l = \frac{\sigma_1 \sqrt{\pi c}}{\sqrt{2\pi r}}$$

Fracture Toughness



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

honeycomb: cell walls bent - fail when applied moment = fracture moment

$M_{app} \propto P\ell$ on wall A $\qquad M_f \propto \sigma_{fs}\, b t^2$

$$M_{app} \propto P\ell \propto \sigma_\ell\, \ell^2 b \propto \frac{\sigma_1 \sqrt{c}\; \ell^2 b}{\sqrt{\ell}} \propto \sigma_{fs}\, b t^2$$

$$(\sigma_f^*)_1 \propto \sigma_{fs} \left(\frac{t}{\ell}\right)^2 \sqrt{\frac{\ell}{c}}$$

$$\boxed{K_{Ic}^* = \sigma_f^* \sqrt{\pi c} = C\, \sigma_{fs} \left(t/\ell\right)^2 \sqrt{\ell}}$$ depends on cell size, ℓ !

C = constant

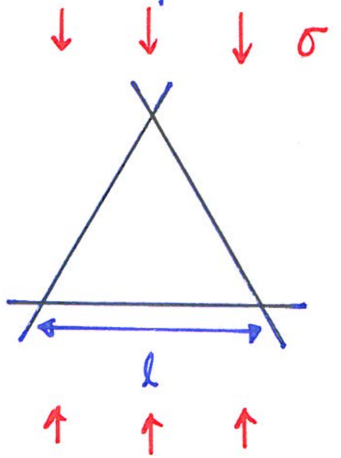
Summary: hexagonal honeycombs, in-plane properties

- linear elastic moduli: E_1^* E_2^* ν_{12}^* G_{12}^*
- plateau stresses (compression) $(\sigma_{el}^*)_2$ elastic buckling

  σ_{pl}^* plastic yield

  σ_{cr}^* brittle crushing
- fracture toughness (tension) K_{Ic}^* brittle fracture

Honeycombs: In-plane behaviour - triangular cells



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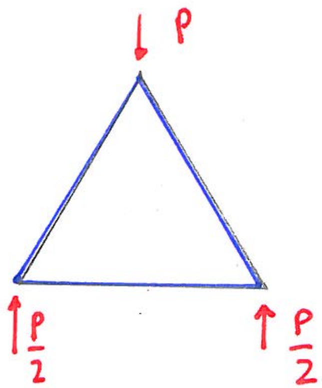
- triangulated structures - trusses
- can analyze as pin-jointed (no moment @ joints)
- forces in members all axial (no bending)
- if joints fixed + include bending, difference ~ 2%
- force in each member proportional to P



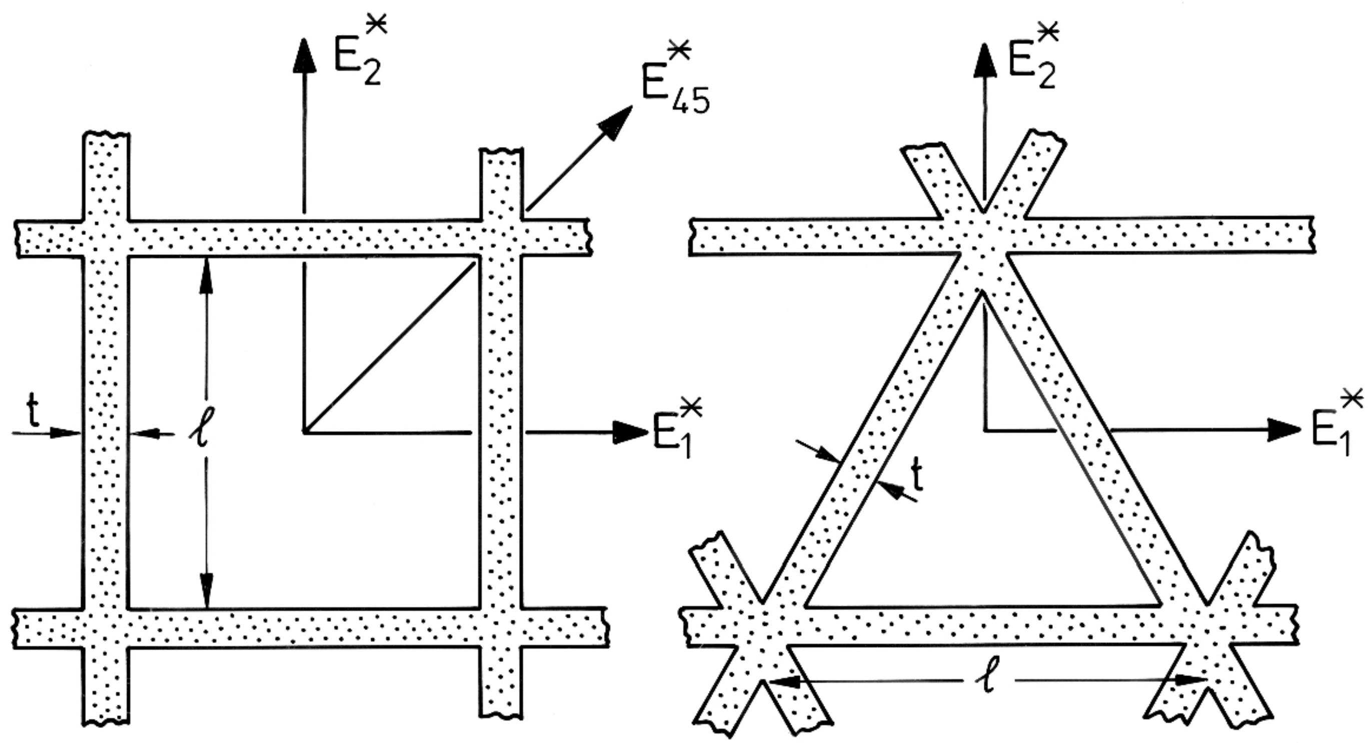
$\sigma \propto \dfrac{P}{l b}$ $\quad \epsilon \propto \dfrac{\delta}{l}$ $\quad \delta \propto \dfrac{P l}{A E_s}$ (axial shortening: Hooke's law)

$$E^* \propto \frac{\sigma}{\epsilon} \propto \frac{P}{lb}\,\frac{l}{\delta} \propto \frac{P}{b}\,\frac{b t E_s}{P l} \propto E_s\left(\frac{t}{l}\right)$$

$$E^* = C E_s (t/l)$$

exact calculation: $E^* = 1.15\, E_s\,(t/l)$ for equilateral triangle

Square and Triangular Honeycombs



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Spring 2015