Energy absorption in foams

- impact protection must absorb the kinetic energy of the impact while keeping the peak stress below the threshold that causes injury or damage
- direction of impact may not be predictable
- impact protection must itself be light eg. helmet

- capacity to undergo large def^n^ ($\epsilon \sim 0.8, 0.9$) at constant σ
- absorb large energies with little increase in peak stress

- foams - roughly isotropic - can absorb energy from any direction
  - light + cheap
- for a given peak stress, foam will always absorb more energy than solid it is made from
- strain rates: Instron typically $\dot{\epsilon} \sim 10^{-8}$ to $10^{-2}/s$

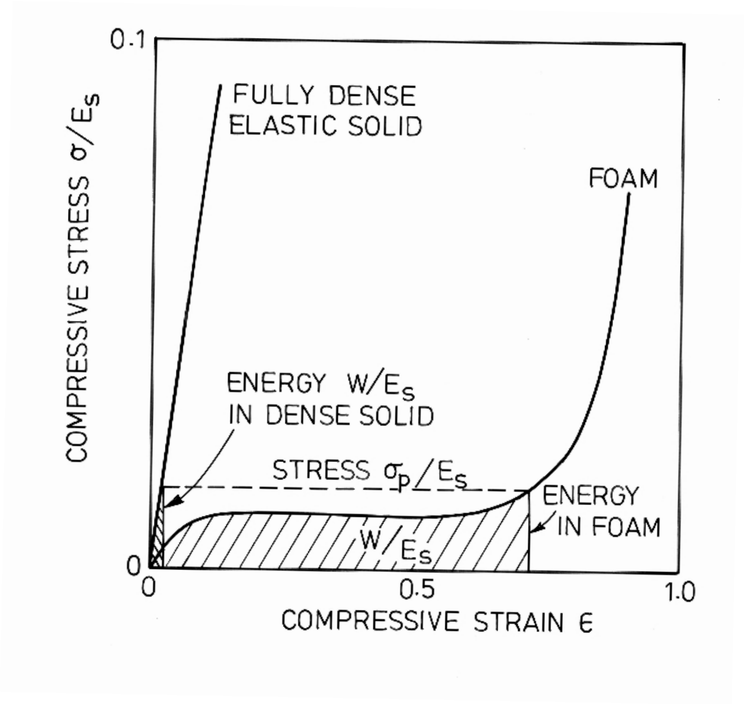
impact eg. drop from height of 1m, if thickness of foam = 100 mm

$V_{impact} = \sqrt{2gh} = \sqrt{2(9.81)(1)} = 4.4 m/s$; $\dot{\epsilon} = \frac{4.4 \, m/sec}{0.1 \, m} = 44/s$

- servo controlled Instrons, drop hammer tests - up to $\dot{\epsilon} = 100/s$

blast: $\dot{\epsilon} = 10^3 - 10^4/s$ - inertial effects import (we won't consider this)

Energy Absorption



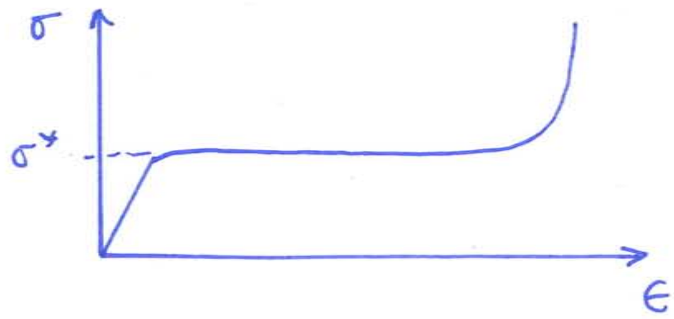
Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

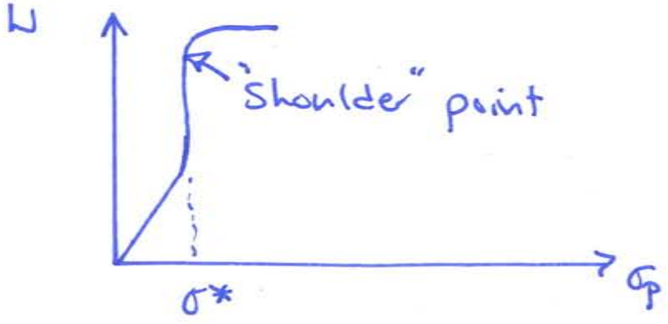
Energy absorption mechanisms

- elastomeric foams - elastic buckling of cells
  - elastic def^{n} recovered ⇒ rebound
  - also have damping - energy dissipated as heat
- plastic foams, brittle foams - energy dissipated as plastic work or work of fracture
  - no rebound
- natural cellular materials - may have fiber composite cell walls
  - dissipate energy by fiber pullout + fracture

- fluid within cells
  - open cell foams - fluid flow dissipation only impt. if fluid is viscous, cells are small or rates are high
  - closed cell foams - compression of cell fluid
    - energy recovered on unloading

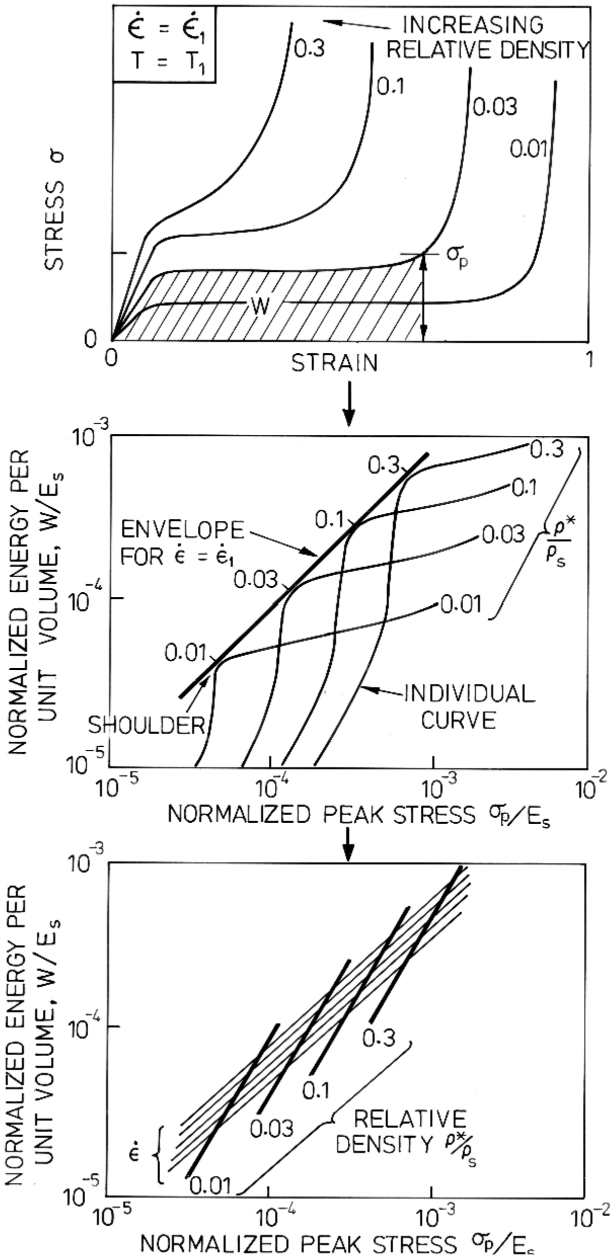
Energy absorption diagrams





- at stress plateau, energy W increases with little increase in peak stress, σ_p
- as foam densifies, W ~ constant & σ_p increases sharply
- ideally, want to be at "shoulder" point

- more generally - see fig
- test series of one type of foam of different ρ^*/ρ_s at constant $\dot{\epsilon}$ + Temp, T.
- plot W/E_s vs σ_p/E_s for each curve (E_s at standard $\dot{\epsilon}$ + T)
- heavy line joins the shoulder points for each curve
- mark ρ^*/ρ_s for each foam on that line
- repeat for varying $\dot{\epsilon}$ ⇒ join lines for constant ρ^*/ρ_s
- build up family of optimum energy absorption curves
- can treat different temperatures, T, in same way



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

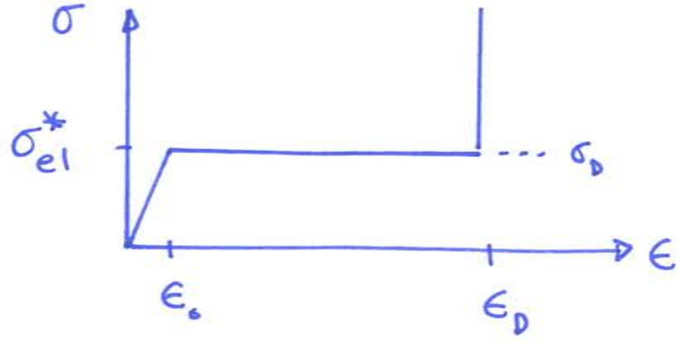
Notes:

- elastomeric foams can all be plotted on one curve since $E^* \propto E_s$ and $\sigma_{el}^* \propto E_s$ (normalize W/E_s & σ_p/E_s)
- figure: polyurethane + polyethylene
- polymethacrylimid: $\sigma_{pl}^* \Rightarrow$ typical of foams with plastic collapse stress with $\sigma_{ys}/E_s = 1/30$
- can generate energy absorption diagrams from data, or use models for foam properties

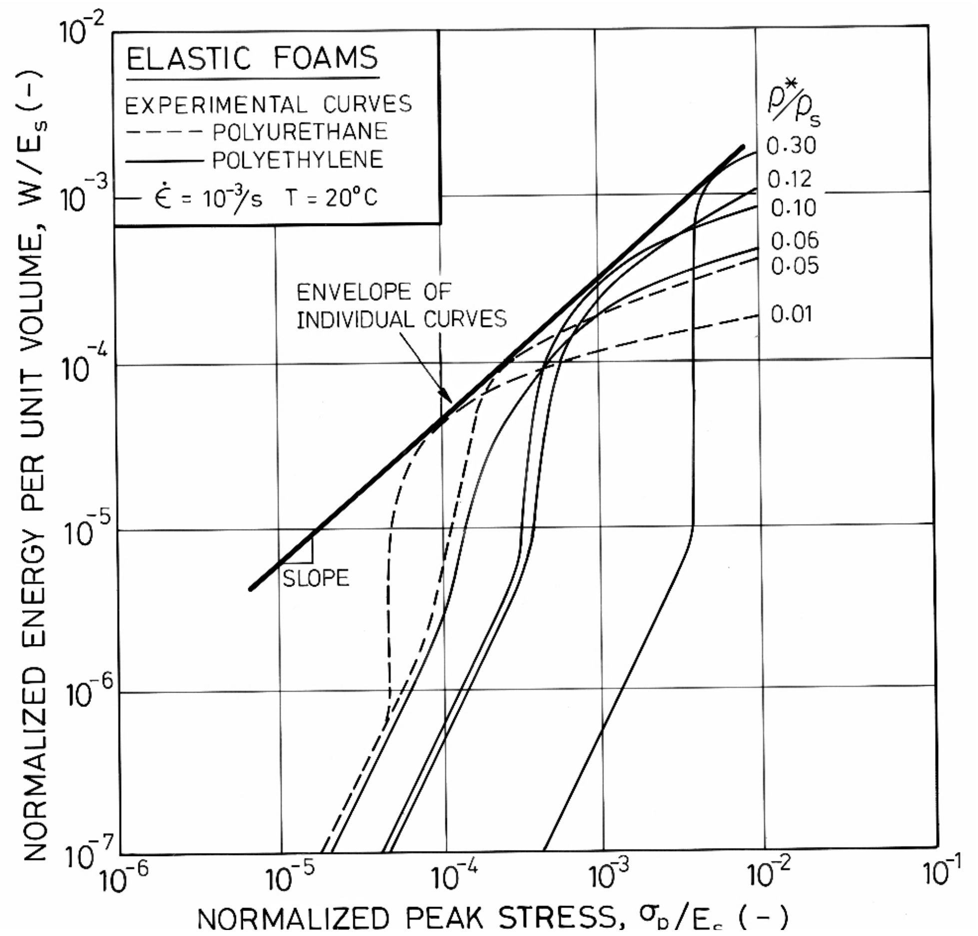
---

Modelling energy absorption diagrams

Open cell elastomeric foams

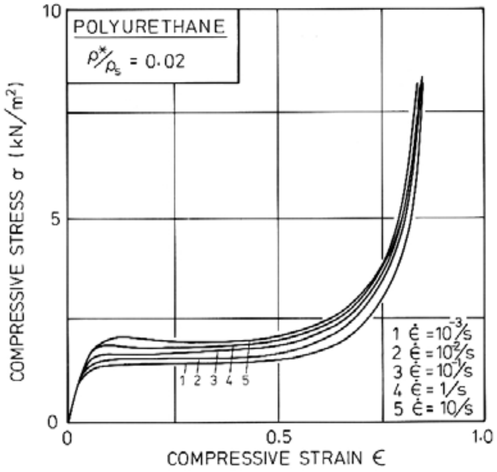


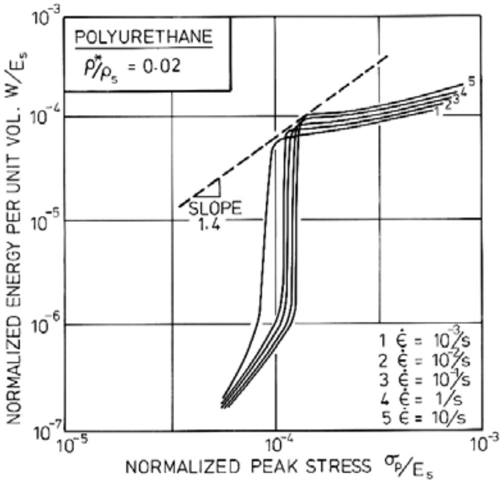
Elastomeric Foams

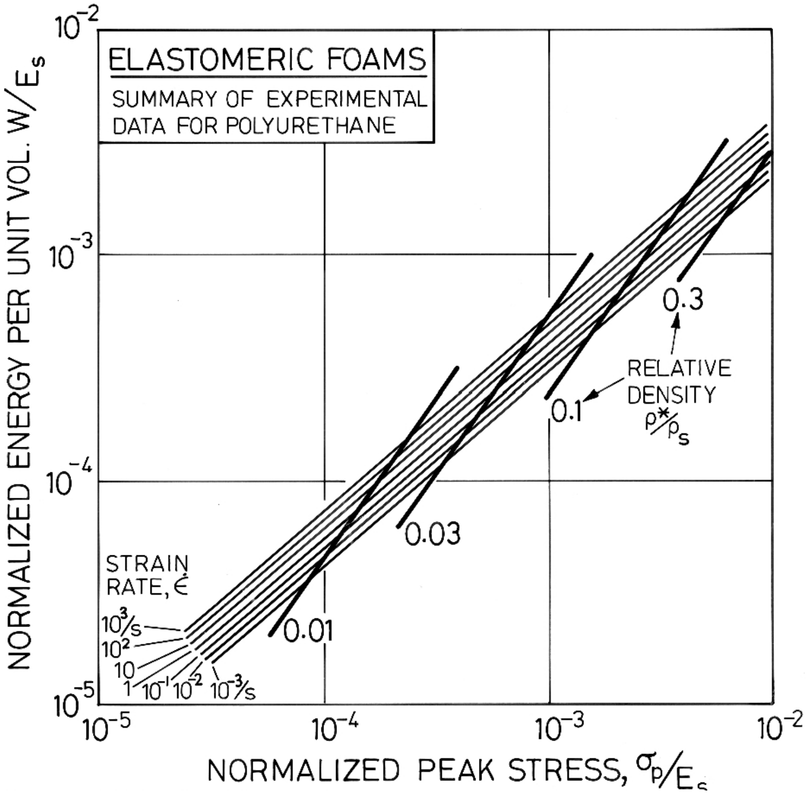


Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Flexible Polyurethane

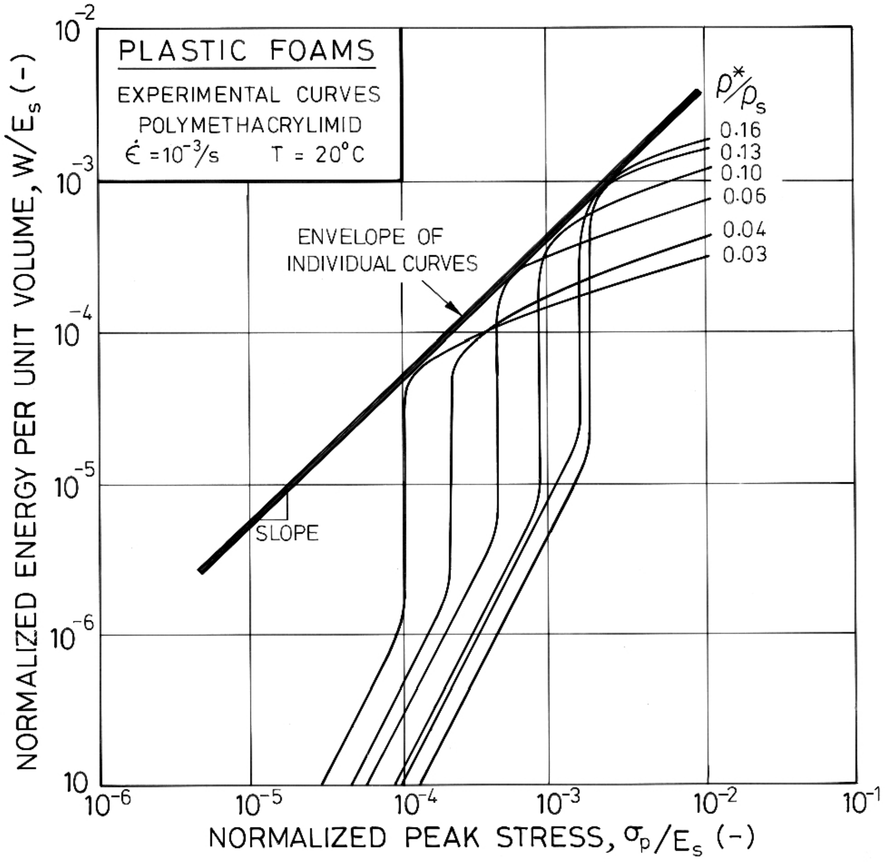


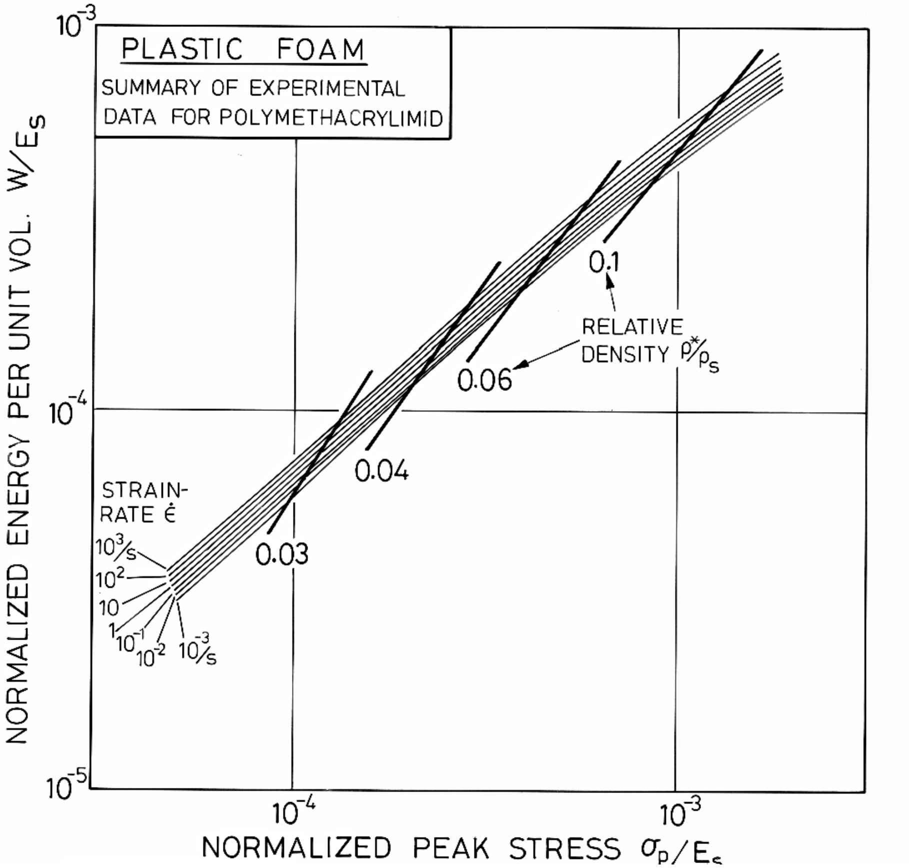




Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

Polymethacrylimid





Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

(a) linear elastic region $\epsilon < \epsilon_o$

$$W = \frac{1}{2}\frac{\sigma_p^2}{E^*} \qquad \frac{W}{E_s} = \frac{1}{2}\left(\frac{\sigma_p}{E_s}\right)^2 \frac{1}{(\rho^*/\rho_s)^2}$$

(b) stress plateau $\epsilon_o < \epsilon < \epsilon_D$

$$dW = \sigma^*_{el}\, d\epsilon \qquad \frac{W}{E_s} = 0.05\,(\rho^*/\rho_s)^2\,(\epsilon - \epsilon_o)$$

- family of vertical lines on figure
- plateau ends at densification strain ϵ_D
- then W/E_s vs. σ_p/E_s becomes horizontal

(c) at end of stress plateau $\epsilon \sim \epsilon_D$

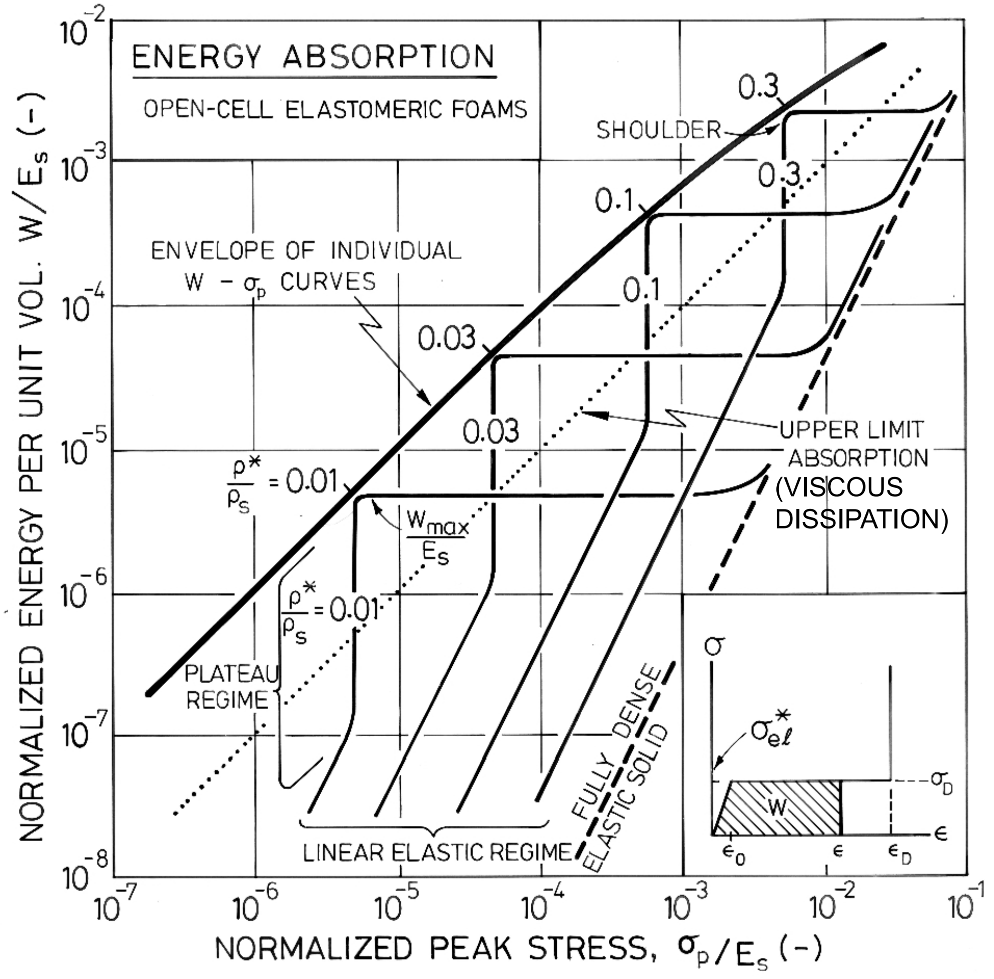
- maximum energy absorbed just before reach ϵ_D (shoulder point)

$$\frac{W_{max}}{E_s} = 0.05\,(\rho^*/\rho_s)^2\,(1 - 1.4\,\rho^*/\rho_s) \qquad \text{(assuming } \epsilon_o \ll \epsilon_D + \text{neglecting } \epsilon_o\text{)}$$

- optimum choice of foam is one with shoulder point that lies at $\sigma_p = \sigma_D$
- envelope of shoulder points, for optimum foams, at:

$$\sigma_p = \sigma_D = 0.05\,E_s\,(\rho^*/\rho_s)^2 \qquad \rho^*/\rho_s = \left(\frac{20\,\sigma_p}{E_s}\right)^{1/2}$$

Open-cell Elastomeric Foams: Modelling



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

substituting into eq'n for W_{max}/E_s:

$$\frac{W_{max}}{E_s} = \frac{\sigma_p}{E_s}\left[1 - 1.4\left(\frac{20\,\sigma_p}{E_s}\right)^{1/2}\right]$$

$$\frac{W_{max}}{E_s} = \frac{\sigma_p}{E_s}\left[1 - 6.26\left(\frac{\sigma_p}{E_s}\right)^{1/2}\right]$$

- line of slope 1 at low stresses, falling to 7/8 at high σ

(d) densification

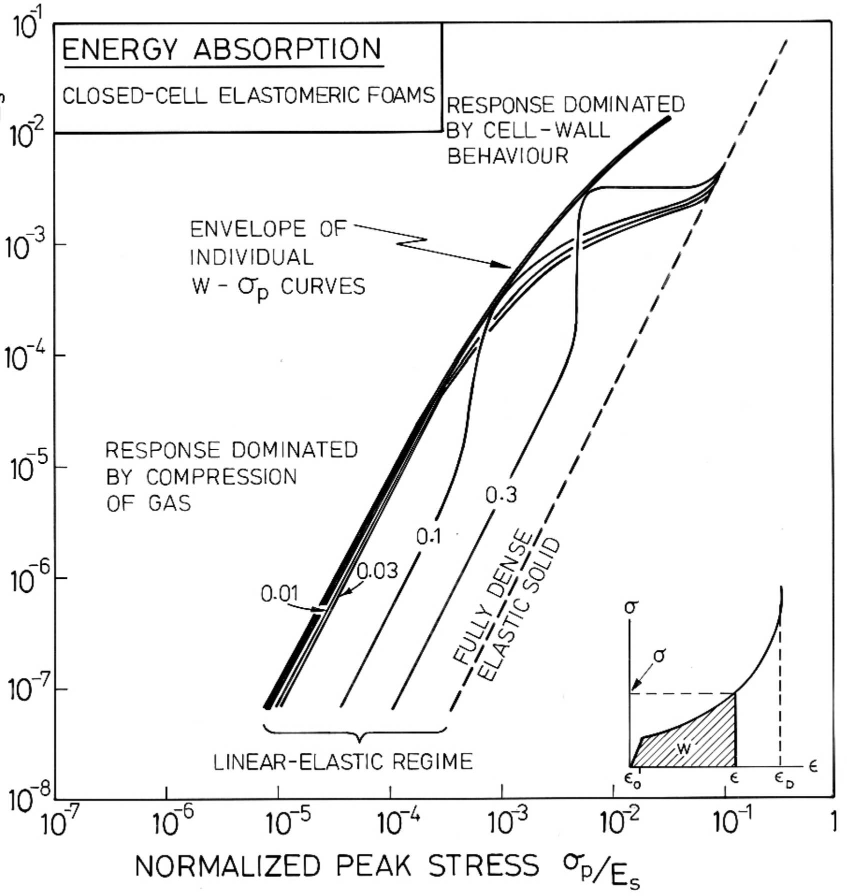
- when foam fully densified + compressed to a solid, then energy absorption curve joins that for the fully dense elastomer

$$\frac{W}{E_s} = \frac{1}{2}\frac{\sigma_p^2}{E_s}$$

Note:

- model curves have same shape as expts.
- model shows W/E_s depends on σ_p/E_s + ρ^*/ρ_s only - one diagram for all elastomer foams
- for a given W/E_s, σ_p/E_s for the foam less than that of the fully dense solid, by a factor of 10^{-3} to 10^{-1}

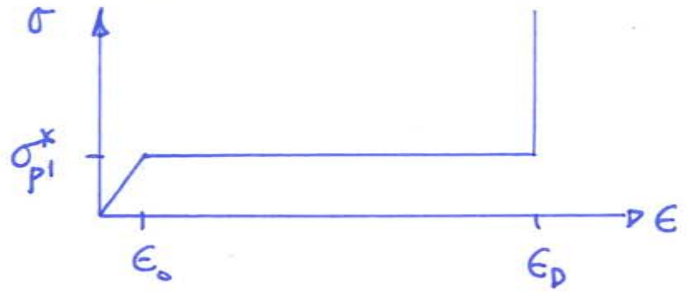
Closed-cell Elastomeric Foams: Modelling



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Modelling: open-cell foams that yield

- analysis similar to elastomeric foams, with σ^*_{pl} replacing σ^*_{el}
- note that some closed cell foams that yield, face contribution to E^* σ^*_{pl} negligible
- neglect fluid contribution



$$\sigma^*_{pl} = 0.3\, \sigma_{ys} (\rho^*/\rho_s)^{3/2}$$

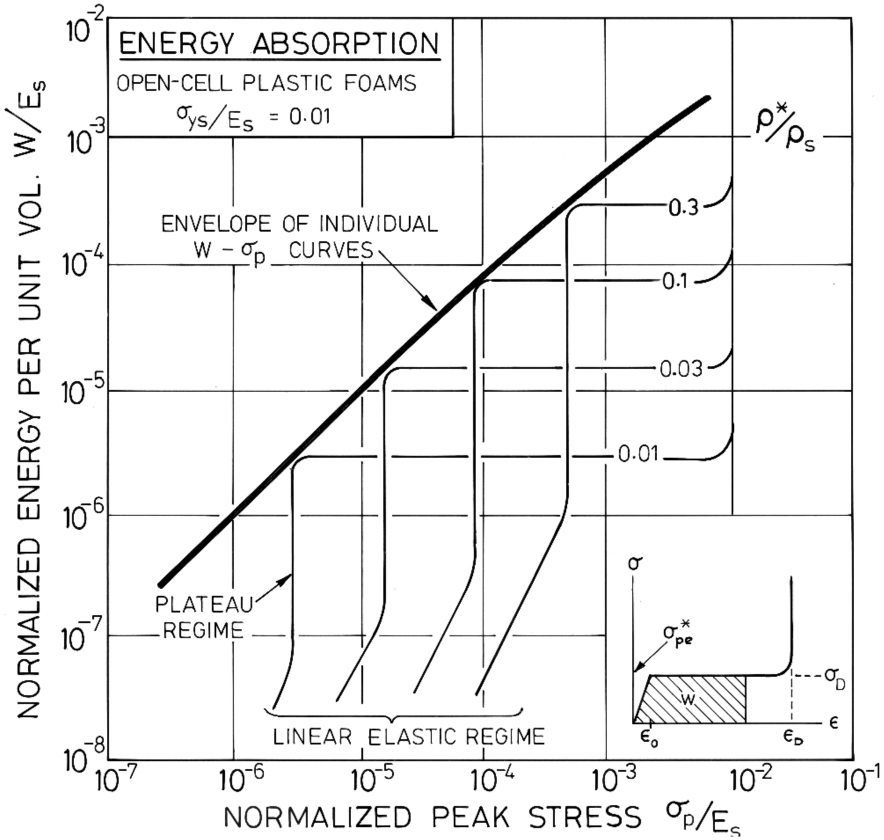
(a) linear elastic regime: same as for elastomeric foam: $\frac{W}{E_s} = \frac{1}{2}\left(\frac{\sigma_p}{E_s}\right)^2 \frac{1}{(\rho^*/\rho_s)^2}$

(b) stress plateau: $\frac{W}{E_s} = 0.3\, \frac{\sigma_{ys}}{E_s} \left(\frac{\rho^*}{\rho_s}\right)^{3/2} (\epsilon - \epsilon_0)$

(c) end of stress plateau: $\frac{W_{max}}{E_s} \approx 0.3\, \frac{\sigma_{ys}}{E_s} \left(\frac{\rho^*}{\rho_s}\right)^{3/2} (1 - 1.4\, \rho^*/\rho_s)$

- optimum choice of foam - absorbs maximum energy without σ_p rising sharply at ϵ_D

Plastic Foams: Modelling



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

- curve of optimum energy absorption (heavy line on figure) is envelope that touches W - σ_p curve at shoulder points
- for σ_p, $\rho^*/\rho_s = \left(\frac{3.3\,\sigma_p}{\sigma_{ys}}\right)^{2/3}$
- substituting in W_{max}/E_s eq'n:

$$\frac{W_{max}}{E_s} = \frac{\sigma_D}{E_s}\left\{1 - 3.1\left(\frac{\sigma_D}{\sigma_{ys}}\right)^{2/3}\right\}$$

- model curves explain general features of experimental curves
- modelling - curves less general than for elastomers
  - this curve for a particular value of $\sigma_{ys}/E_s = 1/100$ (typical value for polymers)

Design + selection of foams for impact protection

- typically know object to be protected + some details about it

mass, m

contact area, A

max drop height, h

(or energy to be absorbed, u

max allowable acceleration, a

(eg. head injury ~100g)

peak stress allowable, σ_p

- variables: foam material, density, thickness

Example 1

Given: mass, $m = 0.5\,kg$

contact area, $A = 0.01\,m^2$

drop height, $h = 1\,m$

max deceleration, $a = 10g$

foam: flexible polyurethane $E_s = 50\,MPa$

Find: optimum foam density

" " thickness

Example 1:
Find Foam Density and Thickness

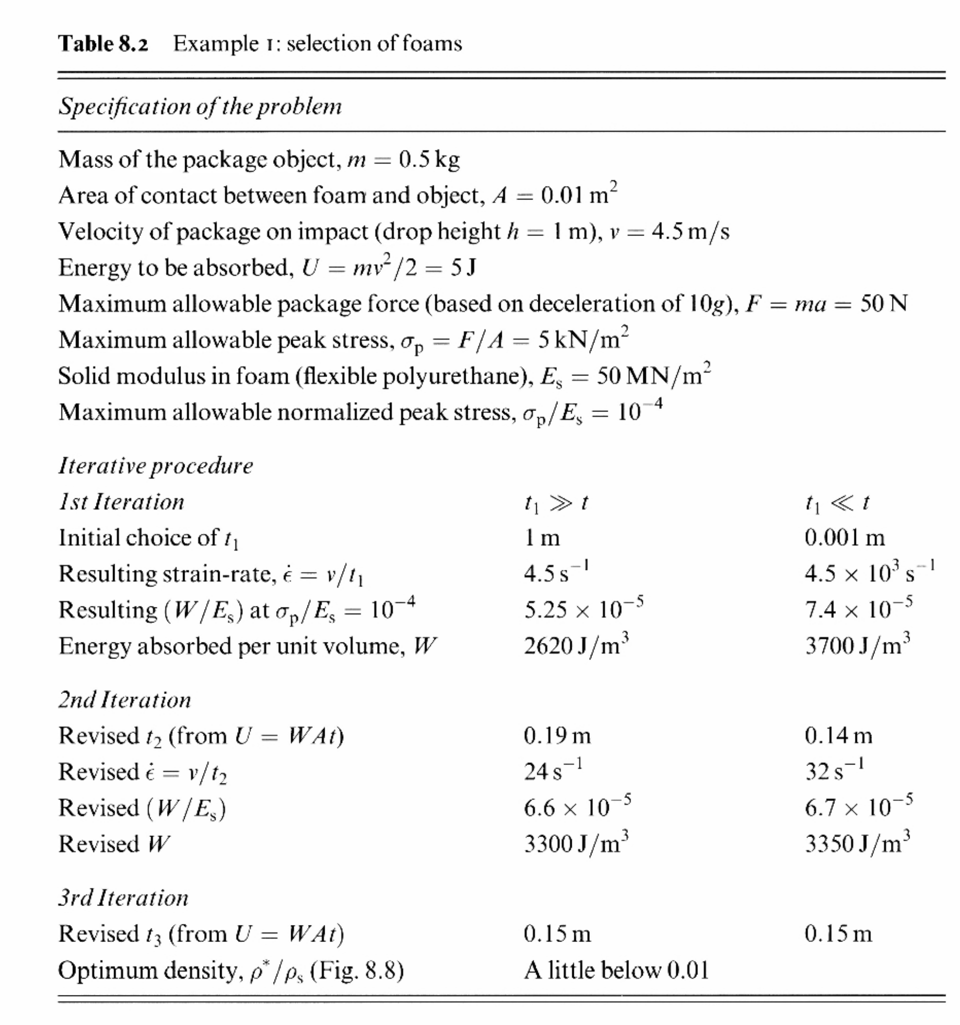
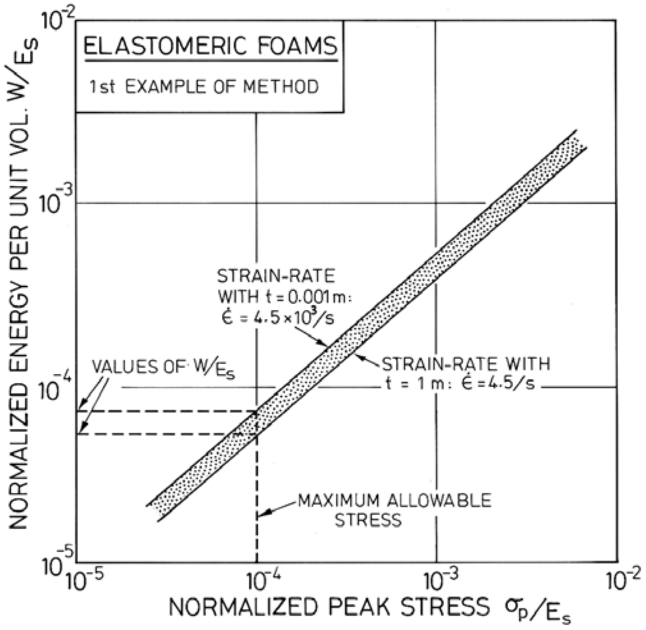


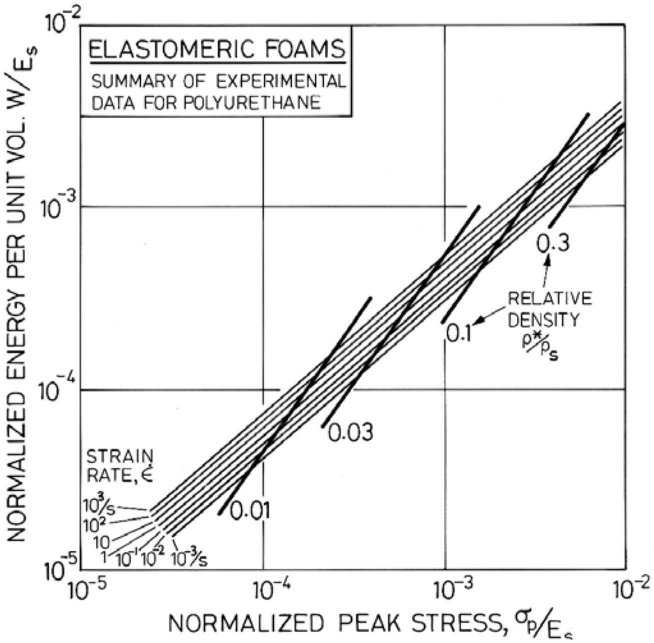
Table 8.2 Example 1: selection of foams

Specification of the problem

Mass of the package object, $m = 0.5\,\mathrm{kg}$
Area of contact between foam and object, $A = 0.01\,\mathrm{m}^2$
Velocity of package on impact (drop height $h = 1\,\mathrm{m}$), $v = 4.5\,\mathrm{m/s}$
Energy to be absorbed, $U = mv^2/2 = 5\,\mathrm{J}$
Maximum allowable package force (based on deceleration of $10g$), $F = ma = 50\,\mathrm{N}$
Maximum allowable peak stress, $\sigma_p = F/A = 5\,\mathrm{kN/m}^2$
Solid modulus in foam (flexible polyurethane), $E_s = 50\,\mathrm{MN/m}^2$
Maximum allowable normalized peak stress, $\sigma_p/E_s = 10^{-4}$

Iterative procedure		
1st Iteration	$t_1 \gg t$	$t_1 \ll t$
Initial choice of t_1	1 m	0.001 m
Resulting strain-rate, $\dot{\epsilon} = v/t_1$	$4.5\,\mathrm{s}^{-1}$	$4.5 \times 10^3\,\mathrm{s}^{-1}$
Resulting (W/E_s) at $\sigma_p/E_s = 10^{-4}$	5.25×10^{-5}	7.4×10^{-5}
Energy absorbed per unit volume, W	$2620\,\mathrm{J/m}^3$	$3700\,\mathrm{J/m}^3$
2nd Iteration		
Revised t_2 (from $U = WAt$)	0.19 m	0.14 m
Revised $\dot{\epsilon} = v/t_2$	$24\,\mathrm{s}^{-1}$	$32\,\mathrm{s}^{-1}$
Revised (W/E_s)	6.6×10^{-5}	6.7×10^{-5}
Revised W	$3300\,\mathrm{J/m}^3$	$3350\,\mathrm{J/m}^3$
3rd Iteration		
Revised t_3 (from $U = WAt$)	0.15 m	0.15 m
Optimum density, ρ^*/ρ_s (Fig. 8.8)	A little below 0.01	





Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

- energy to be absorbed, $u = mgh = (0.5\,kg)(10\,m/s^2)(1\,m) = 5\,J$
- max. allowable force on package $= F = ma = (0.5\,kg)(10\,g) = 50\,N$
- peak stress, $\sigma_p = F/A = 50\,N / 0.01\,m^2 = 5\,kN/m^2$
- normalized peak stress, $\sigma_p / E_s = 5\,kPa / 50\,MPa = 10^{-4}$
- draw vertical line on energy absorption diagram @ $\sigma_p / E_s = 10^{-4}$
- need to know $\dot{\epsilon} \approx v/t$ velocity $v = \sqrt{2gh} = 4.5\,m/s$
- iterative approach - choose arbitrary thickness, t

- e.g. $t_1 = 1\,m$

$\dot{\epsilon} = 4.5/s$

$W/E_s = 5.25 \times 10^{-5}$

$W = 2620\,J/m^3$

$(u = WAt)$ $t_2 = \frac{WA}{u} = 0.19\,m$

$\dot{\epsilon}_2 = 24/s$

$W/E_s = 6.6 \times 10^{-5}$

$W = 3300\,J/m^3$

$t_1 = 0.001\,m$

$\dot{\epsilon} = 4.5 \times 10^3/s$

$W/E_s = 7.4 \times 10^{-5}$

$W = 3700\,J/m^3$

$t_2 = 0.14\,m$

$\dot{\epsilon}_2 = 32/s$

$W/E_s = 6.7 \times 10^{-5}$

$W = 3350\,J/m^3$

Third iteration: $t_3 = 0.15\,m$ (both W).

optimum density (Fig).

$\rho^*/\rho_s \sim 0.01$.

Note: t converges quickly even from very different initial guesses for t

Example 2

Given $m = 2.5\,kg$ Find foam material

foam density

$A = 0.025\,m^2$

$t = 20\,mm$

$h = 1\,m$

$a = 100g$

Calculate W, σ_p, $\dot{\epsilon}$

$$W = \frac{mgh}{At} = \frac{(2.5kg)(10m/s^2)(1m)}{0.025\,m^2\ (0.02m)} = 5\times10^{-4}\ J/m^3$$

$$\sigma_p = \frac{F_{max}}{A} = \frac{ma}{A} = \frac{(2.5kg)(100)(10m/s^2)}{0.025\,m^2} = 10^5\ N/m^2$$

$$\dot{\epsilon} = \frac{v}{t} = \frac{\sqrt{2gh}}{t} = \frac{\sqrt{2(10m/s^2)(1m)}}{0.02\,m} = \frac{4.5\ m/s}{0.02\ m} = 225/s$$

Select arbitrary value of $E_s = 100\ MPa$

Plot $W/E_s = 5\times10^{-4}$ point A

$\sigma_p/E_s = 10^{-3}$

Example 2:
Find Foam Material and Density

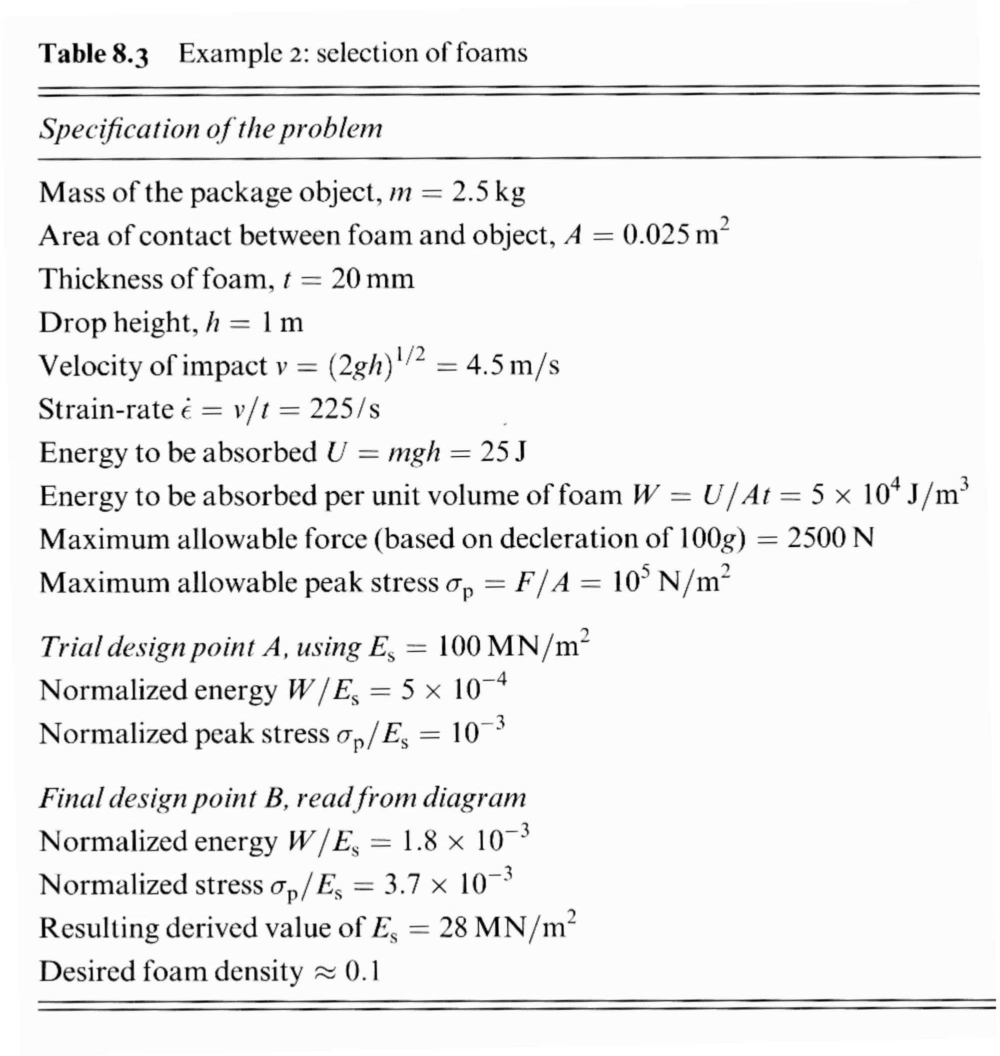


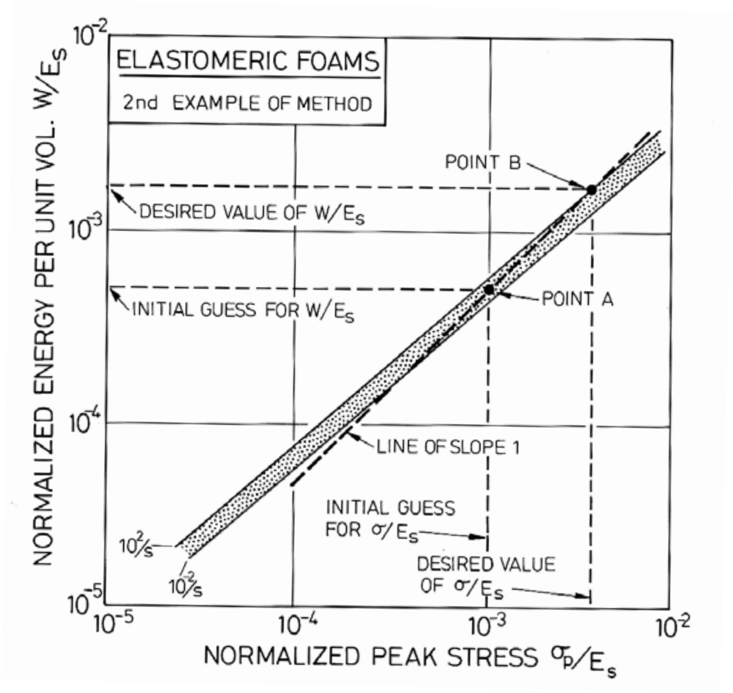
Table 8.3 Example 2: selection of foams

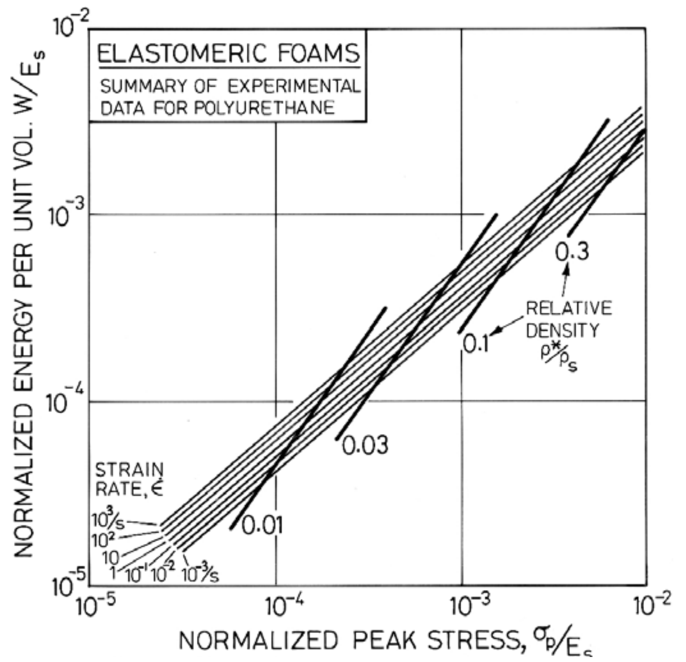
Specification of the problem

Mass of the package object, $m = 2.5\,\mathrm{kg}$
Area of contact between foam and object, $A = 0.025\,\mathrm{m}^2$
Thickness of foam, $t = 20\,\mathrm{mm}$
Drop height, $h = 1\,\mathrm{m}$
Velocity of impact $v = (2gh)^{1/2} = 4.5\,\mathrm{m/s}$
Strain-rate $\dot{\epsilon} = v/t = 225/\mathrm{s}$
Energy to be absorbed $U = mgh = 25\,\mathrm{J}$
Energy to be absorbed per unit volume of foam $W = U/At = 5 \times 10^4\,\mathrm{J/m}^3$
Maximum allowable force (based on decleration of $100g$) = 2500 N
Maximum allowable peak stress $\sigma_\mathrm{p} = F/A = 10^5\,\mathrm{N/m}^2$

Trial design point A, using $E_\mathrm{s} = 100\,\mathrm{MN/m}^2$
Normalized energy $W/E_\mathrm{s} = 5 \times 10^{-4}$
Normalized peak stress $\sigma_\mathrm{p}/E_\mathrm{s} = 10^{-3}$

Final design point B, read from diagram
Normalized energy $W/E_\mathrm{s} = 1.8 \times 10^{-3}$
Normalized stress $\sigma_\mathrm{p}/E_\mathrm{s} = 3.7 \times 10^{-3}$
Resulting derived value of $E_\mathrm{s} = 28\,\mathrm{MN/m}^2$
Desired foam density ≈ 0.1





Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

- construct a line of slope 1 through this point (broken line)
- moving along this line simply changes E_s
- select the point where the broken line intersects the appropriate $\dot{\varepsilon} \sim 10^2/s$ (point B)
- read off values of $W/E_s = 1.8 \times 10^{-3}$

  $\sigma_p/E_s = 37 \times 10^{-3}$
- resulting value of E_s = 28 MPa $\Rightarrow$ low modulus, flexible polyurethane

- replotting on more detailed figure : $\rho^*/\rho_s =$ 0.1

- if point A above all energy contours + line of slope 1 does not intersect them, specification cannot be achieved, A or t has to increase
- if point A below all contours, then A & t larger than need to be - can be reduced

Case study: design of car head rest

- head rest should absorb kinetic energy of head while keeping force less than that which would cause injury.
- example in book:

mass of head $= 2.5\,kg$

Max. deceleration $= a = 50g = 500\,m/s^2$

area of contact, $A = 0.01\,m^2$

thickness of padding $t = 0.17\,m$

max. allowable force $F = ma = 1250\,N$

" " stress $\sigma_p = F/A = 125\,kN/m^2$

energy to be absorbed /vol, $W = \frac{\frac{1}{2}mv^2}{At} = 735\,v^2\ J/m^3$

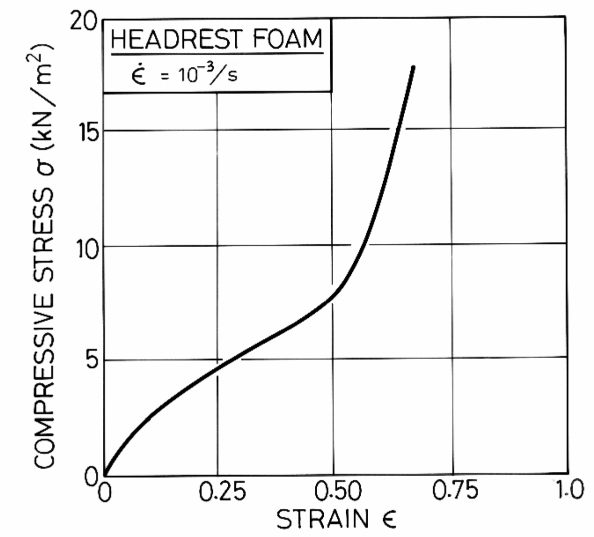
peak strain rate $\dot{\epsilon} = v/t\ [s^{-1}]$

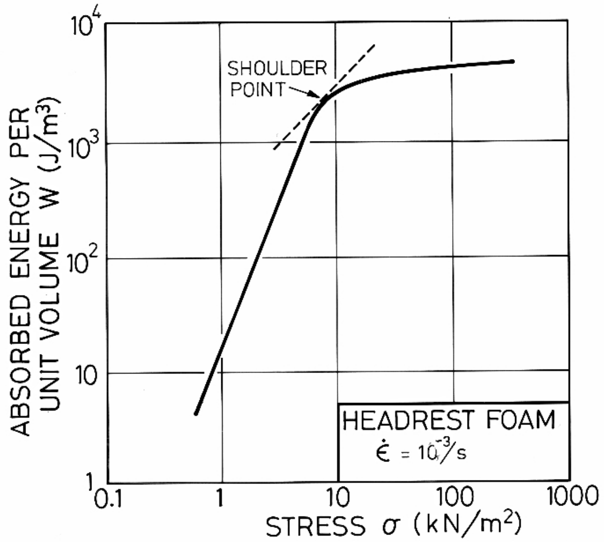
current material - flexible polyester foam $\rho^*/\rho_s = 0.06$

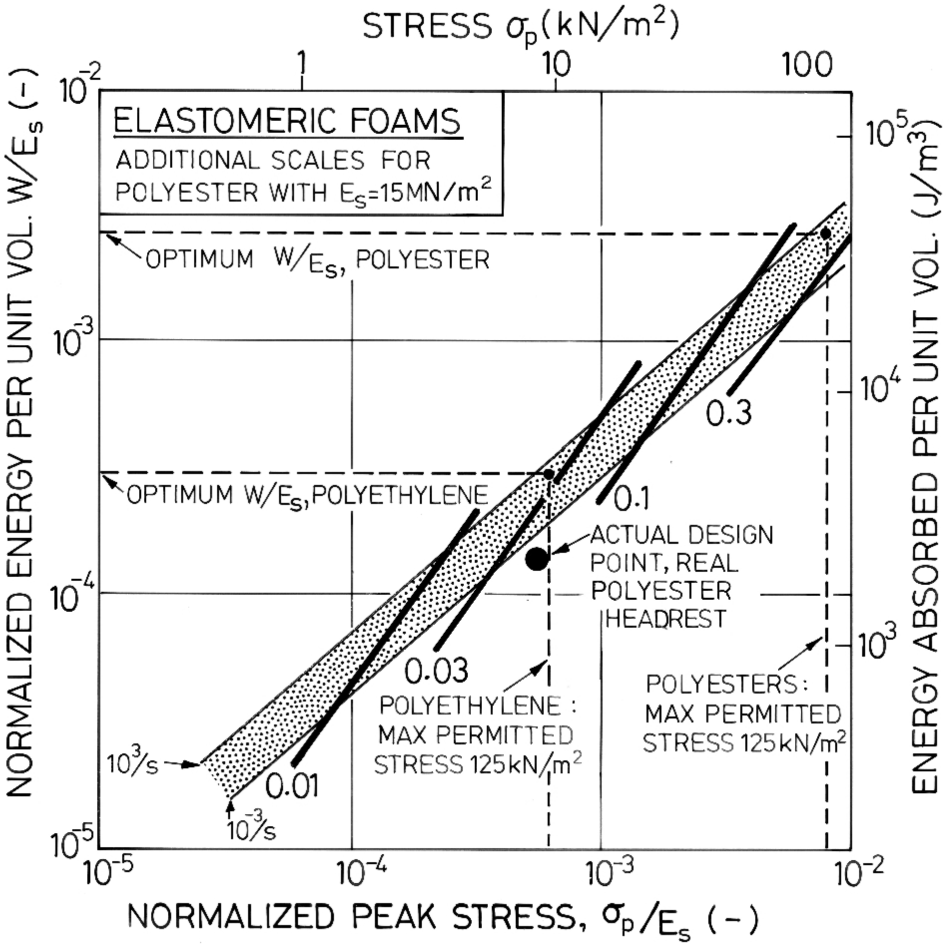
from plot: for $\sigma_p = 125\,kN/m^2$ $W = 5. \times 10^3\,J/m^3$

maximum collision velocity $= v = \sqrt{\frac{W}{735}} = \sqrt{\frac{5\times10^3}{735}} = 2.6\,m/s = 5.8\,mph$

Car Head Rest Design







Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

Alternative design #1

- consider en. abs. diag. for elastomeric foams
- add scales for polyester (using $E_s = 15\,MPa$)
- for $\sigma_p = 125\,kN/m^2$ could use polyester foam $\rho^*/\rho_s = 0.2$

  then $W/E_s = 2.6 \times 10^{-3}$ & $v = 7.3\,m/s = 16\,mph$

Alternative design #2

- use different material e.g. low density open cell polyethylene $E_s = 200\,MPa$

- $\sigma_p / E_s = \dfrac{0.125}{200} = 6.3 \times 10^{-4}$
- at $\dot{\epsilon} = v/t \approx 100/s$ (estimated)

  $W/E_s = 3.2 \times 10^{-4}$ (from fig).

  $W = (3.2 \times 10^{-4})(200\,MPa) = 6.4 \times 10^4\,J/m^3$

  $v = \sqrt{\dfrac{W}{735}} = \sqrt{\dfrac{6.4 \times 10^4}{735}} = 9.3\,m/s = 21\,mph$
- reading from figure: $\rho^*/\rho_s = 0.03$

Case study: foams for bicycle helmets

US: 600-700 bicycle deaths/yr (US Nat. Hwy Traffic Safety Admin
$>90\%$ not wearing a helmet Bicycle Helmet Safety Inst.)
~50,000 cyclists injured (2009)

- helmets consist of solid outer shell + foam liner (eg. expanded PS)
- liner thickness typically 20 mm
- wish to absorb as much energy as possible while keeping peak acc'n less than that to cause head injury

- foam liner
    - redistributes load over larger area, reducing stress on head
    - peak stress on head limited by plateau stress of foam (as long as don't reach densification)
    - max. tolerable acc'n = 300 g (if for a few milliseconds)
    - mass of head ≃ 3 kg

$$F_{max} = ma = (3\,kg)(300)(10\,m/s^2) = 9\,kN$$

- as foam crushes, it distributes load over area ~ A ~ 0.01 m² (may be high)

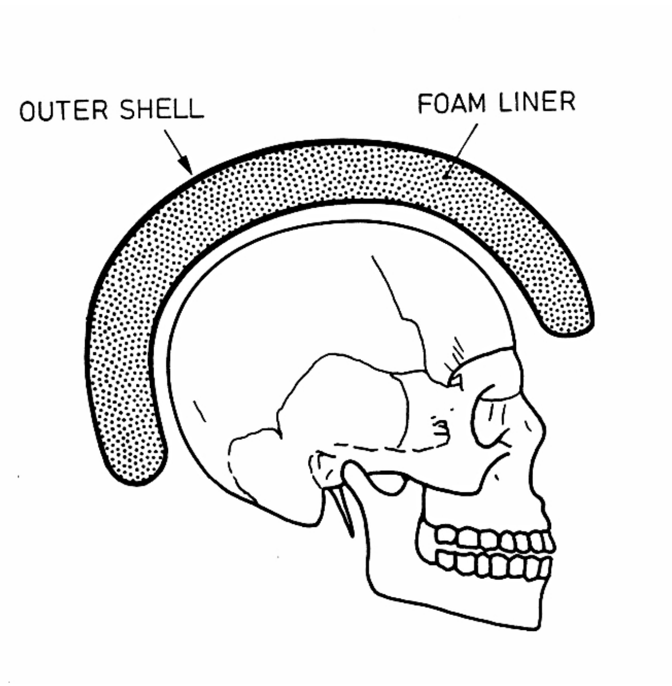
$$\sigma_p = \frac{9\,kN}{0.01\,m^2} = 0.9\,MPa$$

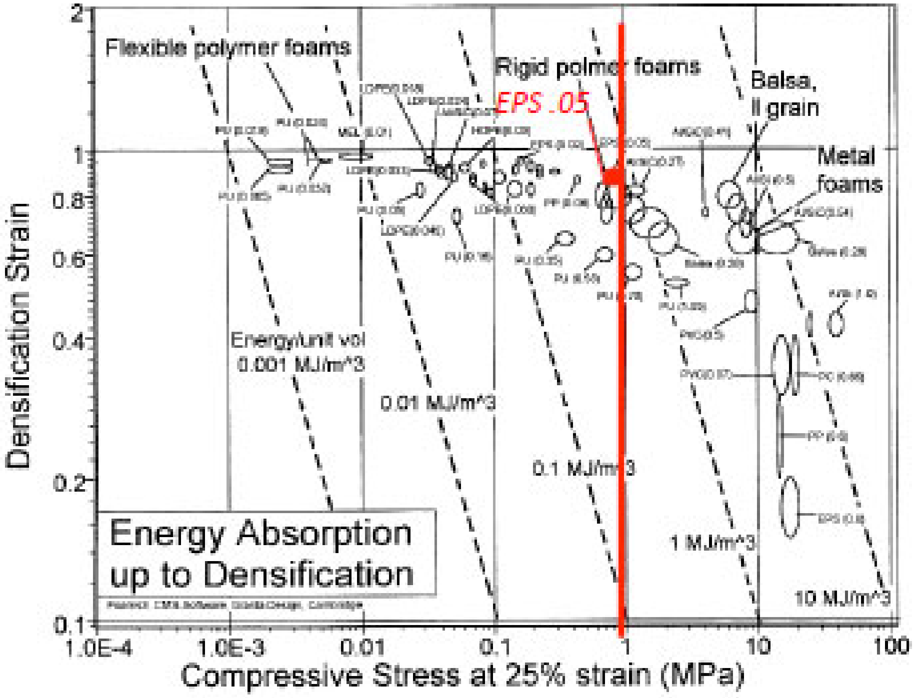
figure → EPS $\rho^* = 0.05\ \mathrm{Mg/m^3}$

absorbs $W = 0.8\ \mathrm{MJ/m^3}$

- diagram allows easy identification of possible candidate materials
- more complete analysis can then be done
- energy absorbed $U = 0.8 \times 10^6 \frac{J}{m^3} \times 0.01\ m^2 \times 0.02\ m = 160\ J$ $(U = WAt)$
- $\frac{1}{2} m v^2 = U$; $v_{max} = \sqrt{\frac{2U}{m}} = \sqrt{\frac{2\ (160\ \cancel{J})}{3\ kg}\ kg\ \frac{m^2}{s^2}} = 10\ m/s \approx 22\ mph.$

Case Study:
Foams for Bicycle Helmets





Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

3.054 / 3.36 Cellular Solids: Structure, Properties and Applications
Spring 2015