Supplement Lecture #10 03/13/07

Integral 1:

$$\int_{x=0}^{x=\infty} \frac{x}{\left[z^2+x^2\right]^{n/2}}\,dx = \frac{1}{(2-n)z^{n-2}}$$

$$u = z^2 + x^2$$

$$du = 2xdx$$

$$\int_{x=0}^{\infty} \frac{x}{(z^2+x^2)^{n/2}}\,dx = \int_{u=z^2}^{\infty} \frac{1}{2u^{n/2}}\,du$$

$$= \frac{1}{2}\left(\frac{1}{(1-n/2)u^{n/2-1}}\right)\Bigg|_{u=z^2}^{\infty}$$

$$= \frac{1}{(2-n)z^{n-2}}$$

Integral 2:

$$\int_{z=D}^{z=\infty} \frac{1}{z^{n-2}}\,dz = \int_{z=D}^{z=\infty} z^{2-n}\,dz = \frac{z^{3-n}}{(3-n)}\Bigg|_{z=D}^{z=\infty} = \frac{1}{(3-n)z^{n-3}}\Bigg|_{z=D}^{z=\infty}$$

$$= \frac{1}{(3-n)D^{n-3}} = -\frac{1}{(n-3)D^{n-3}}$$

Integral 3:

$$\int_{z=0}^{z=2R} \frac{\left(2R-z\right)z}{\left[D+z\right]^{n-3}}\,dz$$

$$For\ D << R: 2R\int_{z=0}^{z=\infty} \frac{z}{\left[D+z\right]^{n-3}}\,dz = \frac{2R}{(n-4)(n-5)D^{n-5}}$$

$$For\ D >> R: \int_{z=0}^{z=2R} \frac{\left(2R-z\right)z}{D^{n-3}}\,dz = \frac{4R^3}{3D^{n-3}}$$