Department of Materials Science and Engineering
Massachusetts Institute of Technology
3.044 Materials Processing – Spring 2013

Exam II

Wednesday, May 1, 2013

The Rules:

1) No books allowed; no computers allowed; etc.

2) A simple calculator is allowed

3) Two hand written 3x5 index cards may be prepared as a crutch.

4) Complete 5 out of the 6 problems. If you do more than 5 problems, I will grade the first 5 that are not crossed out.

5) Make sure that you READ THE QUESTIONS CAREFULLY

6) Supplementary materials are attached to the end of the test (eqns., etc.)

7) WRITE YOUR NAME HERE:

Solutions

$$\nu = \frac{\mu}{\rho}$$

Problem #1: Multiple Driving Forces Conspire to Print out a Film of Fluid
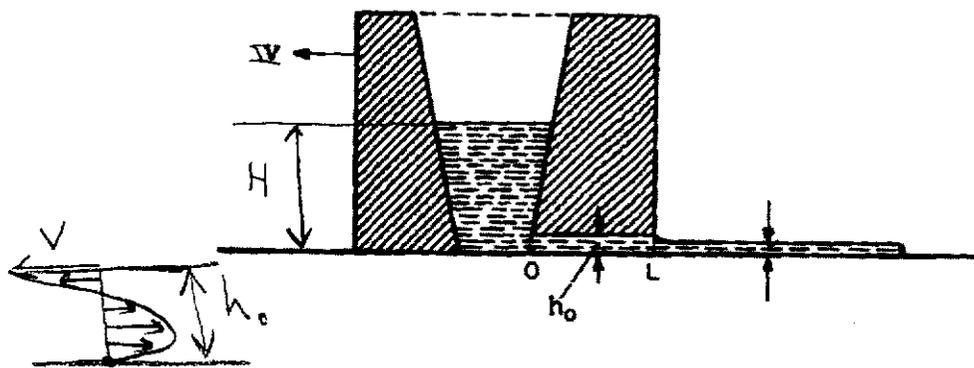
Consider a fluid printing process comprising both pressure and applied velocity. Examine the diagram at the left, which shows how the process works. Fluid is placed into a vessel with a narrow channel releasing it onto a substrate. The whole process is at constant temperature, and the vessel slides across the surface towards the left to emit fluid out the lower right as shown.

We are going to set up a fluid flow model for the channel region (which is the region of thickness h_o in the picture, and which you may assume is thick into the page.) The fluid flow will be assumed to be a laminar, Newtonian 1-D problem in which flow is along the x-axis and momentum transfer is along y.

Part A) In the channel region, both pressure and applied velocity drive flow. Explain, using one sentence each, where these driving forces come from.

PRESSURE: THE HEAD PRESSURE FROM THE HEIGHT OF THE FLUID IN THE RESEVOIR, $\Delta P = \rho g H$, OVER THE DISTANCE $0 \to L$.

APPLIED VELOCITY: THE TOP SURFACE OF THE CHANNEL MOVES RELATIVE TO THE LOWER SURFACE. THE MOTION AT VELOCITY V PROVIDES A SOURCE OF MOMENTUM TO DRIVE

Part B) Draw the channel region, apply a coordinate system to it, and then specify boundary FLOW. conditions needed to solve the problem.
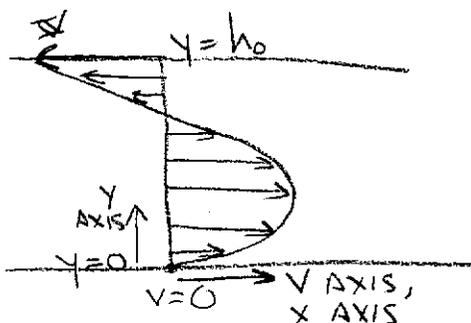
2 BOUNDARY CONDITIONS:

① AT $y = 0$, $V = 0$

② AT $y = h_o$, $V = \underline{V}$

$$\boxed{V_x = \frac{\rho g H}{2\mu L}\left(h_o y - y^2\right) + \frac{\underline{V}}{h_o} y}\ \text{SOLUTION} \Leftarrow$$

Part C) Solve for the flow profile, assuming that the process has reached steady-state and that flow is laminar. (You may try to call on an analogy to provide a solution, but that may prove more challenging than simply solving it).

NAVIER-STOKES:

$$\frac{\partial(\rho V_x)}{\partial t} = \mu \frac{\partial^2 V_x}{\partial y^2} - \frac{(0 - \rho g H)}{(L - 0)} \overset{\Delta P}{\underset{\Delta L}{\Big\downarrow}}$$

$$\frac{\partial V_x}{\partial t} = \frac{\mu}{\rho}\frac{\partial^2 V_x}{\partial y^2} + \frac{\rho g H}{L}$$

STEADY-STATE: $\overset{0}{\frac{\partial V_x}{\partial t}}$

$$\frac{\partial^2 V_x}{\partial y^2} = -\frac{\rho g H}{\mu L}$$

SEPARATE VARS, INTEGRATE

$$\frac{\partial V_x}{\partial y} = -\frac{\rho g H}{\mu L} y + A$$

INTEGRATE

$$V_x = -\frac{\rho g H}{\mu L}\frac{y^2}{2} + Ay + B$$

APPLY ① $\Rightarrow B = 0$.

APPLY ② $\Rightarrow \underline{V} = -\frac{\rho g H}{2\mu L} h_o^2 + A h_o$

$$A = \frac{\underline{V}}{h_o} + \frac{\rho g H h_o}{2\mu L}$$

Problem #2: A Single Question That Addresses Virtually Every Possible Heat Transfer Situation of Relevance to Casting

We are casting a liquid into a mold with a very large cavity of dimension L, which has a finite wall thickness (< L), is not cooled internally, and which is sitting in ambient air.

Part A) Draw a picture of this casting situation along with a temperature profile at a general, arbitrary time after casting, with no additional assumptions.

Part B) There are five possible rate-limiting heat transfer processes in this scenario. Identify them all. For each of these, consider what would happen if it were the controlling process, and indicate how the solidification time should scale with the size of the mold, L.
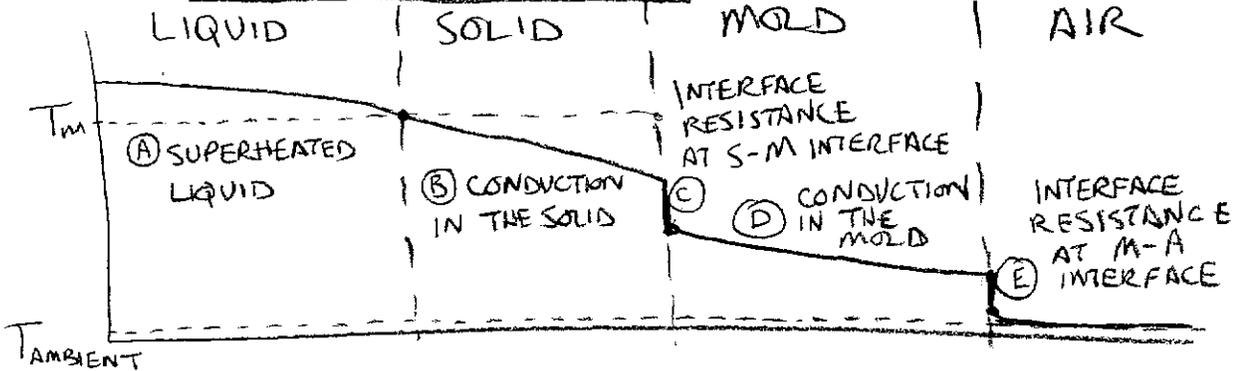
(A) SUPERHEATED LIQUID: IF CONTROLLING, THEN HEAT CONDUCTION THROUGH THE LIQUID IS LIMITING, SO $t_s \propto L^2$

(B) CONDUCTION THROUGH SOLID: IF CONTROLLING, CONDUCTION IS LIMITING, SO $t_s \propto L^2$ BECAUSE THE RATE OF HEAT FLOW IS PROPORTIONAL TO THE TEMPERATURE GRADIENT.

(C) INTERFACE RESISTANCE AT SOLID—MOLD INTERFACE: THE HEAT FLOW IS INDEPENDANT OF GRADIENTS, SO $t_s \propto L$

(D) CONDUCTION IN MOLD: HEAT ACCUMULATES IN THE MOLD WITH TIME, SO $t_s \propto L^2$

(E) INTERFACE RESISTANCE AT MOLD—AIR INTERFACE: LIKE CASE C, $t_s \propto L$

Problem #3: A Casting Microstructure is Analyzed by the Student

Here is a schematic cross-section of a solidified casting of a binary *aluminum-rich* alloy, along with the phase diagram that goes with it.
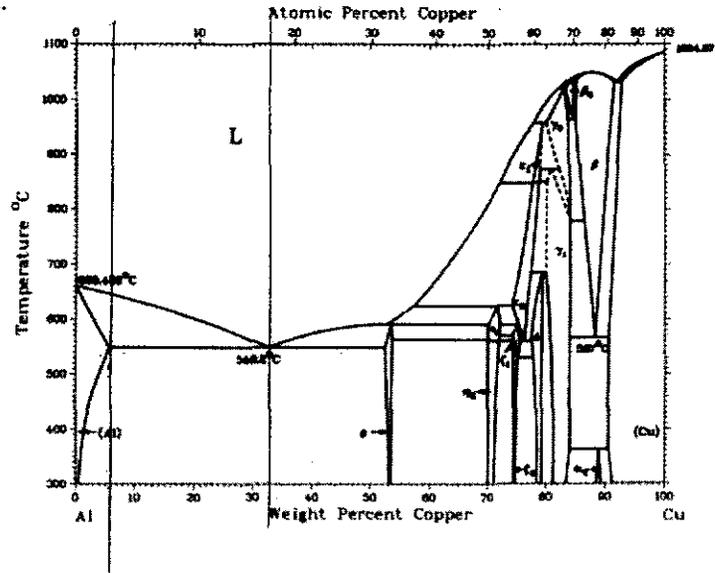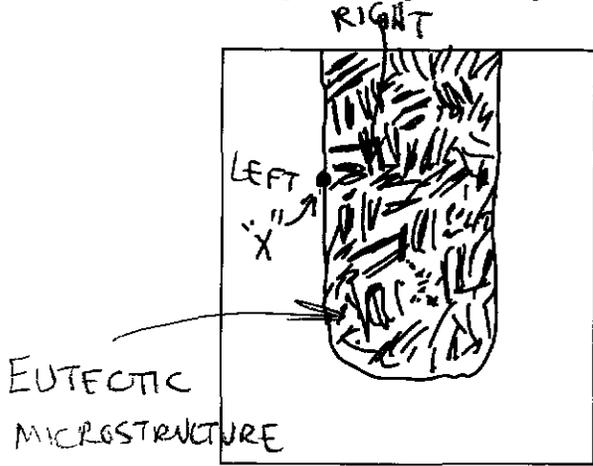
Part A:

By inspecting these exhibits, we can conclude that in this casting, the diffusion must have been rapid in the liquid. Explain why.

THERE ARE NO DENDRITES, SO THE LIQUID MUST HAVE HAD A UNIFORM COMPOSITION THROUGHOUT SOLIDIFICIATION.

Part B:

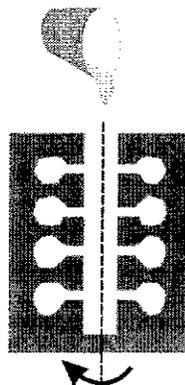What is the composition of the alloy just to the right of position 'X', and just to the left of 'X'?

TO THE LEFT: COMPOSITION IS THAT OF THE SOLIDUS AT THE EUTECTIC TEMP., $C_s = 6$ WT% Cu

TO THE RIGHT: COMPOSITION IS THAT OF THE LIQUIDUS AT THE EUTECTIC TEMP., $C_\ell = 33$ WT% Cu

Problem #4: The Student Recalls that Casting also Involves Fluid Flow
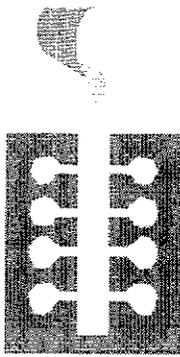
1

2

Consider two investment casting processes, which we shall label 1 and 2:

In process 1, we are using gravity-based pouring of a molten metal to create an investment casting. The mold is a tree of small components, each of which is connected to the central sprue by a small runner tube.

In process 2, the pouring is accompanied by rapid centrifugation.

For each of the following parts, write NO MORE THAN TWO SENTENCES.

Part A:

The production staff working with process 1 have noticed that part quality (especially porosity) seems to depend on a part's position in the tree. Specifically, the parts on the bottom are usually less porous than those on the top. Why?

2

SOLIDIFICATION PROCEDES FROM THE BOTTOM UP, BOTTOM IS IMPURITY-POOR, TOP IS IMPURITY-RICH. POROSITY IS CAUSED BY IMPURITIES. BOTTOM EXPERIENCES HIGHER HEAD PRESSURE, SO BUBBLES RISE OUT.

Part B:

In process 1, what is the kinetically limiting part of the process (i.e., where is fluid flow the slowest) and why?

2

$\leftarrow V \propto \Delta P/L$

THE DRIVING FORCE FOR FLOW IS GRAVITY ONLY, SO AT THE TOP OF THE TREE, LESS PRESSURE HEAD DRIVES MELT IN. THUS, FILLING THE TOP OF THE TREE IS KINETICALLY LIMITING.

Part C:

In light of your answers to Parts A and B, explain how process 2 might lead to better uniformity of parts located differently in the tree.

1

CENTRIFUGATION DRIVES FLUID INTO ALL CAVITIES, OVERCOMING THE KINETIC LIMITATIONS IDENTIFIED IN (B), AND SOLIDIFICATION WILL NOW PROCEDE RADIALLY INWARD STATING IN THE OUTERMOST PORTION OF THE PARTS, OVERCOMING THE LIMITATIONS IDENTIFIED IN (A), DRIVING THE IMPURITIES TO THE CENTRAL SPRUE, WHICH IS "WASTE" ANYWAY.

Problem #5: Blowing Hot and Cold

Below are five scenarios in shape forming. In each case, there is a question about how a change in temperature can impact the process. For each, indicate whether an increase or a decrease is the required change, and explain in ONE SENTENCE why this is the case.

A) A sheet being stretch-formed is developing a local thin region that leads to a rupture. The engineer in charge would like to introduce a local air jet that would either heat or cool the material near the thinning region.

INCREASE TEMPERATURE OF THE SHEET → MORE NEWTONIAN-LIKE BEHAVIOR → STABLE AGAINST RUPTURE ↑ M#

OR DECREASE TEMP LOCALLY WHERE RUPTURE OCCURS TO DECREASE STRAIN RATE / FREEZE PRIOR TO RUPTURE

B) Aluminum is being shape formed at a stress of 1 MN/m^2 and 300° C; the ductility is insufficient to form the desired part geometry. A deformation mechanism map is provided for this material.

DECREASE TEMPERATURE → STAY IN DIFFUSIONAL CREEP REGIME → 300°C m IS 1 → INCREASED DUCTILITY
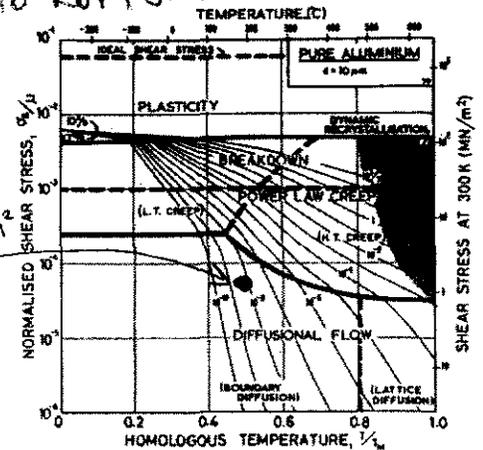
C) A thermoplastic is being continuously film-blown into a large tube. The plastic is coming out cloudy.

CLOUDY = PRESENCE OF CRYSTALLIZATION IN THE PLASTIC. DECREASE TEMPERATURE → STAY BELOW T_g

D) Glass is being poured continuously onto a bed of molten tin. The waves created on pouring are not damping fast enough, and the sheet is coming out non-planar.
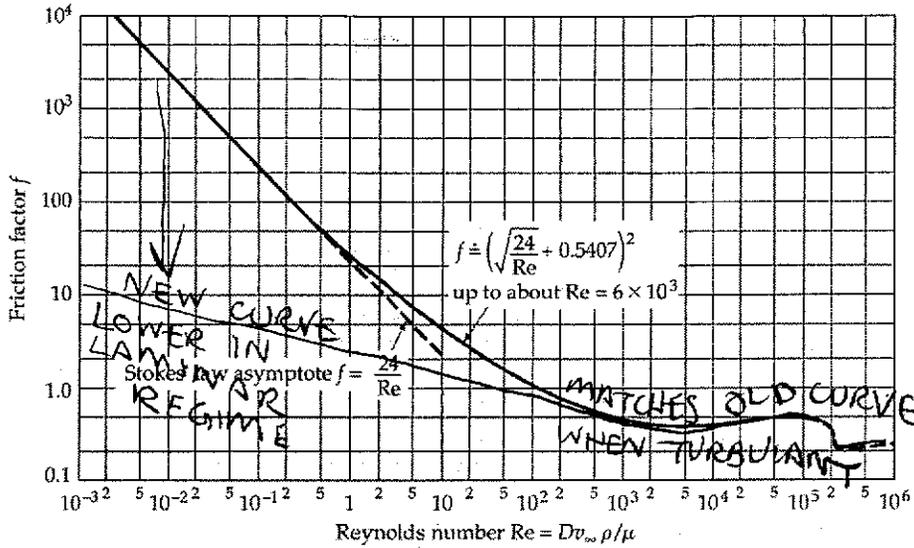
INCREASE TEMPERATURE → DECREASE VISCOSITY OF GLASS → TIME TO REACH STEADY-STATE DECREASES, OSCILLATIONS DAMPED FROM EQN SHEET: $\frac{A}{A_o} = \exp\left(-\frac{\rho g \lambda t}{4\pi \mu}\right)$

E) A melt is being sprayed into an atomization chamber to create powders. These are coming out irregularly shaped instead of spherical.

INCREASE TEMPERATURE → INCREASE TIME TO SOLIDIFICATION → POWDER PARTICLES HAVE TIME TO SHAPE-EQUILIBRATE DUE TO THE CAPILLARY FORCE BEFORE FREEZING → THEY COME OUT SPHERICAL

Problem #6: What a Drag

The graph below indicates how the friction factor (proportional to the drag force) on a spherical particle in a flow changes with the Reynolds number of the flow.

Graph: Friction factor f vs Reynolds number $\text{Re} = Dv_\infty \rho/\mu$

$$f \doteq \left(\sqrt{\frac{24}{\text{Re}}} + 0.5407\right)^2$$
up to about $\text{Re} = 6 \times 10^3$

Stokes' law asymptote $f = \dfrac{24}{\text{Re}}$

(Handwritten annotations: "NEW CURVE", "LOWER LAW IN REGIME", "MATCHES OLD CURVE WHEN TURBULANT")

Now consider a particle of a different shape: a thin disk, which is oriented such that its face is into the flow.

Briefly describe how the friction factor might be different, in both the laminar and turbulent regimes. Draw a schematic curve on the graph above that is consistent with your written comments.

↳ LAMINAR: $f_{\text{FRICTION}} = \dfrac{24}{\text{Re}}$ FOR A SPHERE, BUT THE EDGES OF THE DISK ARE THIN, SO THERE IS NOTHING FOR THE FLOW TO GRIP ON TO, SO f BECOMES VERY SMALL IN THE LAMINAR CASE AS $f_{\text{FORM}} \approx 0$.

(label: RELEVANT AREA FOR MOMENTUM TRANSFER)

↳ TURBULANT: $f_{\text{FORM}} \cong f$ AND $f_{\text{FRICTION}} \approx 0$. THE CROSS-SECTION IS IDENTICAL TO THAT OF A SPHERE, SO f IS UNCHANGED FOR THE TURBULANT CASE.

(label: RELEVANT AREA FOR MOMENTUM TRANSFER)

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