DO NOT TURN THIS PAGE OVER AND START THE QUIZ UNTIL YOU ARE ASKED TO DO SO.

Guidelines:

1. Show all your work on the sheets included in this stapled document.

2. Use partial credit to your advantage. If you're running short on time, solve algebraically and then solve numerically (plugging in numbers) later.

3. If there is not much space given for you to provide an answer, we want you to be brief.

4. You may not need to use all the information given (e.g., dimensions) to reach your conclusions.

5. Enjoy your **dark chocolate truffle** before getting started! It lowers blood pressure (*JAMA*, 2003), contains antioxidants (*Nature*, 2003), and supposedly helps neural synapses responsible for memory to fire faster (*Nature Health*, 2003).

NAME (PRINTED):

I agree that this document represents my own independent work on this quiz, using only my own brain, my allowed single crib sheet of equations and notes, and my pen / pencil / calculator / protractor / compass / ruler / sliderule.

SIGNATURE:

GOOD LUCK!

1. I am often reminded, and with good reason, that MIT students should *not* be treated like "brains on sticks". Fair enough, but let's consider for a moment the mechanical consequences of this mental picture.

   My first impulse is to wonder what such a stick should be made of, so that it should not buckle under the weight of a human brain (mass = 1.4 kg). Let's assume the stick is stuck firmly in the ground (clamped), is 180 cm in height (approximately 6'), and is cylindrical in cross-section with the radius of a rat femur (1.9 mm; Horcajada et al., J. Endocrin. 165: 2000. Strangely, much easier to find documented rat femur radii than human femur radii.)

   (a) What are the magnitudes of all the reaction forces and moments at the bottom of the brain-stick? Be sure to include even those reaction forces and moments that are equal to zero, and state as such.
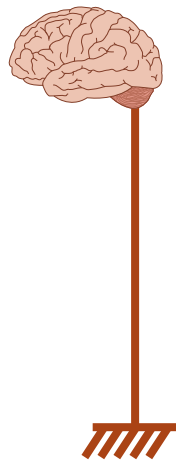

   Figure by MIT OpenCourseWare. Adapted from http://upload.wikimedia.org/wikipedia /commons/6/6d/Brain_stem_normal_human.svg. Courtesy of Patrick J. Lynch, medical illustrator and C. Carl Jaffe, MD, cardiologist.

   (b) What is the minimum Young's elastic modulus E that this brain-stick material must exhibit so that it remains stable under the prodigious weight of the typical brain?

   (c) Given this requirement, what material do you suggest the brain-stick comprise? There are several possible answers, and yours should be well justified by your data and the intended application. Material cost for the brain-stick is not a constraint, as brains are invaluable.

(d) Motivated by the aesthetics of the human body, I decide to make these sticks of 7075 T6 aluminum alloy rod with a radius of 6 mm (about that of the human spinal cord). I then realize that, because I can only fit so many brain-sticks in a lecture hall, I'll probably be making cyclic use of these sturdy brain-sticks, taking off a tired brain and putting on a fresh brain once per week. Luckily, I know the steady-state crack growth behavior of this alloy under these conditions, as indicated in the graph below.
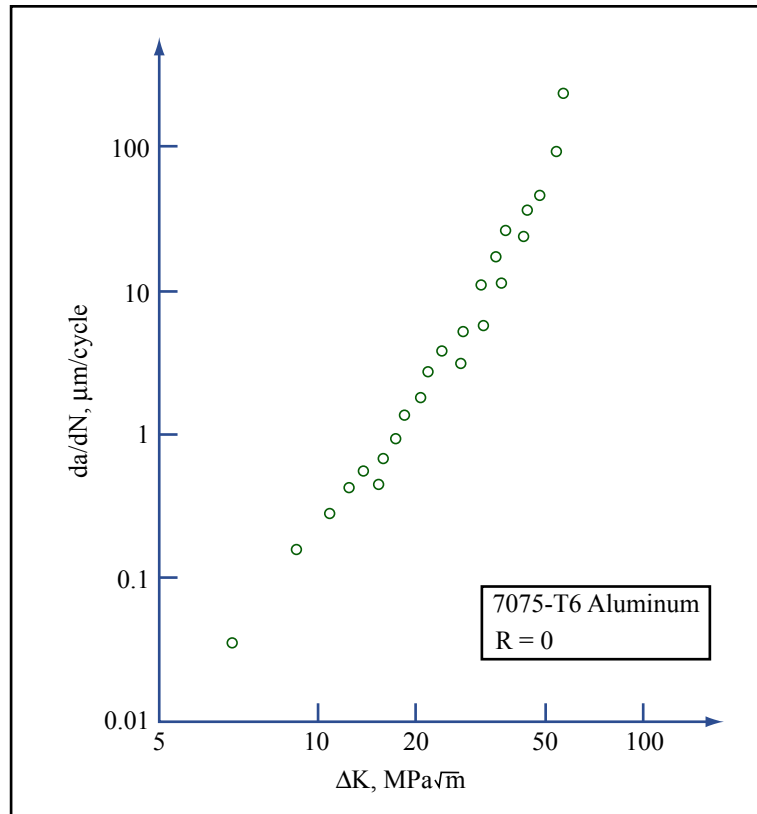

Figure by MIT OpenCourseWare.

Figure 1: Crack growth per fatigue cycle as a function of ΔK for 7075 T6 aluminum.

What is the value of R for this cyclic loading?

(e) What are the values of C and m that characterize the steady-state crack growth for this 7075 Al under this R? Be sure to include units!

(f) I decide I can visually inspect each brain-stick to detect cracks on the surface that are about 100 μm in length. For the cyclic stress involving one brain per brain-stick, am I in the steady-state crack growth regime?

(g) Since I did not acquire these da/dN vs. ΔK data myself, I decide to figure out if I need a safety factor by measuring the fatigue life of one such aluminum brain-stick for a stress range of 1000 MPa. For how many lecture weeks can I use each brain-stick under this loading scheme before I should retire it for fear of fatigue failure?

(h) As the brain-stick is made of 7075 Al and is under cyclic axial loading, how do you expect the fatigue failure surface to appear?

(i) This material is already an alloy, but I want to further extend the fatigue lifetime of the brain-stick. Briefly but completely describe two options to improve the fatigue life of this specific alloy (i.e., further alloying is not an option).

(j) Given that I'm less than 180 cm tall and will have a tough time securing brains to the tops of these brain-sticks, let's take a different tack. It may be easier for the brains if they are stuck onto a wooden beam suspended from the ceiling by wires on each end, a sort of bench that can be set up easily in all lecture halls. The brains will be equally spaced, three to a bench, as shown.
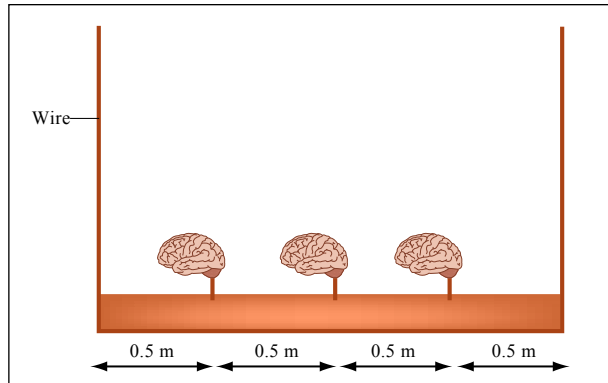

Figure by MIT OpenCourseWare. Adapted from http://upload.wikimedia.org/wikipedia /commons/6/6d/Brain_stem_normal_human.svg. Courtesy of Patrick J. Lynch, medical illustrator and C. Carl Jaffe, MD, cardiologist.

Figure 2: Brain bench: A wooden beam (1.5 m x 1 m x 0.25 m) suspended at each end by wires, along which 3 brains can be spaced equally. Brain graphic source: science.howstuffworks.com/brainport.htm

Determine the reaction forces at the ends of the bench. If the suspension wires are made of hot-rolled AISI 1020 steel with a true (tensile) fracture strength of 800 MPa, what is the minimum wire diameter I can use for this purpose? The density of wood is approximately 900 kg/m^3, and beam dimensions are given in the figure caption.

(k) Determine the shear force and bending moment diagram for this wooden beam, using your method of choice to find the magnitude of V and M at end points, transition points, and minima/maxima along the beam length. The weight of the wooden beam should not be neglected, but partial credit will be given if you do neglect it.

2. In addition to predicting the mechanical responses of linear viscoelastic materials like amorphous polymers, springs and dashpots can be used to created models of creep and recovery in crystalline materials.

   (a) Although springs and dashpots are used to predict the behavior of amorphous polymers and crystalline materials in response to mechanical loading, they are predicting two very different ways of dissipating mechanical energy. Explain the difference, precisely but concisely, in terms of elasticity, plasticity, and atomic / molecular mechanisms.

   (b) Below is a model that accurately captures the creep and recovery response of several micro-crystalline materials. For a step-stress applied at $t=0$ and removed after a long time $t=t_1 \gg 0$, draw how strain in the microcrystalline material evolves as a function of time, up to times much greater than t_1 (i.e., tending toward infinity).
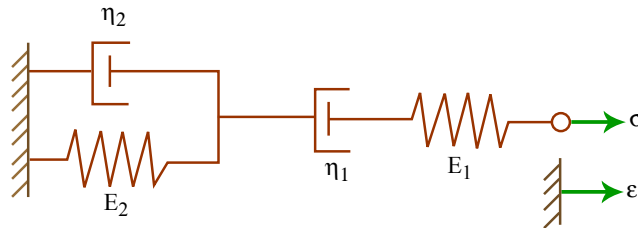

Figure by MIT OpenCourseWare.

Figure 3: Rheological model that is predictive of creep in microcrystalline materials.

Your answer must be in terms of the model's spring and dashpot values E_i and η_i, and you must indicate the magnitude of strain at:

   • $t = 0+$ (i.e., just after the stress is applied);
   • $t = t_1-$ (i.e., right before the stress is removed),
   • $t = t_1+$ (i.e., just after the stress is removed), and
   • $t = \infty$.

(c) Ice is a crystalline material that creeps appreciably, especially under the high stresses and potentially high temperatures characteristic of polar ice caps and glaciers. Consider a portion of a polar ice cap under an in-plane compressive stress of 10 MPa at -50ºC. What is the maximum shear stress imposed on the ice by this compressive stress state?

(d) The shear elastic modulus G of ice is about 3 GPa. Referring to the deformation mechanism map below, what is the (shear) strain rate $d\varepsilon/dt$ we would expect this ice block to exhibit?
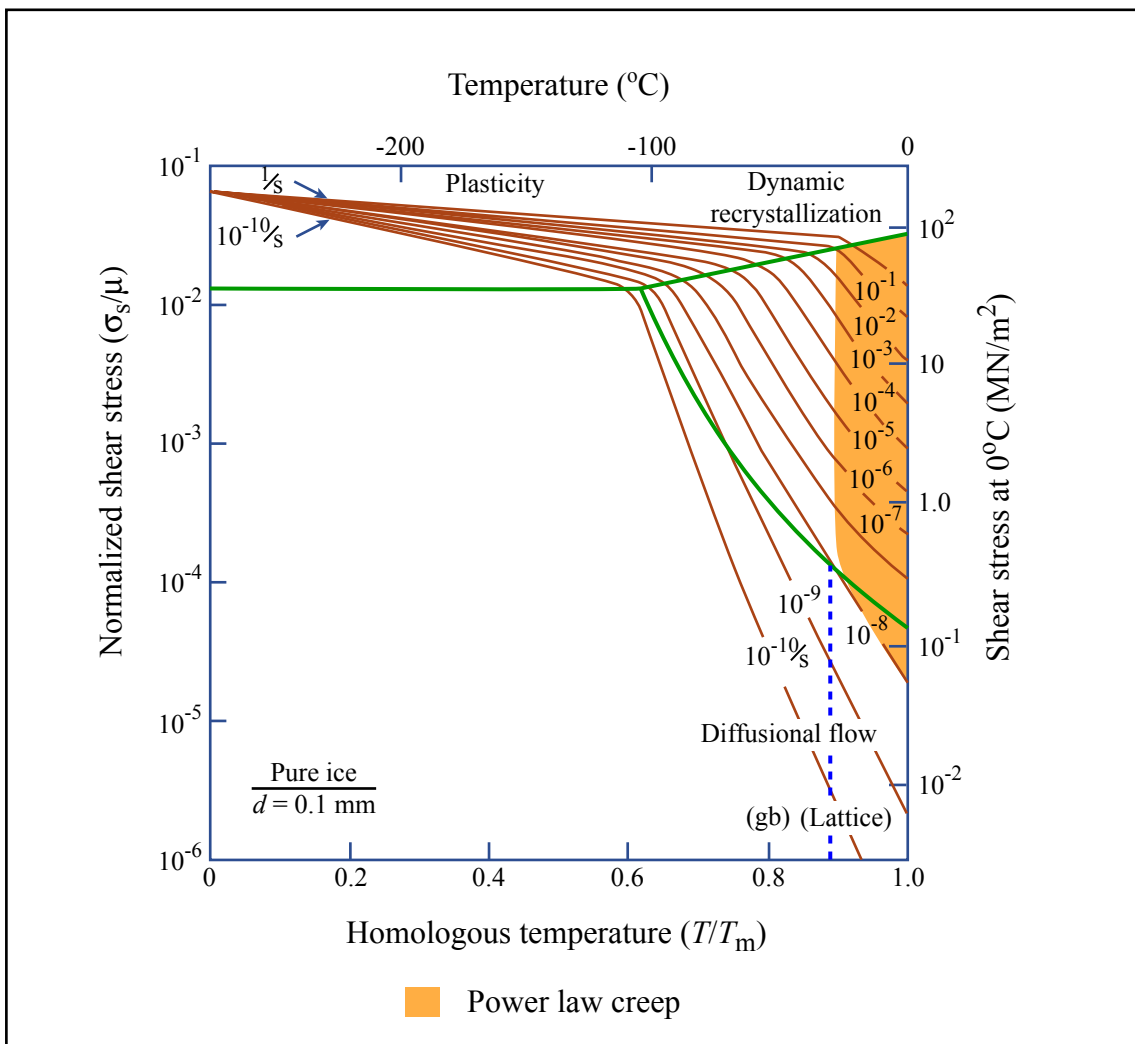

Figure by MIT OpenCourseWare.

Figure 4: Deformation mechanism map of pure ice.

(e) Assuming that the ambient temperature and stress state remain fixed, how do your propose to slow the flow of this polar ice cap? Your answer should include consideration of the dominant and other deformation mechanisms you know to be operative for this ice.

END OF QUIZ 3 AND OF YOUR 3.032 EXPERIENCE. HAVE A RELAXING AND FUN IAP!

USE THIS PAGE FOR EXTRA PROBLEM SOLVING AS REQUIRED.

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