3.032 Problem Set 3
Fall 2007
Due: **Start** of Lecture, 10.01.07

1. Here, we will analyze the stress states of a solid block of aluminum under various loading conditions using the Mohr's Circle construction. We will use the following convention to describe the different components of stress:
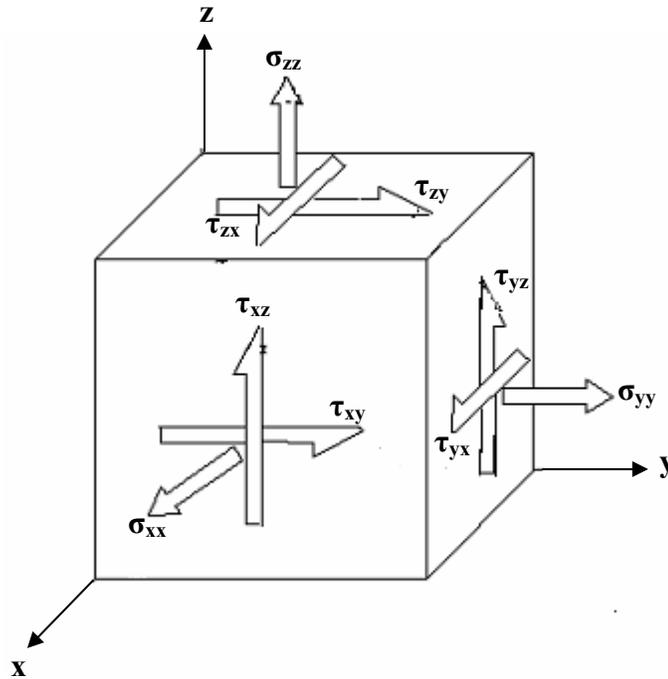
For each of the following situations, describe the stress state in terms of a stress matrix σ_{ij}. Determine the principal normal and shear stresses and give the orientation of the principal axes as well as the orientation of the max shear stress.

(a) One very popular application of aluminum is for use as thin films for metallization layers in integrated circuit (IC) fabrication. In the case of thin-films (or other thin geometries), where one of the dimensions of the material is much smaller than the other two, the stress acting through the smaller dimension is negligible and can be neglected during analysis. This case, where the stresses acting on one of the orthogonal planes is zero, is known as *plane stress*. Let us assume that the thickness dimension of the Al thin-film is along the z-direction (parallel to the x-y plane), so that the plane stress condition is $\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$.

During a bend test of the Al thin film, a point on the surface of the film was found to have high stresses. It was decided that this point of the thin film would be further analyzed. The stresses on the film, with respect to the coordinate system shown

above, were found to be $\sigma_{xx} = 95$MPa, $\sigma_{yy} = 25$MPa, and $\tau_{xy} = 20$MPa. Determine all of the information stated in the problem (in bold italics) and note the value of the maximum normal stress.

(b) Aluminum is also used in electrical transmission lines for power distribution. In order to assess the mechanical integrity of the lines, the lines are loaded in uniaxial tension. It is observed that the lines permanently deform at an applied uniaxial stress of 20 MPa. In addition to the requirements in bold font ($\boldsymbol{\sigma_{ij}}$, magnitude and orientation of principal/max shear stress states), also **draw** the orientation of the material representative volume element (here, a plane) that is under maximum shear stress. Assume the following orientation and axis-set for the Al lines loaded in tension:
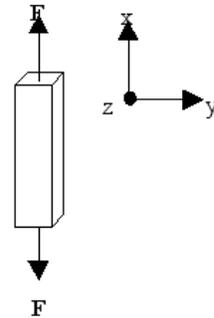
Fig. 1a: Aluminum power line loaded under uniaxial force F.

(c) One common forming process for aluminum parts is known as *extrusion*. In the extrusion process, a billet of material is forced through a die in order to obtain a workpiece with the desired cross-sectional geometry. One type of extrusion process is known as *hydrostatic extrusion*. Unlike many extrusion operations, where pressure to the billet is supplied by a hydraulically-driven ram or pressing stem, the pressure in hydrostatic extrusion is supplied through an incompressible fluid medium surrounding the billet (see Figure 1b below). Hydrostatic extrusion is a very popular operation for ductile materials such as aluminum due to its ability to reduce defects in the extruded part through the compressive environment. Typical pressures exerted on the workpiece are around 1400 MPa.

Figure 1b: Hydrostatic extrusion process

Write the stress state of a representative volume element of the Al billet inside of the extruder under the hydrostatic pressure of 1400 MPa (noted by the grid region in Fig. 1b), and determine all other information requested in bold font.

2. You are responsible for performing uniaxial tensile tests on three very different materials: a 316 stainless steel alloy, alumina (Al_2O_3), and high density polyethylene (HDPE). However, before performing the actual tests, you are asked to predict what the **elastic** stress vs. strain responses of each of the materials based on the mechanical

properties of these materials documented in the literature (e.g., material property databases such as matweb.com, linked on the MIT Server).

(a) Graph the stress [MPa] versus strain [%] response for all three materials on a single graph and on 3 separate graphs, up to an applied strain value of .01 (or 1%) in strain increments of 0.0005.

(b) Remark on the differences in behaviors seen for each of the three materials as related to their relevant mechanical properties. Also, looking at the magnitude of the stresses at the maximum applied strain, do you expect all of these materials to deform elastically up to these strains? If not, what was the fallacy in *solely* using Hooke's law to predict the stress-strain behaviors for each of the materials?

(c) All three samples were given to you as cylinders with identical initial dimensions of 10 cm length and 2 cm diameter. Show whether a uniaxial load frame of maximum load capability = 100 kN (like the ones you used to crush the beverage cans in Lab 1) will be sufficient to deform all three materials to the requested engineering normal strain of 1%. Here, neglect the possibility that the materials might not remain intact to that strain.

3. You and your labmate have been given a joint project by your UROP advisor. He has asked you both to measure all the elements of the stiffness tensor C_{ijkl} of two new proteins he has crystallized and is considering for use in a flexible, organic integrated circuit that must withstand mechanical bending.

Another student has already used x-ray diffraction to determine the crystal structure of the proteins: protein A exhibits a tetragonal unit cell, whereas protein B exhibits an orthorhombic unit cell. Your labmate says, "Let's each measure the elastic constants C_{ijkl} of one protein. Which one do you want to analyze, protein A or B?'"
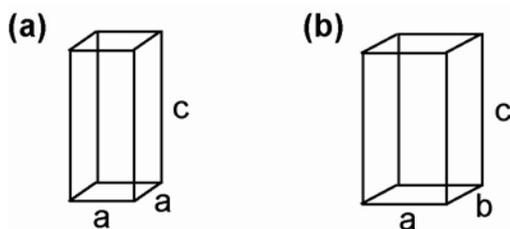
Fig. 3: Unit cells for (a) tetragonal and (b) orthorhombic crystals.

Under the assumption that you'd like to spend as little time as possible on this project so you can get back to your 3.032 studies, what is your answer? Explain concisely, but as fully and accurately as possible, using the elasticity concepts discussed in 3.032.