3.032 Problem Set 2
Fall 2006
Due: Start of lecture, 9/22/06

1. You make a questionable back-to-school purchase: a new pair of platform-sole sneakers that may make you look a bit taller. Much to your dismay, your "friend" tells you that when you put them on she can see that the green soles are compressed to about half their original height (i.e., half of the thickness shown in Figure 1(a)). Looking from the top, you can see that your soles were flush with the shoe upper before you put them on. However, under your weight, the soles stick out about 1 cm on each side.

3 cm
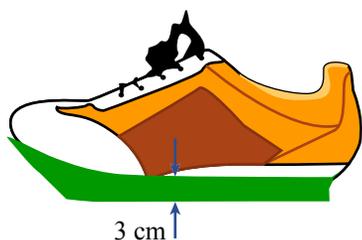
Figure by MIT OpenCourseWare.

1 cm

(a) buy-designer-shoes-online.ishoproute.com.jpg (b) www.istockphoto.com

Figure 1: (a) Sneaker before wearing, top view; (b) Your view of your sneakers while wearing.

   (a) What is the Poisson's ratio of the material from which your sneaker soles are made?

   (b) Based on this back-of-the-envelope calculation, what can you tell your friend about the likely composition and structure of that material?

2. You are hanging from a rope that is of 4 cm diameter and originally of 6 m length. The rope is stationary and clamped tightly at the other end. Treat the rope as a square spring (i.e., Cartesian coordinates are OK) of effective spring constant k = 100 N/m and with a Poisson's ratio of zero.

   (a) At the clamped end, at the point halfway down the rope length, and at the end from which you're hanging, define all the components of the displacement vector $\mathbf{u} = (u_i, u_j, u_k)$.

   (b) Compute all the components of the small-strain, small-rotation strain tensor ϵ_{ij} at the end from which you are hanging.

(c) Compute the normal stress σ_{ii} at the same three points of the rope considered in (a).

(d) What do you notice about the variation in **u** and in σ_{ii} as a function of distance from the clamped end?

3. Concrete, sometimes called "the ultimate material," has enabled impressive achievements in construction for thousands of years. Concrete is also simple to make, consisting–in its simplest form–of only lime (calcium oxide), sand, gravel, and water. An unfortunate limitation of concrete, however, is that it is far, far weaker in tension than in compression. In this problem, you will develop a way around this limitation. The technique you will use, known as pretensioning, is commonly used to create long spans such as those pictured in Figure 3.

Image removed due to copyright restrictions. Please see:
http://www.cement.org/bridges/truckee.htm

Figure 2: The Truckee River Bridge in Truckee, California—a 2002 Concrete Bridge Award Winner (Portland Cement Association, www.cement.org).

(a) Let us model a concrete span as a pin-and-roller-anchored beam with a length $L = 6$ m, a thickness $h = 0.8$ m, and width w (Figure 3(a)). The span is loaded with a distributed load q (force per *area*) of 50 kN/m^2. Determine the values and locations of the maximum tensile and compressive stresses in the beam. (It may be useful to construct shear and bending moment diagrams).

(b) You have decided that it is best to allow no part of the concrete span to be in a state of tensile stress. To accomplish this goal, you will position steel bars along the length of the span before pouring the concrete. You will then stretch the bars, pour the concrete and allow it to harden, and release the tension (Figure 3(b)). The released steel bars will exert an axial compressive stress on the span that is superposed with the bending stresses. Determine the force per meter of span width to cancel the maximum tensile stress you calculated previously. Assume that the load created by the steel bars is distributed evenly across the cross section of the span and that the addition of the bars does not significantly change the cross section.

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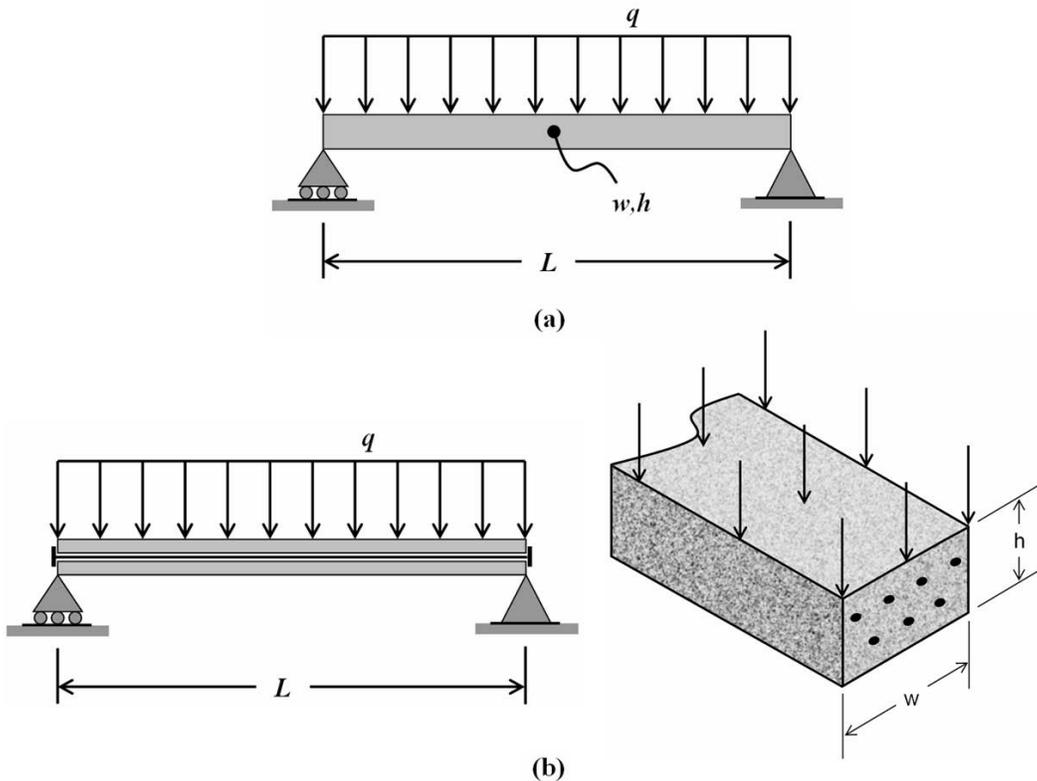
Figure 3: (a) Concrete span under distributed load q; (b) The same span with integrated steel rods to provide pretensioning.

    (c) You are told that the steel you are using has a failure strength $\sigma_F = 400\,\text{MPa}$ and the bars have a diameter of one inch. You wish to avoid failure in the bars with a factor of safety of two. What is the maximum load you can allow in each bar? How many bars are needed per meter of span width to provide the necessary compressive load that you calculated previously?

4. A spiral fracture can occur when long bones such as the femur are exposed to an excessive torsional load (Figure 4(a,b)). The diaphysis (middle portion) of the femur consists of an outer circle of dense cortical bone surrounding a center of much weaker trabecular (meshlike) bone. Biomedical engineers have modeled the stronger cortical bone portion as an annulus, as shown in Figure 4(c). Let us assume that this annulus has an outer diameter of 26 mm and an inner diameter of 20 mm, and that the diaphysis has a length of 30 cm. The shear modulus of cortical bone is approximately 17 GPa.

    (a) The maximum shear stress occurs at the surface of the bone. If a shear stress $\tau_F = 60\,\text{MPa}$ will cause failure in cortical bone, find the maximum torsional

(a)

(c)
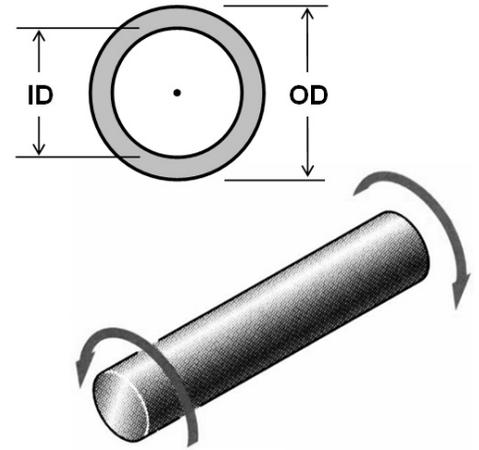
Figure 4: (a) Fracture pattern of femur after excessive torsional loading [Vashishith et al., *J Orthopaed Res* 19:414 (2001)]; (b) X-ray of femoral spiral fracture [Ligier et al. *J Bone Joint Surg* 70B:74 (1988)]; (c) Loading configuration and annular model of cortical bone showing outer and inner diameters.

load that this femur can withstand. What is this load in Newtons, assuming a lever arm of 2 m?

(b) If the dimensions of the cross section (outer and inner diameters) are reduced by 50%, by what percentage is the maximum torsional load reduced?

(c) What is the angle of twist at failure for the original cross section?