Outline:

1. **p-n** Junction Carrier Generation
2. Solar Cells
3. Light-emitting Diodes (LEDs)

1. **p-n** Junction Carrier Generation

$$p_n = p_p e^{\frac{qV_{BI}}{k_B T}}$$

$$n_p = n_n e^{\frac{qV_{BI}}{k_B T}}$$

Forward Bias (applied to n-side)

$$p_n = p_p e^{\frac{q(V_a - V_{BI})}{k_B T}}$$

$$n_p = n_n e^{\frac{q(V_a - V_{BI})}{k_B T}}$$
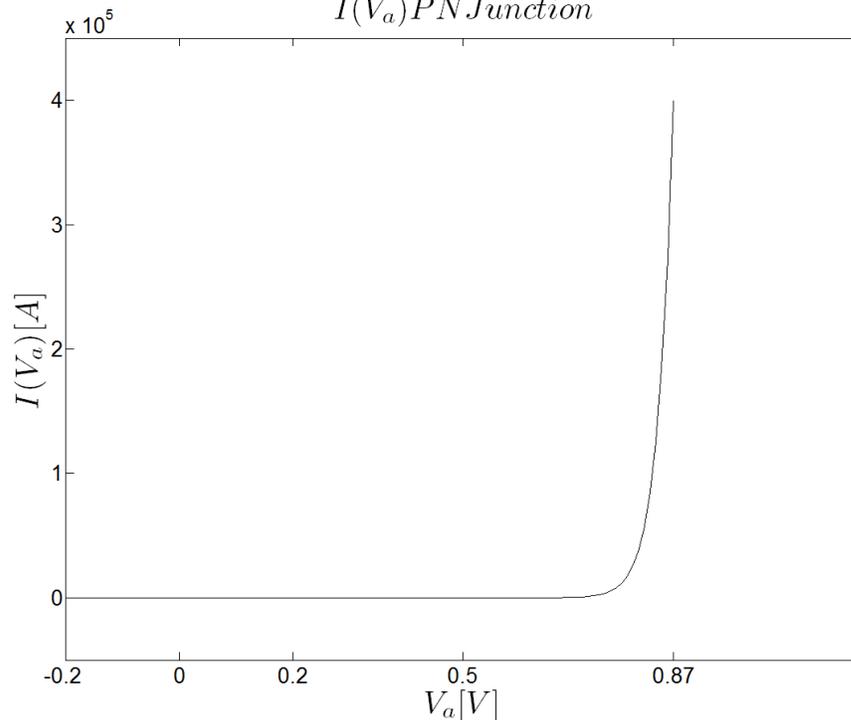
Reverse Bias (applied to n-side)

$$p_n = p_p e^{\frac{-q(V_a + V_{BI})}{k_B T}}$$

$$n_p = n_n e^{\frac{-q(V_a + V_{BI})}{k_B T}}$$

For Diodes:

$$I = I_s \left(e^{\frac{qV}{k_B T}} - 1 \right)$$

$$I(V_a) PN Junction$$

2. Solar Cells

<div align="center">Indirect vs. Direct Band Gaps</div>

Direct band gap semiconductors: Minimum of conduction band is at the same location in k-space as the maximum of the valence band. $E \geq E_g$ will generate electron hole pair.

Indirect band gap semiconductors: Minimum of conduction band is at a different location in k-space as the maximum of the valence band. $E \geq E_g$ in addition to a momentum $\hbar k \geq \hbar \Delta k$, where Δk is the k-space difference of the location of the conduction band and valence band locations of the transition.

<div align="center">For diode Solar Cells:</div>

$$I = I_S \left(e^{\frac{qV}{k_B T}} - 1 \right) - I_{PH}(\lambda)$$

<div align="center">Short Circuit Current</div>

$$V = 0$$
$$I_{SC} = I_{PH}$$

<div align="center">Open Circuit Voltage</div>

$$I = 0$$
$$0 = I_{PH} - I_S \left(e^{\frac{qV}{k_B T}} - 1 \right)$$
$$V_{OC} = \frac{k_B T}{q} \ln \left(\frac{I_{PH}}{I_S} + 1 \right) \cong \frac{k_B T}{q} \ln \left(\frac{I_{PH}}{I_S} \right) \text{ Since } \frac{I_{PH}}{I_S} \gg 1$$

<div align="center">Efficiency</div>

$$P_{MAX} = V_{MP} J_{MP}$$
$$R_{MAX} = \frac{V_{MP}}{I_{MP}}$$
$$FF = \frac{I_{MP} V_{MP}}{I_{SC} V_{OC}}$$
$$\eta_{POWER} = \frac{P_{OUT}}{P_{IN}} = \frac{V_{MP} I_{MP}}{P_{OPTICAL}} = FF \frac{V_{OC} I_{PH}}{P_{OPTICAL}}$$
$$\eta_{INT} = \frac{Electrons\ Out}{Photons\ Absorbed}$$
$$\eta_{EXT} = \frac{Electrons\ Out}{Photons\ In} = \frac{\frac{I}{q}}{\frac{P\lambda}{hc}} = \frac{Ihc}{P\lambda q}$$

e.g. 1 "Green" Solar Cell

You are designing new solar cell material to meet the needs of the "green" energy economy. Taking the concept literally, you decide it is best to make a **p-n** junction solar cell that only absorbs green light at 550 nm. Using a semiconducting material that is made of a complex alloy of rare earth elements and group IV elements, the valence and conduction bands have a uniform dispersion relationship (i.e. E_g is the same value in all k-space). If the absorption coefficient $\alpha = \frac{10^{-7}}{\lambda}$ and the carrier generation is given by $G = \alpha \frac{P_{PH} \lambda}{hc}$, calculate the maximum power your device can produce, the corresponding resistance of the load at this power, and the external quantum efficiency η_{EXT}. Assume the saturation current $I_S = 10^{-9}$ A, $P_{PH} = 100$ mW, and we're operating at room temperature $T = 300$ K.

The photocurrent I_{PH} is simply given by $I_{PH} = qG$, where q is the charge of the electron. The net current in the device is then

$$I = I_S \left(e^{\frac{qV}{k_B T}} - 1 \right) - I_{PH}(\lambda)$$

The actual current generated by the device when connected to a load is the negative of this (i.e. the current in the load):

$$I = I_{PH}(\lambda) - I_S \left(e^{\frac{qV}{k_B T}} - 1 \right)$$

Here the voltage V is roughly the voltage drop across the resistor, thus the power considering the load behaves according to Ohm's law is:

$$P = IV = V \left(I_{PH}(\lambda) - I_S \left(e^{\frac{qV}{k_B T}} - 1 \right) \right)$$

To find the max power, we can take the derivative of P with respect to V and set this equal to 0.

$$\frac{dP}{dV} = I_{PH}(\lambda) - I_S \left(e^{\frac{qV}{k_B T}} - 1 \right) - \frac{V I_S q}{k_B T} e^{\frac{qV}{k_B T}} = 0$$

$$q\alpha \frac{P_{PH}\lambda}{hc} + I_S = I_S e^{\frac{qV}{k_B T}} \left(1 + \frac{qV}{k_B T} \right)$$

Numerically solving this transcendental equation, we find $V \cong 0.343$ V and the max power is $P_{MAX} = 2.56$ mW.

The resistance this corresponds to is given by:

$$P = \frac{V^2}{R}$$

$$R = \frac{V^2}{P} = \frac{(0.343 \text{ V})^2}{0.00256 \text{ W}} = 45.9$$

$$\eta_{EXT} = \frac{Ihc}{P_{PH}\lambda q} = \frac{Vhc}{RP_{PH}\lambda q} = 16.9\%$$

3. Light-emitting Diodes (LEDs)

When a forward-bias is applied to a *p-n* junction, light will be emitted once electrons are injected into the conduction band in the depletion region. The minimum forward bias voltage needed to inject these electrons is based on the band gap energy.

Therefore, if $V \geq \frac{E_g}{q}$, light will emit as the electrons are swept out by the electric field in the depletion region and the electrons recombine with the holes. This recombination rate determines the efficiency of the LED, and the details of recombination are covered in the class lecture on LEDs.

The resulting spectrum of the LED peaks at $h\nu = E_g + \frac{k_B T}{2}$.

e.g. 2 LED Thermometer
You are designing a new semiconducting material to operate as an LED thermometer. By applying a constant voltage $V \gg E_g + \frac{k_B T}{2}$ to the LED, you measure the frequency of the peak light intensity using a photodetector and use that value to calculate the temperature T. Find a relationship for your device of how a change in temperature affects the wavelength of the light observed. If your device has a $T = 0$ K band gap energy that corresponds to green light of 550 nm, at what temperature would the LED emit blue light instead (450 nm blue)?

We want a relationship of a change in temperature dT to a change in wavelength $d\lambda$.
Since $h\nu = E_g + \frac{k_BT}{2}$ is the peak frequency, in terms of λ this is $\frac{hc}{\lambda} = E_g + \frac{k_BT}{2}$. To find the
relationship, we take the derivative of λ with respect to T.

$$\frac{hc}{\lambda} = E_g + \frac{k_BT}{2}$$

$$\lambda = \frac{hc}{\left(E_g + \frac{k_BT}{2}\right)}$$

$$\frac{d\lambda}{dT} = -\frac{hck_B}{2\left(E_g + \frac{k_BT}{2}\right)^2}$$

This relationship says that as T increases, the rate of change of the wavelength with temperature
decreases with the inverse square of temperature and as T increases, the wavelength decreases.

If we want to decrease the wavelength by 1 nm, we have to increase the temperature by ~100 K.
For $T = 0$ K:

$$dT = -\frac{2\left(E_g + \frac{k_BT}{2}\right)^2}{hck_B}d\lambda = -\frac{2\left(\frac{1240 \text{ eV nm}}{550 \text{ nm}} + 0 \text{ K}\right)^2}{1240 \text{ eV nm } 8.617 \times 10^{-5}\frac{\text{eV}}{\text{K}}}(-1 \text{ nm}) = 95 \text{ K}$$

For $T = 99$ K:

$$dT = -\frac{2\left(E_g + \frac{k_BT}{2}\right)^2}{hck_B}d\lambda = -\frac{2\left(\frac{1240 \text{ eV nm}}{550 \text{ nm}} + \frac{1}{2}8.617 \times 10^{-5}\frac{\text{eV}}{\text{K}}1000 \text{ K}\right)^2}{1240 \text{ eV nm } 8.617 \times 10^{-5}\frac{\text{eV}}{\text{K}}}(-1 \text{ nm}) = 99 \text{ K}$$

If the band gap corresponds to 550 nm green, the peak intensity would become blue at the following
temperature T:

$$\frac{hc}{\lambda} = E_g + \frac{k_BT}{2}$$

$$\frac{1240 \text{ eV nm}}{450 \text{ nm}} = \frac{1240 \text{ eV nm}}{550 \text{ nm}} + \frac{8.617 \times 10^{-5}\frac{\text{eV}}{\text{K}}T}{2}$$

$$T = 11628 \text{ K}$$

This temperature is clearly not reachable by modern semiconducting materials.

3.024 Electronic, Optical and Magnetic Properties of Materials
Spring 2013