

140 Points Total

1. (35 points) You're working as a post-doctoral researcher debating whether you should go into industry or academia when you notice very thin 1D crystals forming on your substrates during chemical vapor deposition. Upon further analysis, you realize that the crystal is a true 1D semiconductor made up of germanium atoms (atomic number 32, band gap energy of $E_g = 0.67$ eV).

a) What is the general form of the single electron wavefunction in this material (5 points)?

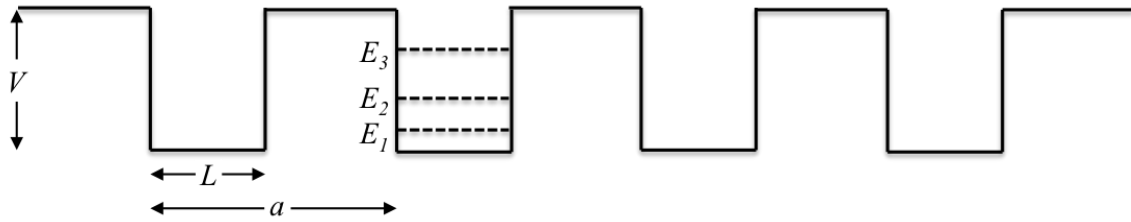
b) Assuming the conduction electrons in this material behave as free electrons, derive the density of states $g(E)$ for this 1D crystal. Don't forget that electrons are Fermions (10 points).

c) At a given temperature T , what is the probability of an electron having energy E inside this crystal (5 points)?

Draw this function vs. E at $T=0$ and qualitatively at $T=300$ K (5 points).

- d) Sketch the electron concentration in the conduction band as a function of energy (5 points).
Based on your sketch set up an integral expression you can use to calculate the electron concentration in the conduction band (lowest energy is E_c). You do not have to evaluate this integral (5 points).

2. (40 points) You're now working for IBM and have discovered a new way of storing information using particle confinement. In your experiment particles are trapped in 1D finite potential wells of length L periodically spaced with a lattice constant a . Each 1D well has a finite number of energy levels for the particle, and each level can be correlated to a unit of information ($E_1 \rightarrow 0, E_2 \rightarrow 1, \text{etc.}$). The number of states per well is controlled by an external potential barrier V that can be raised or lowered. The information is written onto this device by a precise photonic tip that transfers the particles between the energy levels by emitting or absorbing excess energy in the form of photons.

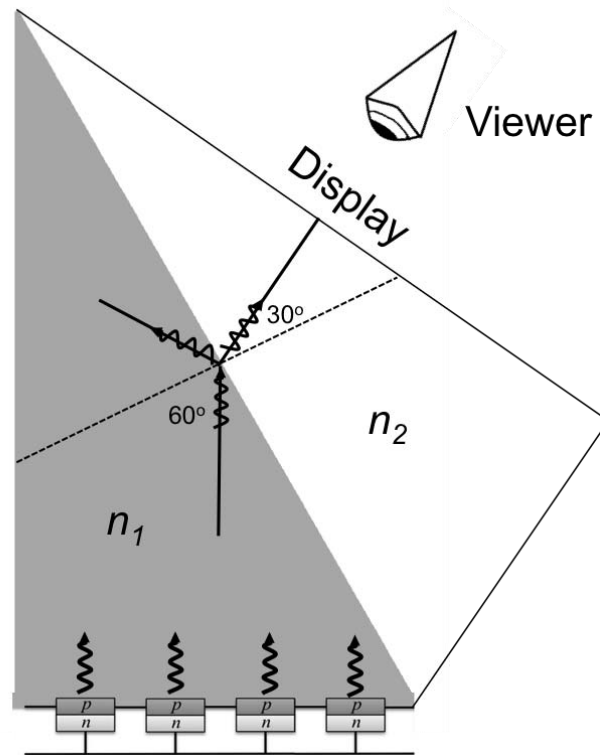


- a) Assuming the potential V is raised to infinity and a also goes to infinity, sketch the system and write down the time-independent Schrödinger equation for the particle inside a well (5 points).
- b) For the well spanning $0 \leq x \leq L$ find the energy eigenvalues and eigenfunctions of the system in part a) (8 points). Remember to normalize the eigenfunctions (2 points).

- c) If the potential barrier is reduced to a finite level V_0 while the particle is in the energy eigenstate $E_2 > V_0$, qualitatively what do you expect to happen to the particle (4 points)? What will happen to the particle if $E_2 < V_0$ (4 points)? Make sketches to aid your reasoning. What major potential problem do you see with this proposed device that might keep it from functioning the way you intend at finite potentials (2 points)?

- d) Having realized the problems with your setup you have decided on a back-up plan which involves using magnetic materials for information storage. By drawing hysteresis loops for hard and soft magnetic materials motivate which material will be better suited for information storage (10 points).
- e) Having Mn, Fe, Co and Ni at your disposal, which of the materials would you pick for your memory device. Explain in terms of crystal structure (5 points).

3. (35 points) You ended up transferring from IBM to a professorship at MIT because of your theoretical work in display devices. You now are managing a research group of your own trying to get tenure in a few years. One of your students is working on a project in novel photonic display devices. The device works by coupling series of *p-n* junction LEDs of different materials and a high index of refraction glass to direct the photons out of the display.



- a) Your industry sponsor requested to design LEDs emitting photons with wavelengths of 900 nm, 550 nm, and 450 nm (corresponding to energies of 1.38 eV, 2.26 eV, and 2.76 eV respectively). Your equipment allows you to process the following elements with listed atomic number: Ga (32), In (49), N (7), and As (33). List the possible compounds you can make with these elements to form a compound semiconductor (5 points).

b) Rank in order the expected band gap size of these compound materials you listed in part a). Please explain your reasoning (5 points).

c) You are reviewing your student's lab notebook and discover their energy bandgap measurements for the different compounds from (a), but they are not labeled. The measured average values of the band gap energies are: 1.4 eV, 3.4 eV, 0.65 eV, and 0.35 eV.

By alloying your compounds, you should be able to reach the bandgap values suitable for the LEDs with desired values wavelengths. By matching your rankings from part (b) to the measured values, choose three pairs of compounds that you could use to reach each of the desired wavelengths. Only consider alloys consisting of two metals atoms and one non-metal atom within a compound (10 points).

- d) You now dope your materials with Si (*n*-type dopant for these materials) and Mg (*p*-type dopant for these materials) at a concentration of 10^{16} cm^{-3} . You may assume that the intrinsic carrier concentrations of your materials are negligible (these materials essentially act as insulators at room temperature without doping). Qualitatively draw the band diagram of the system when you connect a Si doped piece with a Mg doped piece and label the bands, Fermi energy, band gap, and built-in potential (10 points).
- e) You now turn on your device and the photons go from an index of refraction in the material of ~ 1.5 through the high refractive index glass. If the light enters the high refractive index glass at an incident angle of 60° and you want it to go through the glass at an angle of 30° because of the orientation of your display (30° refraction angle will correspond to 0° incidence to the air), what must the index of refraction of the high refractive index glass be (5 points)?

4. (30 points) After getting tenure at MIT you have decided to “make the world a better place” by refocusing your research lab from displays to photovoltaics.

a) What material makes a better solar cell crystalline Ge or GaAs (5 points)?
Explain using qualitative sketches of absorption spectra (5 points).

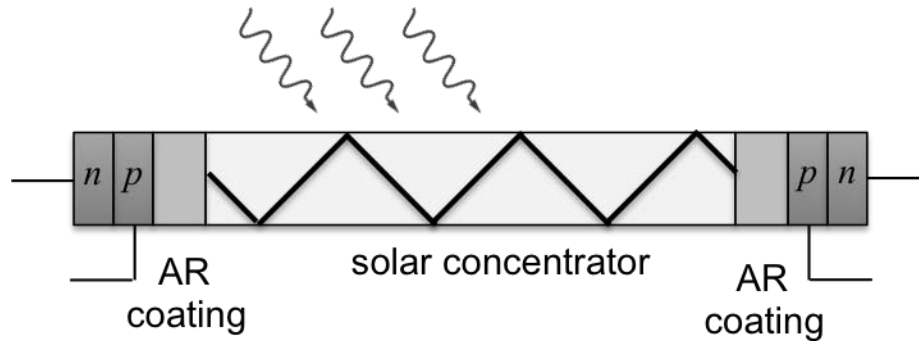
b) By comparing the current voltage characteristics for *p-n* junction solar cell with and without illumination describe their mechanism of operation (10 points).

- c) As crystalline solar cells are expensive your graduate students have decided to use the smallest possible area devices (and spend the rest of the money on new computers and international conference trips). But they don't want you to get angry so they have devised a clever scheme to collect all the light and focus it onto their small area *p-n* junction solar cells. They are using a solar concentrator, which consists of a piece of wave-guiding plastic doped with luminescent molecules and positioned the solar cells on the edges of the concentrator. When the light hits the plastic it gets absorbed and re-emitted by the luminescent molecules and guided towards the solar cells on the edges.

What is the minimum incidence angle for the re-emitted light to remain inside the plastic with refractive index of 1.5?

Hint: What is the wave guiding condition for this plastic (5 points)?

- d) Your clever students realized that despite having a solar concentrator they still cannot reach the efficiency that they promised, because a significant portion of wave-guided light gets reflected off the interface between the plastic and the solar cell. To minimize the reflection, they decided to use an antireflection coating between the concentrator and the solar cell.



What is the refractive index of the anti-reflection coating (2.5 points)? What thickness of antireflective coating should they choose knowing that the solar spectrum intensity is peaked at 555 nm (2.5 points)?

Note: refractive indexes for Ge and GaAs are 4.0 and 3.4 respectively. Use the material you chose in (a).

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