Subject 24.242. Logic II. Sample problems from the fifth homework, due Thursday, April 15

Here are some theorems of PA that we derived either in class or in the notes. You may want to use them in the problems below:

$(\forall x)(\forall y)(x + y) = (y + x)$
$(\forall x)(\forall y)(\forall z)((x + y) + z) = (x + (y + z))$
$(\forall x)(x \cdot [1]) = x$
$(\forall x)(\forall y)(x \cdot y) = (y \cdot x)$
$(\forall x)(\forall y)(\forall z)((x \cdot y) \cdot z) = (x \cdot (y \cdot z))$
$(\forall x)(\forall y)(\forall z)((x + y) \cdot z) = ((x \cdot z) + (y \cdot z))$

1. Show that "$(\forall x)(\forall y)(\forall z)(x \mathrm{E} (y+z)) = ((x\mathrm{E}y) \cdot (x\mathrm{E}Z))$" is a theorem of PA.

2. Show that, where $\mathfrak{A}$ is a nonstandard model of PA, there isn't any formula $\phi(x)$ of the language arithmetic that is satisfied by all the standard numbers in $\mathfrak{A}$ but isn't satisfied by any of the nonstandard elements of $\mathfrak{A}$.