On the Brink of Paradox:
List of known errors, as of April 17, 2019

Chapter 1

- p. 10, table: "At most~~,~~ as many members in A as in B".
- p. 11: "select. [For example,] [i]t will be answered negatively".
- p. 18: "Let the range of that function be the set $S^{\mathbb{N}} = [\{]s_0, s_1, s_2, \ldots [\}]$".
- p. 22: Exercise 1 (and its answer on page 30) uses "B_0" for the set that is introduced as "B" in the main text.

Chapter 2

- p. 48: the claim that $|A| + |B| = |A|$ when at least one of A and B is infinite and $|B| \leq |A|$ assumes the axiom of choice. (The minimal correction here is to delete "The claim that $|A| \otimes |B| = |A|$ whenever A is infinite, B is nonempty, and $|B| \leq |A|$ assumes" and replace with "Nerdy Observation: Here I assume".)

Chapter 3

- p. 75: "~~a~~ high table"

Chapter 4

- p. 100: "correspond to points on the ~~dotted~~ [horizontal] line".

Chapter 5

- p. 134, last equation: "$1BE$" should be "$2BE$".
- p. 135, first indented conditional: "she failed to do so" should be "she failed to take the trip".

Chapter 6

- There is a serious omission in section 6.1.1. The Objective-Subjective connection is only plausible when one presupposes that a perfectly rational agent is always certain about the connection between events before t and the objective probabilities at t. Here is a proposed fix:

  > (*Nerdy observation:* Here I am tacitly presupposing that a perfectly rational agent is always certain about the connection between events before t and the objective probabilities at t. So, in particular, for each complete history of the world up to t, H_t, there is a specification P_t of the objective probabilities at t such that the agent assigns credence one to the proposition [if H_t then P_t]. This assumption is potentially controversial but adds simplicity to our discussion.)

  With this fix in place on can give a formal proof—given assumptions—of the Principal Principle. Here is a proof for a particular isntance:

  Assume that x's half life is $7.04 \cdot 10^8$ years. Let D be the proposition that x will decay sometime within the next $7.04 \cdot 10^8$ years. We show that you should believe D to degree 0.5.

  It follows from the fact that x's half life is $7.04 \cdot 10^8$ years that the objective probability of D is 0.5. It then follows from the Objective-Subjective Connection that a perfectly rational agent with perfect information about the past (and none about the future) would assign credence 0.5 to D.

  Now suppose you are perfectly rational and that—although you have not quite learned the full truth about the past—the information you have acquired, E, is entirely about the past. Suppose, moreover, that a rational agent would take E to be compatible with the proposition that $p(D) = 0.5$, were p is objective probability.

  Because E is entirely about the past, it is equivalent to some disjunction $H_t^1 \vee H_t^2 \vee \ldots$ of possible histories-up-to-t. (We must assume that the conjunction is either finite or countably infinite, to ensure Conglomerability later on.) Because perfectly rational agents are always certain about the connection between events before t and the objective probabilities up to t, each H_t^j is equivalent to $H_t^j P_t^j$, where P_t^j is a complete specification of the objective probabilities at t.

Because E (and therefore $H_t^1 \vee H_t^2 \vee \ldots$) is compatible with $p(D) = 0.5$, there are some $H_t^{k_1}, H_t^{k_2}, \ldots$ amongst the H_t^1, H_t^2 such that each $P_t^{k_i} H_t^{k_i}$ entails $p(D) = 0.5$. (Note that every H_t^j outside this list entails something incompatible with $p(D) = 0.5$.) So $(p(D) = 0.5)E$ is equivalent to $H_t^{k_1} \vee H_t^{k_2} \vee \ldots$:

$$c(D|(p(D) = 0.5)E) = c(D|H_t^{k_1} \vee H_t^{k_2} \vee \ldots)$$

But, for each i, we know that $c(D|H_t^{k_i}) = 0.5$. So, by Conglomerability,

$$c(D|(p(D) = 0.5)E) = c(D|H_t^{k_1} \vee H_t^{k_2} \vee \ldots) = c(D|H_t^{k_1}) = 0.5$$

And how do we know the Conglomerability holds? Here is a proof for the finite case. (The result also holds in the countably infinite case but requires Countable Additivity.)

$$\begin{aligned}
p(A|B_1) &= p(A|B_2) \\
\tfrac{p(AB_1)}{B_1} &= \tfrac{p(AB_2)}{B_2} \\
p(B_2) \cdot p(AB_1) &= p(B_1) \cdot p(AB_2) \\
p(B_2) \cdot p(AB_1) &= p(B_1)(p(AB_2) + p(AB_1) - p(AB_1)) \\
p(B_1) \cdot p(AB_1) + p(B_2) \cdot p(AB_1) &= p(B_1) \cdot p(AB_2) + p(B_1) \cdot p(AB_1) \\
p(AB_1)(p(B_1) + p(B_2)) &= p(B_1)(p(AB_2) + p(AB_1)) \\
\tfrac{p(AB_1)}{p(B_1)}(p(B_1) + p(B_2)) &= p(AB_2) + p(AB_1) \\
p(A|B_1)(p(B_1) + p(B_2)) &= p(AB_1) + p(AB_2) \\
p(A|B_1) &= \tfrac{p(AB_1)+p(AB_2)}{p(B_1)+p(B_2)} \\
p(A|B_1) &= \tfrac{p(AB_1 \vee AB_2)}{p(B_1 \vee B_2)} \\
p(A|B_1) &= \tfrac{p(A(B_1 \vee B_2))}{p(B_1 \vee B_2)} \\
p(A|B_1) &= p(A|B_1 \vee B_2)
\end{aligned}$$

We have now shown that $c(D|(p(D) = 0.5)E) = 0.5$. But the only restrictions on E are that it be entirely about the past and that it be compatible with $p(D) = 0.5$. So if you're fully rational, then as long as everything you've learned is entirely about the past and compatible that $p(D) = 0.5$, Update by Conditionalizing entails that you should believe D to degree 0.5.

- p. 169, indented paragraph: "k dollars, say. [If k is odd, I should definitely switch. What about the case in which k is even? In that case] ~~This means that~~ the other envelope [...]". (Also, replace two occurrences of "outcomes" in that paragraph with "scenarios".)

Chapter 8

- p. 209, "T" and "B" labels on diagram should be "U" and "D", respectively.