Omega-sequence Paradoxes (Part II)

1 The Bomber's Paradox[1] [Paradox Grade: 6]

There are infinitely many bombs:

Bomb	When bomb is set to go off
B_0	12:00pm
B_1	11:30am
B_2	11:15am
$\vdots$	$\vdots$
B_k	$\frac{1}{2^k}$ hours after 11:00am
$\vdots$	$\vdots$

Should one of the bombs go off, it will instantaneously disable all other bombs. So a bomb goes off if and only if no bombs have gone off before it:

(0) B_0 goes off $\leftrightarrow$ B_n fails to go off $(n > 0)$.

(1) B_1 goes off $\leftrightarrow$ B_n fails to go off $(n > 1)$.

(2) B_2 goes off $\leftrightarrow$ B_n fails to go off $(n > 2)$.

$\vdots$

$(\boldsymbol{k})$ B_k goes off $\leftrightarrow$ B_n fails to go off $(n > k)$.

$(\boldsymbol{k+1})$ B_{k+1} goes off $\leftrightarrow$ B_n fails to go off $(n > k+1)$.

$\vdots$

Will any bombs go off?

[1] This paradox is due to Josh Parsons, who was a fellow at Oxford until shortly before his untimely death in 2017. (It is a version of Bernadete's Paradox.)

2 Yablo's Paradox[2] [Paradox Grade: 8]

There are infinitely many sentences:

Label	Sentence
S_0	"For each $i > 0$, sentence S_i is false"
S_1	"For each $i > 1$, sentence S_i is false"
S_2	"For each $i > 2$, sentence S_i is false"
$\vdots$	$\vdots$
S_k	"For each $i > k$, sentence S_i is false"
$\vdots$	$\vdots$

The meanings of our sentences guarantee that each of the following must be true:

(0) S_0 is true $\leftrightarrow$ S_n is false ($n > 0$).

(1) S_1 is true $\leftrightarrow$ S_n is false ($n > 1$).

(2) S_2 is true $\leftrightarrow$ S_n is false ($n > 2$).

$\vdots$

(*k*) S_k is true $\leftrightarrow$ S_n is false ($n > k$).

($\boldsymbol{k+1}$) S_{k+1} is true $\leftrightarrow$ S_n is false ($n > k+1$).

$\vdots$

Which sentences are true and which ones are false?

3 Bacon's Problem[3] [Paradox Grade: 7]

- An omega sequence of prisoners: $P_1, P_2, P_3, \ldots$. (P_1 is at the end of the line, in front of her is P_2, in front of him is P_3, and so forth.)

[2] This paradox was discovered by Steve Yablo, who is a famous philosophy professor at MIT (and was a member of my dissertation committee, many years ago).

[3] This paradox is due to USC philosopher Andrew Bacon.

- Each person as assigned a red or blue hat, based on the outcome of a coin toss.
- Everyone can see the hats of people in front of her, but cannot see her own hat (or the hat of anyone behind her).
- At a set time, everyone has to guess the color of her own hat by crying out "Red!" or "Blue!".
- People who correctly call out the color of their own hats will be spared. Everyone else will be shot.

Problem: Find a strategy that $P_1, P_2, P_3, \ldots$ could agree upon in advance and that would guarantee that at most finitely many people are shot.

4 The Three Prisoners[4] [Paradox Grade: 2]

- Three prisoners. Each of them is assigned a red or blue hat, based on the outcome of a coin toss.
- Each of them can see the colors of the others' hats but has no idea about the color of his own hat.
- The prisoners are then taken into separate cells and asked about the color of their hat. They are free to offer an answer or remain silent.
  - If all three prisoners remain silent, all three will be killed.
  - If one of them answers incorrectly, all three will be killed.
  - If at least one prisoner offers an answer, and everyone who offers an answer answers correctly, then all three prisoners will be set free.

Problem: Find a strategy that the prisoners could agree upon ahead of time which would guarantee that their chance of survival is above 50%.

[4] I don't know who invented it, but I learned about it thanks to philosopher and computer scientist Rohit Parikh, from the City University of New York.

Prisoner A	Prisoner B	Prisoner C	Result of following Strategy
red	red	red	Everyone answers incorrectly
red	red	blue	C answers correctly
red	blue	red	B answers correctly
red	blue	blue	A answers correctly
blue	red	red	A answers correctly
blue	red	blue	B answers correctly
blue	blue	red	C answers correctly
blue	blue	blue	Everyone answers incorrectly

Figure 1: The eight possible hat distributions, along with the result of applying the suggested strategy.

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