

Assignment #4: The quantum mechanical representation of states (due 3/7/05)

Read the appended passage from van Fraassen's *The Scientific Image*, beginning with "In quantum mechanics we can find..." and ending with "The observable phenomena which are actual, however, are the same." Write a short but careful critique of his reasoning in this passage (1 or 2 pages should suffice), drawing on what you now know about how states are represented in quantum mechanics. Specifically, you should answer all of the following questions: What, precisely, is the claim that van Fraassen is trying to establish? What argument does he give for this claim? Assuming his argument is a good one, is its conclusion in fact confined to the special case van Fraassen considers (namely, that of an operation on vectors corresponding to a rotation through 2π radians)? Or could the same argument be used to establish a much more general claim about when two vectors represent different states? *Is* his argument any good—and if not, why not?

Hint: Pay special attention to van Fraassen's reasoning in the second-to-last paragraph, and keep the following sermon in mind: One commits a "use/mention" fallacy when one conflates a property of a *representation* with a property of the thing it *represents*—as, for example, when we infer from the claim that "Ned" has three letters in it to the claim that Ned has three letters in him. (The fallacy gets its name because in the first kind of claim the name is *mentioned* whereas in the second it is *used*.) There is good reason to think that in the following passage van Fraassen commits something like the use/mention fallacy: "If ψ and $R_{2\pi}\psi$ really represented exactly the same physical state, then the superposition ($k\phi + m\psi$) would represent the same state as ($k\phi + mR_{2\pi}\psi$)." Why would anyone think that this claim is true?

In quantum mechanics, we can find a similarly simple, telling example. First, I must make some preliminary remarks. The states are represented by vectors in a Hilbert space, and simple mathematical operations can be performed on these vectors. To calculate the probability of a measurement outcome, the theory tells us to proceed as follows. First we represent the state of the system by means of such a vector in a Hilbert space. Then we multiply that vector by a positive scalar, so that the result is a new vector just like the first, except that it has unit length. Next, we express this unit vector ψ in terms of a family of vectors (eigen-vectors) specially associated with the physical magnitude we are measuring, in this form:

$$\psi = c_1\psi_1 + \dots + c_i\psi_i + \dots$$

Each vector ψ_i corresponds to one possible measurement result r_i . The probability that the result will be r_k equals the square of coefficient c_k (or what corresponds to the square for complex numbers, if that coefficient is complex).

In view of this, it is often said that all positive multiples of ψ represent the same state. For if you begin with $k\psi$ or with $m\psi$, your first step will be to 'normalize', that is, multiply by a scalar

so as to arrive at unit vector ψ . There is ‘no physical difference’, ‘the phase has no physical significance’ people say; and the reason they give is that the probabilities for measurement outcomes are the same.

Now consider a simple physical operation, rotation. Rotating a system changes its state. There are corresponding operations on vectors, to change the vector that represented the system before, to the one that represents it after the rotation. Let us call the vector operation that corresponds to a rotation through angle x by the name R_x . If the old state was ψ , the new state, after this rotation, is $R_x\psi$. In general, the probabilities for measurement outcomes are very different in this new state, so there is here in general a genuine physical difference.

One special case is the rotation through 2π radians, a complete circle. Physically, it brings the system back to its original position, in the classical, macroscopic examples we know so well. In the quantum analogue, the operation $R_{2\pi}$ is also quite simple: multiplication by the scalar -1 . Hence $R_{2\pi}\psi = -\psi$. If we now expand this new vector in terms of the eigen-vectors ψ_i , we get the coefficients $-c_i$. But if we then calculate the probabilities of measurement results, we square these, so the minuses disappear. Those probabilities are therefore exactly the same for the new state as for the old.

Following the same reasoning as before, we should now say that $R_{2\pi}$, like positive scalar multiplication, simply produces a vector representing the same physical state as the original vector. But there has been a good deal of discussion of this case in the literature, and that apparently easy way out is not available to us. To explain this, we must look at another operation on vectors, namely superposition. If ϕ and ψ are two vectors, then $(k\phi + m\psi)$ is a superposition of them, which is again a vector in the same space, and also represents a physical state. We can sum up the argument in the literature as follows: if ψ and $R_{2\pi}\psi$ really represented exactly the same physical state, then the superposition $(k\phi + m\psi)$ would represent the same state as $(k\phi + mR_{2\pi}\psi)$. That the latter is not so, is easily seen by calculating probabilities for various observables. Klein and Opat designed an experiment on a beam of neutrons in which the observable differences between the two sorts of superpositions were verified: a Fresnel diffraction experiment in which the diffracting object was the boundary between two regions carrying opposite magnetic fields.

What should we conclude from this? The case is quite similar to that of classical mass. If in one possible world, an isolated system is in state ψ and in another it is in state $R_{2\pi}\psi$, no amount of empirical information actually available can tell the observer which of these two worlds he is in. But there is a counterfactual statement we are inclined to make about the case: if the system had interacted with another one in such and such a way, the results would have been different in the two cases. The observable phenomena which are actual, however, are the same.