ESD: Recitation #7

Assumptions of classical multiple regression model

- Linearity
- Full rank
- Exogeneity of independent variables
- Homoscedasticity and non autocorrellation
- Exogenously generated data
- Normal distribution

Hypothesis Testing

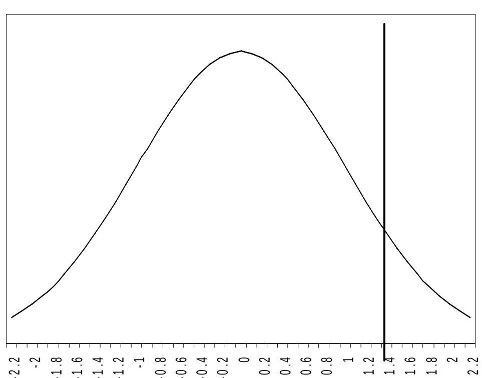
- Method:
  1. Formulate the null hypothesis H_0 and the alternative hypothesis H_A.
  2. Identify a test statistic to assess the truth of the null hypothesis.
  3. Compute the P-value, which is the probability that a test statistic as least as significant as the one observed would be obtained, assuming that the null hypothesis were true.
  4. Compare the P-value to an acceptable significance level, α. If $p \leq \alpha$, the null hypothesis is ruled out.

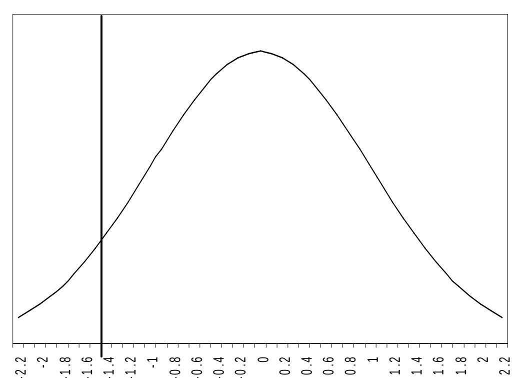
α levels and P-values

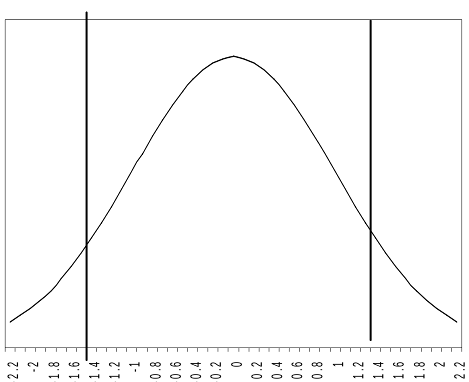
- Examples of confidence levels:
  - Proportions: $$\hat{p} \pm z_{\alpha/2} \sqrt{\hat{p}\left(1-\hat{p}\right)/n}$$
  - Means: $$\bar{x} \pm z_{\alpha/2}\, \sigma/\sqrt{n}$$
  - Differences between two population proportions (large samples): $$\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2} \sqrt{\hat{p}_1\left(1-\hat{p}_1\right)/n_1 + \hat{p}_2\left(1-\hat{p}_2\right)/n_2}$$
  - Differences between two population means (large samples): $$\left(\bar{x}_1 - \bar{x}_2\right) \pm z_{\alpha/2} \sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}$$

One tailed or two tailed tests

One tailed (right)	One tailed (left)	Two tailed
$\mathbf{H_{0:}}\ \mu \leq \mu_{\mathbf{o}}$	$\mathbf{H_{0:}}\ \mu \geq \mu_{\mathbf{o}}$	$\mathbf{H_{0:}}\ \mu = \mu_{\mathbf{o}}$
$\mathbf{H_{A:}}\mu > \mu_{\mathbf{o}}$	$\mathbf{H_{A:}}\mu < \mu_{\mathbf{o}}$	$\mathbf{H_{A:}}\mu = \mu_{\mathbf{o}}$







μ_{o} is a specified value of the population mean

Z-test

	One-tailed (right)	One-tailed (left)	Two-tailed
$H_{0:}$ $H_{A:}$	$\mu_1 \leq \mu_2$ $\mu_1 > \mu_2$	$\mu_1 \geq \mu_2$ $\mu_1 < \mu_2$	$\mu_1 = \mu_2$ $\mu_1 \neq \mu_2$
Test statistic	z	z	z
Decision Rule	Reject if $z > z_\alpha$	Reject if $z < -z_\alpha$	Reject if $z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$

Z-test (2)

- Large sample proportion: $$p = p_o, z = \frac{x - np_o}{\sqrt{np_o(1 - p_o)}}$$
- Large sample mean: $$\mu = \mu_o, z = \frac{\overline{x} - \mu_o}{s/\sqrt{n}}$$
- Large sample difference between two population proportion: $$p_1 = p_2, or, p_1 - p_2 = 0, z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}}$$
- Large sample difference between two population means: $$\mu_1 = \mu_2, z = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^{\;2}}{n_1} + \frac{s_2^{\;2}}{n_2}}}$$

T-test

- Same method
- Tests:
  - Small sample mean: $\mu = \mu_o, t = \dfrac{\bar{x} - \mu_o}{s/\sqrt{n}}$
  - Small sample difference: $\mu_1 = \mu_2, t = \dfrac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$

(S_p is the pooled standard deviation)

Example: Medical treatment

Is your treatment more successful than control?

Test at the 1% significance level

- What are we comparing?
- Small or large sample?
- One or two tailed?

	Treatment	Control
Sample mean (%): μ	85	83
Sample SD: s	3	2
N	75	60

Medical treatment (2)

Comparing means, z test statistics, one tailed (right tailed)

$H_{0:}\ \mu_t \leq \mu_c$ $H_{A:}\ \mu_t > \mu_c$

Test Statistic: z-test

$$z = \frac{\mu_t - \mu_c}{\sqrt{\frac{s_t^2}{n_t} + \frac{s_c^2}{n_c}}}$$

Decision Rule: For significance level α = 0.01, reject null hypothesis if computed test statistic value:

z=4.6291 > z_α =2.33, p=.000002 (from z-table)

Conclude: reject null hypothesis.

ANOVA

- ANOVA allows for comparing points estimates for more than 2 groups.
- ANOVA separates the total variability of outcome in two categories: variability within or between groups.
- H_0: same average for each group

  H_A: all averages are not the same

ANOVA: Method

1) Calculate variation between groups.
   - Compare mean of each group with mean of overall sample: between sum of squares (BSS):

$$BSS = \sum_{i=1}^{m} n_i(\overline{x_i} - \overline{x})^2$$

   - Divide BSS by number of degree of freedom (m-1)
   - Get estimate of variation between groups

ANOVA: Method (2)

2) Calculate variation within groups.

- Find sum of squared deviation between individual results and the group average, calculating separate measures for each group
- Sum the group values: obtain the within sum of squares (WSS):

$$WSS = \sum_{i=1}^{m} (n_i - 1) \times \sigma_i^2$$

- Divide WSS by number of degrees of freedom ($\left(\sum_{i=1}^{m} n_i\right) - 1$): get estimate of variation within groups.

ANOVA: Method (3)

3) Calculate F-statistics.

– F = BSS/WSS

– Compare value with standard table for the F distribution: calculate significance level of F value

– If null rejected: use z- and t-tests between each pair of groups.

Example: shift productivity

$H_{o:}\ \mu_{morning} = \mu_{afternoon} = \mu_{night}$

The null indicates that all groups have the same average score and by assumption the same standard deviation

$H_{A:}\ \mu_{morning} \neq \mu_{afternoon} \neq \mu_{night}$

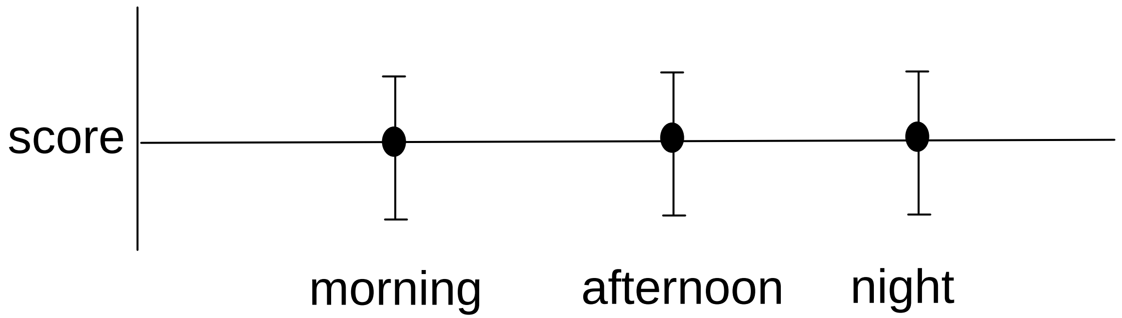
The alternative is that all means are not the same

Note: the alternative is not that all means are different.

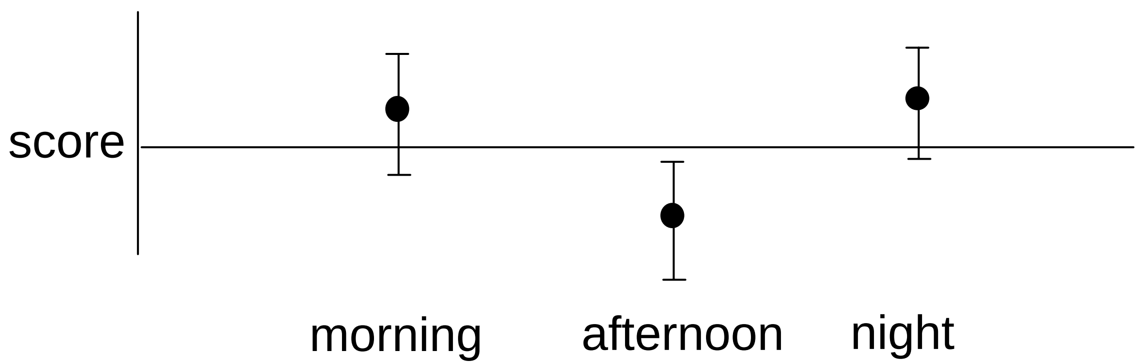
It is possible that some of the means could be the same, yet if they are not <u>all</u> the same, we would reject the null.

Shift productivity (2)

$H_{o:}\ \mu_{morning} = \mu_{afternoon} = \mu_{night}$



$H_{A:}\ \mu_{morning} \neq \mu_{afternoon} \neq \mu_{night}$



Shift productivity (3)

	Av. Prod.	SD	# of workers
Morning	4.12	1.3	313
Afternoon	3.99	1.3	340
Night	4.37	1.3	297
Average	4.15		950

1.

BSS = $n_1\ (x_1\text{-}x)^2 + n_2\ (x_2\text{-}x)^2 + n_3\ (x_3\text{-}x)^2$

$= 313\ (4.12\text{-}4.15)^2 + 340\ (3.99\text{-}4.15)^2 + 297\ (4.37\text{-}4.15)^2$

$= 23.36$

Between Mean Squares = BSS/v = 23.36/2 = 11.69

Shift productivity (4)

2.

$$WSS = (n_1 - 1)\, SD_1^2 + (n_2 - 1)\, SD_2^2 + (\eta_3 - 1)\, SD_3^2$$

$$= (313-1)\, 1.3^2 + (340-1)\, 1.3^2 + (297-1)\, 1.3^2$$

$$= 1600.43$$

Within Mean Squares = WSS/v = 1600.43/947

$$= 1.69$$

Shift productivity (5)

3)

F = (Between Mean Squares/Within Mean Squares)

= (11.68/1.69) = 6.91

Compare value to standard table for the F distribution

In this case, significance level is less than .01

Reject the null, students’ performance varies significantly across the three classes

Testing for heteroscedasticity

- White's test
- Goldfeld-Quandt test
- Lagrange Multiplier test

➔ If $E[\varepsilon\varepsilon|\Omega]=\sigma^2.\Omega$ is known: Weighted least squares

Maximum Likelihood Estimation

- Definition:
  - PDF of a random variable y, conditioned on a set of parameters $\vec{\theta}$: $f(y \mid \vec{\theta})$
  - Joint density of n *independent and identically ditributed* observations from this process:

$$f(y_1, \ldots, y_n \mid \vec{\theta}) = \prod_{i=1}^{n} f(y_i \mid \vec{\theta}) = L(\vec{\theta} \mid \vec{y})$$

  - The joint density is the *likelihood function*.
  - Maximize $L(\theta \mid \vec{y})$ with respect to θ:

$$\frac{\partial L(\theta \mid \vec{y})}{\partial \theta} = 0 \quad \text{and} \quad \frac{\partial^2 L(\theta \mid \vec{y})}{\partial \theta^2} < 0$$

Maximum Likelihood Estimation (2)

- Conditions:
  - Parameter vector $\vec{\theta}$ identified $\Leftrightarrow \forall \vec{\theta}* \neq \vec{\theta}, L(\vec{\theta}* \mid \vec{y}) \neq L(\vec{\theta} \mid \vec{y})$
- Properties:
  - Asymptotically efficient