ESD.71 / 1.146 / 3.56 / 16.861 Engineering Systems Analysis for Design
Fall 2008

ESD 71 Assignment   Lattice Development

Summary: This exercise gives you the opportunity to use a lattice and explore its implications.

Learning Objective: to help you understand how to develop and use a lattice to project possible uncertainties.

Note Carefully: You do not have to do this exercise if you are doing essentially the same as part of your Application Portfolio exercise. (Some students may not be working with time series data and so would not fall into this category.) **If in doubt, discuss with instructors.**

Tasks:

1.   **Calibrate the Lattice to a situation**
         You are to prepare a lattice for the movement of price of copper. Assume that:
- The price of copper in October 2005 was about $3700/ton. (In fact the price of copper fluctuates significantly – in 2004, for example, it rose about 50 % and recently it reached the highest price ever, of over $1.85 per pound);
- Assume for this exercise that its long-term annual rate of appreciation is 5% (in general line with inflation – however, this is not a documented trend); and
- The standard deviation of its price is twenty times the sum of the number of the day and month of your birth (i.e, if you were born on May 6, you would use a standard deviation = 20 (5 + 6) = 220). The purpose of this is to enable you to do your own individual analysis.

         You are to use this information to calculate u, d and p. [1]

2.   **Develop the Lattice model of the projected fluctuation of the price of copper over the next 6 periods**
         Use binomial lattice.xls as provided on course web site to develop the outcomes, the associated probabilities, and the resulting distribution.

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[1] Note: If your standard deviation is small, the factor (average/standard deviation) $(\text{Delta } T)^{1/2}$ = $(v / \sigma) (\text{Delta } T)^{1/2}$ may be greater than 0.5, and lead you to think you have to use a probability greater than 1.0 – which makes no sense. The solution, of course, is to work with small increments of time, so that the factor $(\text{Delta } T)^{1/2}$ is smaller. For example, you could work with a (Delta T) equal to 3 months or ¼ of a year.
Further Note carefully: The time period T needs to correspond to that associated with the rate of change and the volatility (specifically, the v and the σ). Thus if the rate of growth is 40% per year and the σ is 32% so that the factor $(v / \sigma) (\text{Delta } T)^{1/2}$ = 1.25 on a yearly basis, if we go to T = ¼ year, we do not change either the v or the σ; they stay the same. The new factor, applied in the lattice of 3 month intervals, would be $(v / \sigma) (\text{Delta } T)^{1/2} = (1.25) (1/4)^{1/2}$ = 0.625.