Public Switched Telephone Networks:
A Network Analysis of Emerging Networks

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A recent focus in the area of network analysis has been on the comparison of technological, informational, social and biological networks (Newman 2003). Within the technological networks category, one comparison of interest is among infrastructure networks such as the power generation and distribution networks, the public switched telephone networks (PSTN), the Internet, and various transportation networks that we have come to rely heavily upon. In this paper we will use network analysis to study the PSTN.

Our analysis of the PSTN is focused on wired (copper and fiber) networks. It does not entail wireless networks such as microwave, satellite or other radio links. In the United States, telecommunications service providers that operate the PSTN fall under three categories: interexchange carriers (IXCs) that own networks for long-distance calling, incumbent local exchange carriers (ILECs) that own networks for inter and intrastate calling, and competitive local exchange carriers (CLECs) that own networks within a state[1]. For our analysis, we looked at the networks of a CLEC and an ILEC serving one US state.

Nomenclature and Data Sources

For the purpose of our analysis, we will refer to the CLEC whose network we are analyzing as a *Nano Bell*; the ILEC whose network we are analyzing as a *Mini Bell*; and IXCs, which are *not* a part of our analysis, *Maxi Bells*. We have decided to use this nomenclature in order to protect the identity of the companies that shared the data with us, as per our agreement with them. An example of a Nano Bell (CLEC) is Mid-Maine Communications in Maine, where Verizon is the Mini Bell (ILEC) and the Maxi Bell (IXC) is a long distance company such as AT&T.

The networks we will analyze – a Nano Bell network and a Mini Bell network – have two types of switches: *tandem switches* and *central office switches*. A tandem switch switches traffic between central offices and forms the core of the network. A central office switch has connections to homes' and offices' end systems, such as phones, faxes, etc. A central office switch often has two parts – the *host switch* and the *remotes*. The host is the part of the switch that carries out all of the switching functions, whereas the remotes only provide geographical coverage but rely entirely on the host for any switching functions its connections may require. A tandem switch is equivalent to a Class 4 of AT&T's original product line. A central office switch is typically a relatively smaller Class 5 switch.

Our data comes from two primary sources. The Mini Bell data was obtained from their public website: http://www.qwest.com/iconn/. The Nano Bell data comes from network plans a Nano Bell operating in one US state has shared with us under the aforementioned confidentiality agreement. The assumptions (discussed later) we made about the Nano Bell's network come from our interviews with an enthusiastic contact person at the Nano Bell.

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[1] Our focus on the ownership of the network is deliberate since it is no longer possible to separate the three types of providers by the service they provide. Today an IXC, ILEC or a CLEC is allowed to offer local or long-distance service.

History and Political Economy of PSTN

With a technology more than a century old, the US PSTN has a rich history of rapid changes in technology, regulation and industry structure. However, keeping in mind the focus of this paper, we will discuss the history and political economy of PSTN from the network analysis perspective. For our analysis, we have defined telephone switches as *nodes* and the connections between them as *edges*. Therefore, we will discuss the historical events that changed the characteristics of the nodes, the links and the connections between them.

The period of monopoly (until 1984)

The telephony, as we know it today, was born with a key breakthrough in 1875-76 when Alexander Graham Bell developed transducers from voice to electrical signal and vice versa. Bell had quickly realized the tremendous potential of his invention and formed the Bell Telephone Company in 1877. Although Bell himself stopped working on telephony one year after his invention, he left the company to his father-in-law Gardiner Hubbard and Watson (Boettinger 1977). Watson invented the ringer, the first switchboard, and many other things essential to transforming a laboratory toy into a commercial product. As the company grew, they hired Theodore Vail to manage the company. Vail worked hard to ensure that the Bell Telephone Company controlled a substantial portion of the telephone service in the United States even after the expiration of Bell's patent in 1894. He also used the ever-increasing capital to buy out other telephone companies. By the time he was done, the American Telephone and Telegraph Company (AT&T) owned every telephone instrument, every telephone switch, and every telephone pole in the country. Vail made sure that AT&T would survive the antimonopoly sentiments by promising that every American would have access to the telephone network.

AT&T's dominance continued for several decades before it was regulated as a natural monopoly with the creation of a federal regulatory body, the Federal Communications Commission (FCC). The FCC was established by the Communications Act of 1934 and was charged with regulating interstate and international communications by radio, television, wire, satellite and cable.

The Telecommunications Act imposed universal service obligations on AT&T, which led to planned growth (*designed network*) of PSTN in the subsequent years. Until then, the PSTN had been growing (*grown network*) in areas that were population centers and where the installation made business sense. Today, as most of the PSTN is modernized, it would be difficult to recreate the picture of what the network looked like during this pre-1934 era of grown networks.

The initial overriding obstacle to providing universal service was attenuation in copper lines, known as the challenge of conquering distance (Fagen, Joel et al. 1975). The improvement and general availability of vacuum tubes had a major impact on solving the distance challenge. With vacuum tubes, it became possible to interconnect widely separated cities with low loss and good quality circuits. In 1925, it was possible to call any big city in the continental United States over a circuit of good quality. The switching technology during the first half of the twentieth century was dominated by manual switching for both local and

long-distance calls (Fagen, Joel et al. 1975). **Figure 1** shows the 1928 graph of PSTN. Initially, improvement in switching technology kept it ahead of the demand. Over time, incremental improvements in manual switching hit a limit of reduction in labor costs and call-completion time. Around 1925, large volumes of calls began to experience delays while waiting for a transmission facility. This made electro-mechanical switching popular during 1925 through 1950s. The quality of electro-mechanical switches also steadily improved.
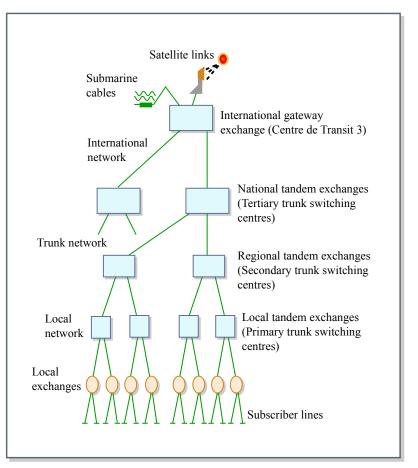
Image removed for copyright reasons.

Two breakthroughs that first necessitated hierarchical switching were: the advent of automatic crossbar switching and the ability to do statistical multiplexing on transmission lines. Around 1913, the seeds were planted for the next generation of automatic switching equipment, what we know as the crossbar switches, when patents for automatic coordinate switching were granted to Western Electric[2].Crossbar switches initiated the direct distance dialing by two customers with no operator in the middle in 1951. The notion of carrying traffic other than voice (data and video) also began with the automatic, crossbar switches (of course, using appropriate wiring). The point-to-point links could no longer scale to accommodate the additional traffic from customer direct dial. A statistical aggregation scheme using time division multiplexing (TDM) of voice channels was on the horizon.

Availability of solid state electronics post World War II enabled the pulse code modulated (PCM) transmission – what we now know as digital transmission – over ordinary wire. The PCM used Harry Nyquist's sampling rate – a discovery he made at Bell Labs in 1927 (Bellamy 1982) – at which an analog signal must be sampled for its recovery at the receiver. The Nyquist rate of sampling enabled the statistical aggregation of voice channels. In 1962, the first T1 carrier that multiplexed voice channels of 24-channels over 1.5 Mbps was installed and was immediately successful. Subsequently came the higher levels of aggregation with T2 (96 channels), T3 (672 channels) and T4 (4032 channels).

The combination of automatic crossbar switching and statistical multiplexing on transmission lines led to the design of hierarchical networks. By 1970, AT&T had a five-level

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[2] Unlike the manual system that worked by moving the contact over an appreciable distance to make a connection between two lines, the cross-bar switches worked by closing electrical contacts at points in an x-y coordinate array.

hierarchical network shown in **Figure 2**(a) (Chapuis 1982). **Figure 2**(b) illustrates AT&T's network with the United States and Canada divided into twelve regions.. By this time the challenge of distance was replaced with that of efficient operations and management of the complex network. Two major considerations governed the economics of transmission systems then: minimum capacity below which economies-of-scale was not possible and minimum distance below which multiplexing was not economical.

Figure by MIT OCW.

REGIONAL CENTERS

Figure by MIT OCW.

Figure by MIT OCW.

While these developments were underway, the invention of the laser in Bell laboratories in 1958, and the simultaneous rise of Nyquist and Claude Shannon's work on information theory, had kindled interest in optical transmission. Two other important complementary inventions that occurred around that time were: first, the advent of the transistor by Bardeen, Shokley and Brattain, and second, the advent of the general purpose computer, ENIAC, by Eckert and Mauchly. Ultimately, the manufacture of fiber optics by Corning Glass was what led to the next revolution in transmission (Keshav 1997).

The five-level hierarchy has never remained purely a hierarchical tree structure. First, level-skipping was exercised as shown in **Figure 3** in order to cut down on losses from every electro-mechanical contact. This was done by ensuring that there are not more than two electro-mechanical contacts in placing a long-distance call. Later, additional alternate routes were introduced across levels with the installation of the automatic crossbar switches in 1950s to cater to the higher call volume resulting from customer direct dial.

Five-level toll switching plan in use from the 1950s. A variety of routings was possible with a maximum of nine trunks in tandem.

Figure by MIT OCW. After Andrews & Hatch, 1971.

The breakup of the monopoly (after 1984)

In the late 1970s, the anti-monopoly sentiments were strong in America. By the early 1980s there was an alternate long-distance carrier, MCI, but it was believed that AT&T's market power over MCI came from its ownership of the local loops (the last mile connection from central office to home). In 1984, the US Gove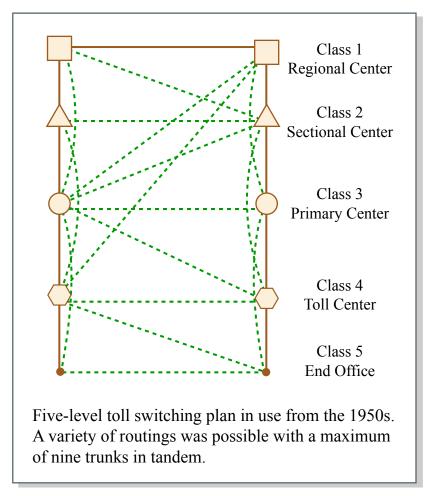rnment decided to breakup AT&T into the long-distance company (AT&T), and seven regional bell operating companies (RBOC). At the time of its creation, the RBOCs inherited AT&T's network, including the regional centers (primary centers), tandem switches (toll centers) and central (end offices) shown in Figure 3, Figure 4 shows the original breakup of AT&T into seven RBOCs and the changes they have undergone over time.

An inter-carrier compensation (also known as access charge) regime was established defining the per-minute charges AT&T (now an IXC) paid an RBOC (now an ILEC) to help the RBOCs can operate and maintain the local networks. AT&T was disallowed to offer local service, while the RBOCs were prohibited from offering the long-distance service after the breakup.

As the competition in local and long distance service began to emerge, AT&T began to use innovative routing schemes to maintain its competitive edge following the breakup of 1984. In 1987, AT&T installed a dynamic routing system called dynamic non-hierarchical routing (DNHR). The objective of this system was to decrease the number of dropped calls, especially on high-traffic days like holidays. Originally, the static paths from the five-level hierarchy would simply drop a call if it reached a link that was blocked. A new "originating call control feature" allowed a call to be "cranked back" to the original switch if the direct path was blocked. Then, with new technology coming in the form of 4ESS™ switches,

alternate paths that may not be considered the most direct paths were tried until all possible 'two-hop' routes were exhausted. Only then would a call be dropped (Ash and Oberer, 1989).

Image removed for copyright reasons.
See: http://en.wikipedia.org/wiki/Image:RBOC_map.png

      As an example, imagine a call wanting to go from Lincoln NE to Seattle WA. In the five-level hierarchy, the call from the Plains states would be forced to go to one of the Regional Center switches at Denver, St. Louis, or Chicago. If the link from the chosen regional center to Seattle was blocked, the call was dropped. There was no way to switch to a new Regional Center switch without starting the call over. With DNHR, the call would initially try to connect directly from Lincoln to Seattle (i.e. from A to B in Figure 5a). If this direct connection was blocked for some reason, the call could "crank back" to Lincoln, which could then send it to any other directly connected city (not just the Regional Centers) and ultimately on to Seattle via the direct connection from this intermediate city (i.e. from A to C to B in Figure 5a). The path could ultimately be Lincoln NE to Dover DE to Seattle

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http://en.wikipedia.org/wiki/Regional_Bell_operating_company

WA, as an example, even though this seems counterintuitive. Quality loss over large distances was no longer a constraint on the system. Hence, the DNHR system was able to be implemented to handle a different quality constraint: dropped calls.
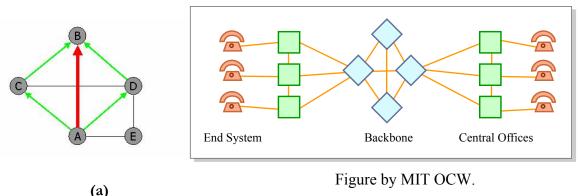

Figure by MIT OCW.

(a)

Figure 5 (a) Dynamic non-hierarchical routing (DNHR)

Ultimately, the smaller carriers liked the new dynamic routing systems as well. After the breakup, AT&T and MCI still controlled the long-distance market and charged access charges for RBOCs to use their network. With fixed paths from the five-level hierarchy, RBOCs would be forced to pay the access charges. Thanks to schemes like DNHR, RBOCs could set up their dynamical routing to try all of their own paths between cities before jumping to one of the IXCs. The PSTN network today only has two or three levels of central offices connecting to local and regional tandem as shown in Figure 5b.

Jumping ahead to 1996, the concern for local loop ownership being a bottleneck to the growth of local PSTN markets led to the unbundling act of 1996, which required the ILECs to share their access tandems with the competitive local exchange carriers (CLECs) for FCC-determined usage charges. In the following sections, we will see that due to various constraints, networks the ILECs inherited from AT&T compared to the new ones built by the CLECs have turned out to be different in structure and hence in capabilities.

Summary of constraints and the current state of PSTN

Before we begin the network analysis discussion, in this section we will summarize the constraints that had to be overcome during the evolution of the PSTN to understand the current state of the art in wired networks.

The distance constraint is addressed today using optical fibers that can go up to 70 km (compared to 1 km for copper) without using a repeater. They can carry up to a few Gbps (compared to 100 Mbps for copper) of traffic. A low-end optical fiber costs nearly the same as a high-end copper cable; however, what stalls the installation of fiber is the cost of switching electronics. A fiber network interface card costs approximately three to four times higher than the electronics to run copper (Barnett, Groth et al. 2004).

Losses in electromechanical contacts used to be a constraint before the 1950s. This was overcome by level-skipping that ensured not more than two electromechanical

connections were made to complete a call. With the introduction of automatic crossbar switches in the 1950s that used solid state electronics for switching, the losses in switching became secondary to the concern for the ability to handle the additional call volume, as the automatic switches enabled customer-to-customer direct dialing with no operator in the middle. This introduced routes additional to the existing level-skipping routes.

From the regulatory perspective, two constraints affect the evolution of the network. First, the access charges imposed for using another carrier's network leads to routing schemes that are non-hierarchical such as DNHR. Second, the equal access obligation requires Mini Bells to open their Access tandems to competitors[4]. On the one hand, upgrading the Access tandem switch is difficult for Mini Bell to coordinate as all of the connecting carriers must upgrade their side of the network simultaneously. On the other hand, Mini Bell has little incentive to upgrade the Access tandem switch since doing so would help all of its competitors using the same switch.

Many of the other constraints today are operational. For instance, obtaining the right-of-way and digging is more expensive than the cost of fiber. This makes it attractive to lay dark fiber once a company pays for digging. Also, reliability of electronics in the nodes is less of a concern compared to the physical breaks in fiber. This has led to the deployment of physically separate redundant fiber rings (discussed later). Finally, legacy is a big constraining factor. The network of Mini Bell, having evolved from the legacy AT&T network, has much different constraints on their choice of what they can change, as compared to the networks of new companies such as the Nano Bells.

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[4] Unbundling Act of 1996 required ILECs to designate several Access tandems that other competing carriers such as CLECs and IXCs can use to reach local customers. Since in any given state, there was a single Mini Bell (ILEC) that owned all local loops to homes, it was believed that it had a potential to stifle competition in lieu of such unbundling obligations.

Network Analysis

In this section we will carry out a network analysis of a network of a CLEC, a network of an ILEC and the interconnection between the two.

Network Modeling Decisions and Assumptions

- Nodes in the networks are tandem switches and central offices (aka host offices) in both the Mini Bell and Nano Bell networks.
- Our data for Nano Bell included both remote offices and host offices. Host offices connect with each other and the tandem switches to create the main structure. Approximately 3 remote offices connect to each host office in a parent node with three child node structure. Remember, a host office switches traffic for remotes connected to them, so for the purposes of our analysis, a cluster of one host and three connecting remotes constitutes a single node.
- Bandwidth capacities in the links of the network were not modeled, largely due to time constraints.
- When creating Nano Bell's network in 2005, leased lines from Mini Bell were assumed to connect disconnected host offices via the most direct path.

Call scenarios and the networks to analyze

Although collecting data proved somewhat difficult for this project, we still wanted to represent as many call scenarios as possible with the network data we received. The table below illustrates the possible call scenarios.

	Nano	Mini
Nano	In-network Local Calls	Inter-network Local & Long-distance Calls
Mini	Inter-network Local & Long-distance Calls	In-network Local Calls

Table 1: Call scenarios represented by phone company interactions

Various call scenarios, combined with the data sets we received, led us to run a network analysis on five networks:
1. Nano Bell's network in 2005
2. Nano Bell's projected network in 2010
3. Mini Bell's interconnections of tandem switches and central offices
4. Mini (#3) combined with Nano 2005 (#1)
5. Mini (#3) combined with Nano 2010 (#2)

Our reasoning behind these five networks is as follows. Today (after 1999), all companies are allowed to offer both local and long distance calls. The question becomes whose wires are used to make the call? Smaller companies, like Nano Bell, have a limited number of wires that they control. Thus, they can usually provide local and intrastate calls on their own network, but they must rely on Mini and Maxi Bell companies to provide interstate long-distance services. Since we were able to obtain detailed data for the Nano Bell network, they became our focus of in-network calls. Our Mini Bell is large enough to control a large network that allows them to provide interstate long-distance calls, but our data is sparse at best. However, all companies still must interact when the two callers have different service providers. Hence, we needed at least a portion of Mini Bell's network to represent the interconnection of Nano Bell and Mini Bell for inter-network calls.

Our initial focus was on Nano Bell's current and future networks. Looking at the 2005 network shown in Figure 6, Nano Bell currently has many branches extending away from a core that is connected by a few rings. This is indicative of its current network arrangement, where some of its isolated nodes are connected by lines leased from Mini Bell. Without complete control of its own network, Nano Bell focuses on connecting all of its cities as directly to a tandem switch as possible. By 2010, however, Nano Bell plans to connect all of its host offices via interconnected rings of double-fiber connections without leasing any fiber from Mini Bell. This redundant, fully interconnected ring structure is easily visible in the 2010 graph in Figure 6. The benefit of the additional capital necessary to create these doubly-linked rings comes in terms of network robustness. Table 2 shows the network parameters for both 2005 and 2010 Nano Bell networks. In a later section, we will discuss the simulations we ran to illustrate how robust the 2010 network is in comparison to the 2005 network, but for now, we'll illustrate conceptually why the doubly-linked ring structure is more robust.

Figure 6 Nano Bell Networks: Current (2005) and Future (2010)

Figure 7 below illustrates that a singly-linked fiber ring (a.k.a. collapsed rings) can have a host office isolated with the failure of only two links. The link failures are represented by the little x's, whereas the X over the H1 node represents the fact that the node is now isolated from the rest of the network. By physically separating two rings that connect the same set of nodes, two link failures is no longer enough to isolate the host office, as shown

on the right-hand side of Figure 7. It now takes the failure of four links before a host node is isolated from the network. So from the viewpoint of a single node, intuitively the node can sustain double the number of link failures when the number of links attached to the node doubles. However, our interest is in the network as a whole. By connecting all nodes with this doubly-linked ring structure (a.k.a. separated rings), what is the increase in the number of link failures the network can withstand before it becomes disconnected? Through simulation, we will show later that the factor of link failures the network can withstand is more than doubled when all nodes are connected via doubly-linked rings.
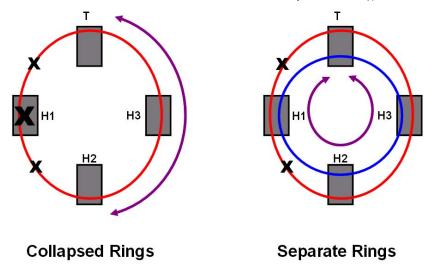

Figure 7 Robustness in redundant fiber rings

Figure 8 shows Mini Bell's network. In the center are the fully-connected tandem switches (Class 4 switches). There are four types of tandem switches: Access, Local, Toll and 911. Access tandems are the equal access tandems used by other carriers for reaching local customers. Local tandems are for routing the intrastate local calls. Toll tandems are for the long-distance traffic, whereas 911 tandems are for routing emergency access calls. Connected to the tandems are the clusters of central offices (Class 5 switches). As we can see, Mini Bell's network appears much more hierarchical. Our conjecture, based on domain knowledge, is that there are three reasons for this. First, it is difficult and costly to change the legacy network they have inherited from AT&T. Second, the regulatory obligation of equal access that forces them to share the Access tandems with other carriers makes coordination of any upgrade difficult. Finally, there is the fact that Mini Bell built its network with low bandwidth voice communication in mind, whereas the new companies such as Nano Bell have engineered their network to handle high bandwidth voice and data traffic.
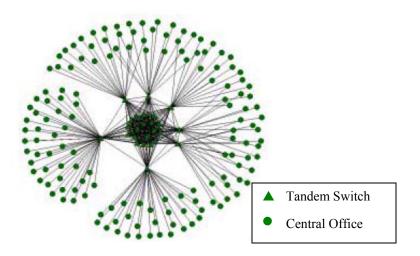
Figure 8 Current Mini Bell network

Shifting back to the focus on rings, it was not surprising to find a low clustering coefficient for Nano Bell's networks. In both cases, the clustering coefficient was approximately 0.025. Our contention is that for understanding clustering in a network with rings, one needs a new statistic that is not counting the number of triangles. This differs from Mini Bell's network that has a clustering coefficient of 0.12, due to its fully connected set of tandem switches that increases the number of triangles in the network.

When considering the two networks of Nano Bell and Mini Bell, the Pearson correlation coefficient (r), seen in Table 2, would suggest something is fundamentally different. For Nano Bell's networks, 'r' is strongly positive, while Mini Bell's 'r' is strongly negative. However, the above network structure for Mini Bell does not have any central offices connected to each other, which we believe is highly unlikely in the true situation. Unfortunately, the interconnections between central offices constitute a highly proprietary part of the information and hence could not be obtained due to competitive reasons. Knowing that central offices are likely connected due to the adoption of level-skipping in early years, and then the DNHR, we ran a simulation that added links to Mini Bell's network to see its effect on 'r.' These results are discussed later, but ultimately, adding connections drove 'r' towards a positive value.

For completeness, we connected Nano Bell's network to Mini Bell's network, illustrated in Figure 9. This was modeled by adding a fixed number of connections, 123, between Nano and Mini Bell networks. The 123 number was given to us during an interview with our source at Nano Bell, but we were not told where these connections were located. Thus, to simulate the connections, we first connected some of Nano Bell's host offices (central offices) to geographically close tandem switches and central office switches in Mini Bell's network. Next, we connected Nano Bell's tandem switches to a larger number of various geographically close tandem switches and central office switches in Mini Bell's networks. Each Nano Bell tandem switch has upwards of 6 or 8 connections to Mini Bell's network, whereas the Nano Bell host offices that are connected to Mini Bell's network only have 1 or 2 connections. We gathered from our interviews that , about 1/3rd of Nano Bell to Mini Bell connections are between their central offices. For the tandem connections, only

Access and Local tandems in Mini Bell's network are connected to Nano Bell. While we were unable to verify their accuracy, we feel that spreading out the 123 connections over our connection space should provide a reasonably accurate model of the interconnection of the two networks.
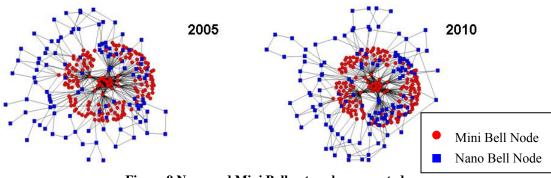

Figure 9 Nano and Mini Bell networks connected

As we expected, most of the key network analysis variables (average node degree $<k>$, average path length l, and Pearson's correlation coefficient r) for the interconnected Nano and Mini network fell in between the Nano values and the Mini values shown in Table 2. We found that the clustering coefficient (C), discussed earlier, did not follow this pattern. For the combined network, the C value increased beyond the Mini Bell network's C value. This is attributed to the connections made in particular with the previously singly-connected Mini Bell central offices. Mini Bell nodes that were geographically close to Nano Bell nodes were favored. Thus, the sharp increase in the clustering coefficient value should not be considered to be indicative of anything critical to the network analysis. Ultimately, we were unable to take this part of the analysis further than this at this time.

Parameter	Nano 2005	Nano 2010	Mini Only	Mini+Nano 2005	Mini+Nano 2010
N	104	123	171	275	295
M	121	152	446	667	714
z (<k>)	2.327	2.452	5.216	4.85	4.84
l	7.308	8.729	2.582	3.71	4.275
log n/ log <k>	5.499	5.365	3.113	3.557	3.606
C	0.0262	0.0206	0.1179	0.196	0.2136
<k>/n	0.022	0.020	0.031	0.018	0.016
r	0.2196	0.3277	-0.6458	-0.1882	-0.1552

Table 2: Network analysis metrics for the five networks modeled

Network Experiments: Dynamically changing Pearson's Correlation Coefficient

As shown in Figure 8, we constructed the Mini Bell network for the U.S. state that we modeled. There are 7 central office clusters geographically distributed in the state. Each central office within a cluster connects to the same tandem. Since there are 7 clusters, there are 7 of these "front tandems." In total, there are 25 tandems, and all are assumed to connect to each other, making this part of the network fully connected as per Keshav 1997. In Mini Bell's online database only the connections from central offices to tandems are provided. The tandems themselves are assumed to be fully connected (a conventional assumption), whereas the connections between central offices are not provided for competitive reasons. The clusters show the connections we found between a tandem switch to each central office connected to them. Knowing that other connections amongst central offices and tandems exist due to the level-skipping structure of the past, we created the following three cases to test how the Pearson's correlation coefficient changes under different wiring assumptions. The assumption of tandem switches being fully connected was included a priori in all three cases.

Case 1: Randomly add edges within each cluster of Central Offices (COs)

Assumptions:
- Maintain current hierarchy structure
- Only randomly add edges between central offices (COs) within each cluster
- The COs within each cluster are fully connected at the end of simulation

From these assumptions, the maximum number of edges that could be added is 1756. For each iteration, we randomly select an edge, add it to the Mini Bell network, and then calculate the corresponding Pearson's degree correlation coefficient. We continuously add edges until all possible edges are added. Since this process is stochastic, we repeat the simulation 50 times and take an average of the degree correlation coefficient over all iterations. The plot in Figure 10 shows how the correlation coefficient changes at each step. The blue line is the average result, and the red line is a sample case. We can see the process is very stable. Based on the simulation, the average number of additional edges needed for achieving a zero degree correlation coefficient is 185, which is only about 10% of the total number of edges to be added. Overall, the Pearson's degree correlation coefficient rises from -0.6458 to 0.7403 as we progress from adding 0 to 1756 edges. Figure 11 shows what the network looks like when the degree correlation becomes zero.
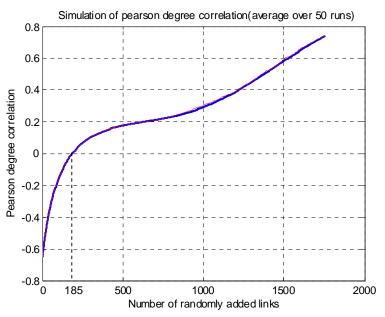
Figure 10 Randomly add edges within central offices' clusters
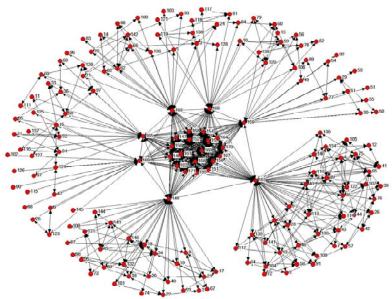

Figure 11 Mini Bell network when r=0

Case 2: Randomly add edges between all Central Offices

Assumptions:
- Cross cluster linking is possible even when two COs are located far away from each other
- Edges are added randomly only between COs
- All COs are fully connected at the end of simulation

Therefore, there are 146*145/2= 10585 edges that could be added. The simulation results are shown in Figure 12. One interesting observation is that while we are adding more edges, the degree correlation coefficient does not increase monotonically. There are three points with zero degree correlation coefficients corresponding to adding 183, 1599, and 2325 edges on average, respectively. We think this is due to network architecture constraints. Although we keep randomly adding edges between COs, the hierarchical architecture between tandems and COs is unchanged. Each CO still only connects to its one "front tandem." In this case, the degree correlation coefficient rises from -0.6458 to 0.8419, which is higher at the end of the simulation than it is in Case 1. Figure 13 through Figure 14(b) show three network structures corresponding to three points highlighted in Figure 12, respectively. The network in Figure 13 still shows the hierarchical structure since only 190 edges have been added. The upper left hand clusters in Figure 14(a) and (b) are the fully connected tandem switches.
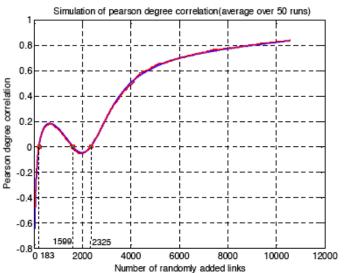

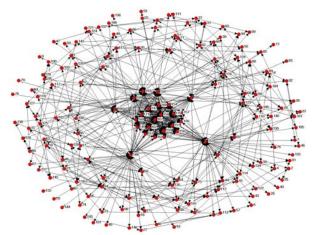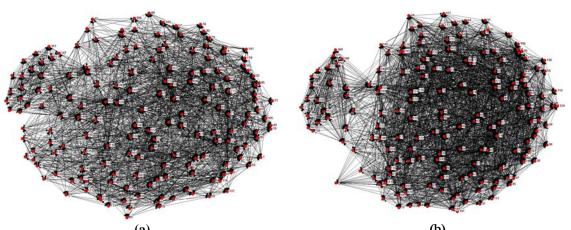**Figure 12 Randomly add edges for all central offices**

Figure 13 Mini Bell when r = 0 with 190 added edges

Figure 14 Mini Bell (a) when r = 0 with 1599 added edges and (b) when r = 0 with 2305 added edges

Case 3: Completely randomly add edges to the Mini Bell network

Assumptions:
- Central offices can connect to any other central office
- Central offices can connect to any tandem switch
- All COs and Tandems are fully connected at the end of the simulation

The total number of links that can be added is $171*170/2- (146+25*24/2) = 14089$ in this case. The simulation results are shown in Figure 15. From the plot, we observe that the Pearson's degree correlation coefficient increases and reaches a peak at 0.1611, and then decreases to -0.0118 (Note: r=-0.0118 when the network has its last pair of edges still unconnected. The Pearson Matlab routine does not work for a fully connected network; the return of Pearson's degree correlation for a fully connected network is NaN). Figure 16(a) and (b) show the corresponding network structures at the first two points highlighted in Figure 15. The third point in Figure 15 is the second time when r =0. The reason why r reaches 0 and then decreases to -0.0118 is because of undirected network, which also means the random links in the adjacent matrix are added pairwise. The fully connected network should have 0 degree correlation, which is not shown in this figure due to the Pearson's routine's failure for fully connected network. Since the network with 8640 edges added is very dense, we can not gain much insight by looking at the network, so we do not show it in this paper. One observation from this experiment is that the fully randomly added links gradually destroys the original network hierarchical structure between tandems and COs. Also, as the number of edges added reaches the maximum number, driving the network to be fully connected, the Pearson's correlation coefficient approaches 0, as expected analytically.
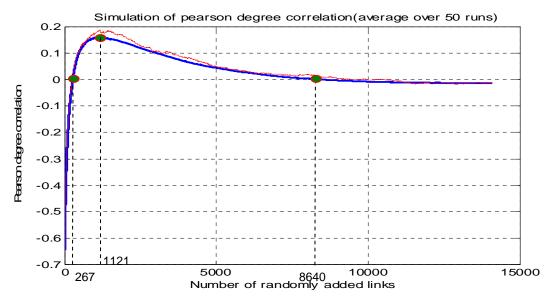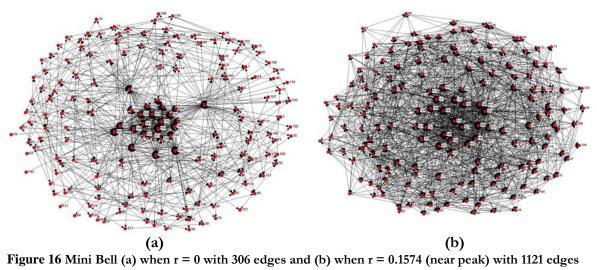

Figure 15 Randomly add edges for all central offices and tandems

(a) **(b)**

Figure 16 Mini Bell (a) when r = 0 with 306 edges and (b) when r = 0.1574 (near peak) with 1121 edges

Summary of Pearson's degree correlation experiments

	Maximum number of edges can be added	Final Pearson' degree correlation	First time when r = 0 (# edges added)	Second time when r = 0 (# edges added)	Third time when r=0 (# edges added)
Case 1: add edge within CO clusters	1756	0.7403	185	-	-
Case 2: add edges for all COs	10585	0.8419	183	1599	2325
Case 3: add edges to COs and Tandems (Complete random)	14089	-0.0118	267	8640	-

Table 3 Summary of Pearson's degree correlation experiments

Network Experiments: Robustness Analysis for Nano Bell

One of the key design goals of Nano Bell's 2010 network is to achieve robustness by introducing separate rings. In order to see how robust the 2010 network is compared to 2005, we did several simulation experiments in Matlab by randomly removing nodes or edges in both networks. Here we define two metrics for evaluating the robustness of the Nano Bell networks shown in Figure 6.

Metric 1: Number of node/edge failures the network can tolerate, which also means how many nodes or edges can be randomly removed before the rest of the network becomes disconnected.

Metric 2: Number of clusters existing in the rest of the network by randomly removing nodes or edges.

Case 1: Randomly remove nodes

In the simulation, we randomly remove nodes from the network until the rest of the network becomes disconnected or reaches the maximum number of node failures we predefine. We replicate the experiment 500 times and take the mean values of the number of node failures the network can tolerate. Since the results also depend on the maximum number of node failures, we ran a set of experiments with different predefined maximum node failures. Table 4 shows the results. The number in the table means the average number of node failures that the networks can tolerate. Clearly, the numbers of Nano 2010 are higher than Nano 2005, which means Nano 2010 is more robust in terms of node failures. Figure 17 shows the probability distribution of the number of node failures that the Nano 2005 and 2010 networks can tolerate. Table 5 shows the average number of clusters existing in the rest of the network after randomly removing the predefined number of nodes listed in the first column of the table. We changed the replication to 50 runs because of the intensive computational effort required to calculate the number of clusters in the networks. From the number of clusters remaining in the rest of the network, we see that Nano 2010 has fewer clusters than Nano 2005, making Nano 2010 more robust.

Maximum number of node failures	Nano 2005	Nano 2010
1	0.5620	0.9040
10	1.2500	3.8560
20	1.4000	3.6440
50	2.0700	3.0400

Table 4 Number of node failures that Nano 2005 and 2010 can tolerate (500 runs)

Number of node failures	Nano 2005	Nano 2010
1	1.4200	1.2000
10	6.2200	3.5200
20	12.4800	8.4800
50	23.1400	23.0200

Table 5 Number of clusters existing in the rest of the network with random node failures (50 runs)
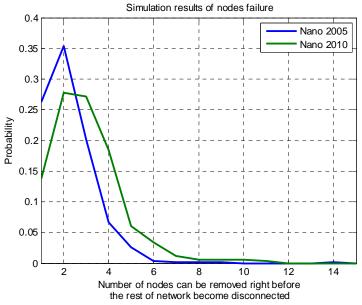
Figure 17 Probability distribution of number of node failures that Nano 2005 and 2010 can tolerate

Case 2: Randomly remove edges

The same experiment procedures are applied to randomly removing edges. The results are shown in Table 6, Table 7 and Figure 18. Again, it is obvious that the Nano 2010 network is more robust than the 2005 network, this time in terms of randomly removing edges.

Maximum number of edge failures	Nano 2005	Nano 2010
1	0.5520	0.9800
10	1.3340	7.7540
50	1.2400	13.0280
100	1.1940	11.7820

Table 6 Number of edge failures that Nano 2005 and 2010 can tolerate (500 runs)

Number of edge failures	Nano 2005	Nano 2010
1	1.3600	1.0400
10	5.6800	1.4000
20	10.6600	2.8000
50	26.7000	7.3000

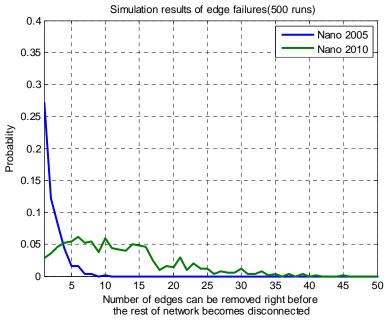**Table 7 Number of clusters existing in the rest of the network with random edge failures (50 runs)**

Figure 18 Probability distribution of number of edge failures that Nano 2005 and 2010 can tolerate

Contributions to the ESD.342 Project Portfolio

We believe our analysis adds the following points to the current understanding of network analysis of the PSTN:

- The new hierarchy is flat: from 5 to ~3 levels
- The new wired network is a hybrid of copper and fiber
- The new architecture is a tree structure with rings
- The new routing scheme is DNHR (Dynamic Non-Hierarchical Routing) instead of level skipping
- The Pearson's correlation coefficient has been changing from negative to positive as the network evolves
- The network analysis confirms the increased robustness of the new fiber network architecture of separate rings

Recommendations for Future Work

We have the following recommendations for future work. For the immediate, our analysis can be enhanced by coding in the link and node characteristics data. Link characteristics such as bandwidth and traffic loads can be obtained by a few more interviews with our Nano Bell contact. Node information such as switching capacity, number of customers served and the characteristics of traffic switched can be obtained by a combination of interviews and some approximation.

We also observed similarities between our network and the HOT network (Lun, Alderson et al. 200) if remote offices and homes (or access lines) are included. Verifying this by modeling all offices and access lines is therefore the other step that could be taken with relative ease.

For deeper analysis, more data is necessary. The richest data source for the US PSTN is Telcordia's LERG[5] database. LERG offers time series data showing network deployments. With such data, the impact of legacy on the evolution of Mini Bell's network may be possible to analyze.

Finally, it would be interesting to do a joint analysis of the Internet and the PSTN network. However, this will require much triangulation and clean up of various data sources before a meaningful snapshot of data can be created.

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[5] http://www.telcordia.com/products_services/trainfo/catalog_details.html

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