Inventory Management
Time Varying Demand
Fixed Planning Horizon

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Assumptions: Basic FPH Model

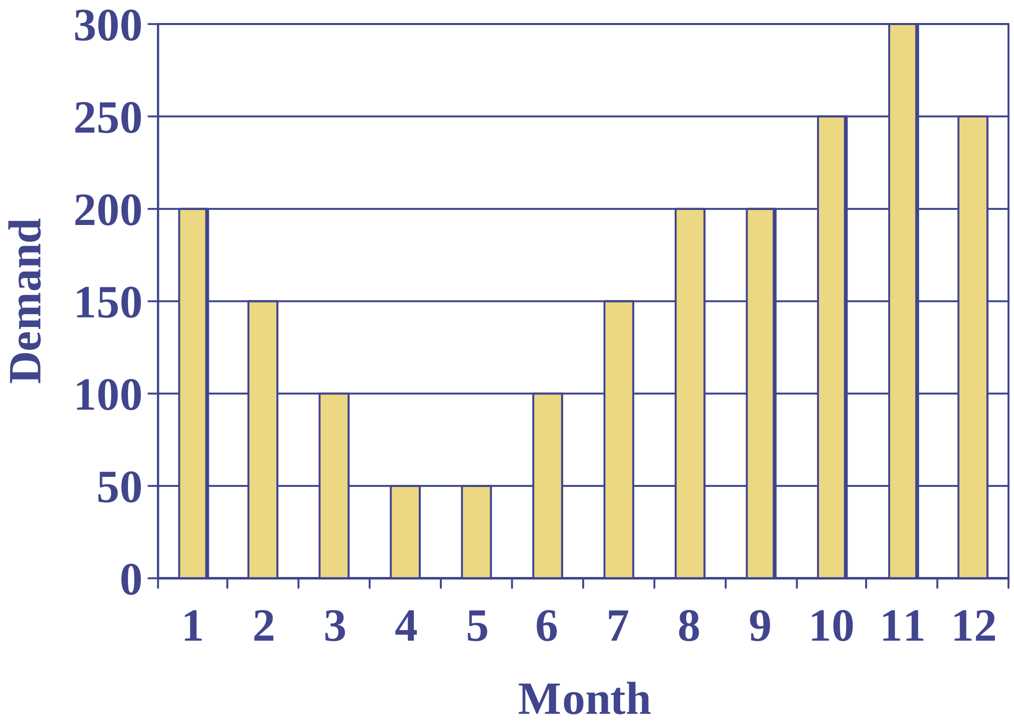
- Demand
  - Constant vs **Variable**
  - **Known** vs Random
  - **Continuous** vs Discrete
- Lead time
  - **Instantaneous**
  - Constant or Variable (deterministic/stochastic)
- Dependence of items
  - **Independent**
  - Correlated
  - Indentured
- Review Time
  - **Continuous**
  - Periodic
- Number of Echelons
  - **One**
  - Multi (>1)
- Capacity / Resources
  - **Unlimited**
  - Limited / Constrained
- Discounts
  - **None**
  - All Units or Incremental
- Excess Demand
  - **None**
  - All orders are backordered
  - Lost orders
  - Substitution
- Perishability
  - **None**
  - Uniform with time
- Planning Horizon
  - Single Period
  - **Finite Period**
  - Infinite
- Number of Items
  - **One**
  - Many
- Form of Product
  - **Single Stage**
  - Multi-Stage

Example

When should I order and for how much?

Costs

- D = 2000 items per year
- A = $500.00 per order
- v = $50.00 per item
- r = 24% per item per year
- C_{hm} = rv/N = 1 $/month/item
- N = number of periods per year



More Assumptions

- Demand is required and consumed on first day of the period
- Holding costs are not charged on items used in that period
- Holding costs are charged for inventory ordered in advance of need

Methods Used

Different Approaches

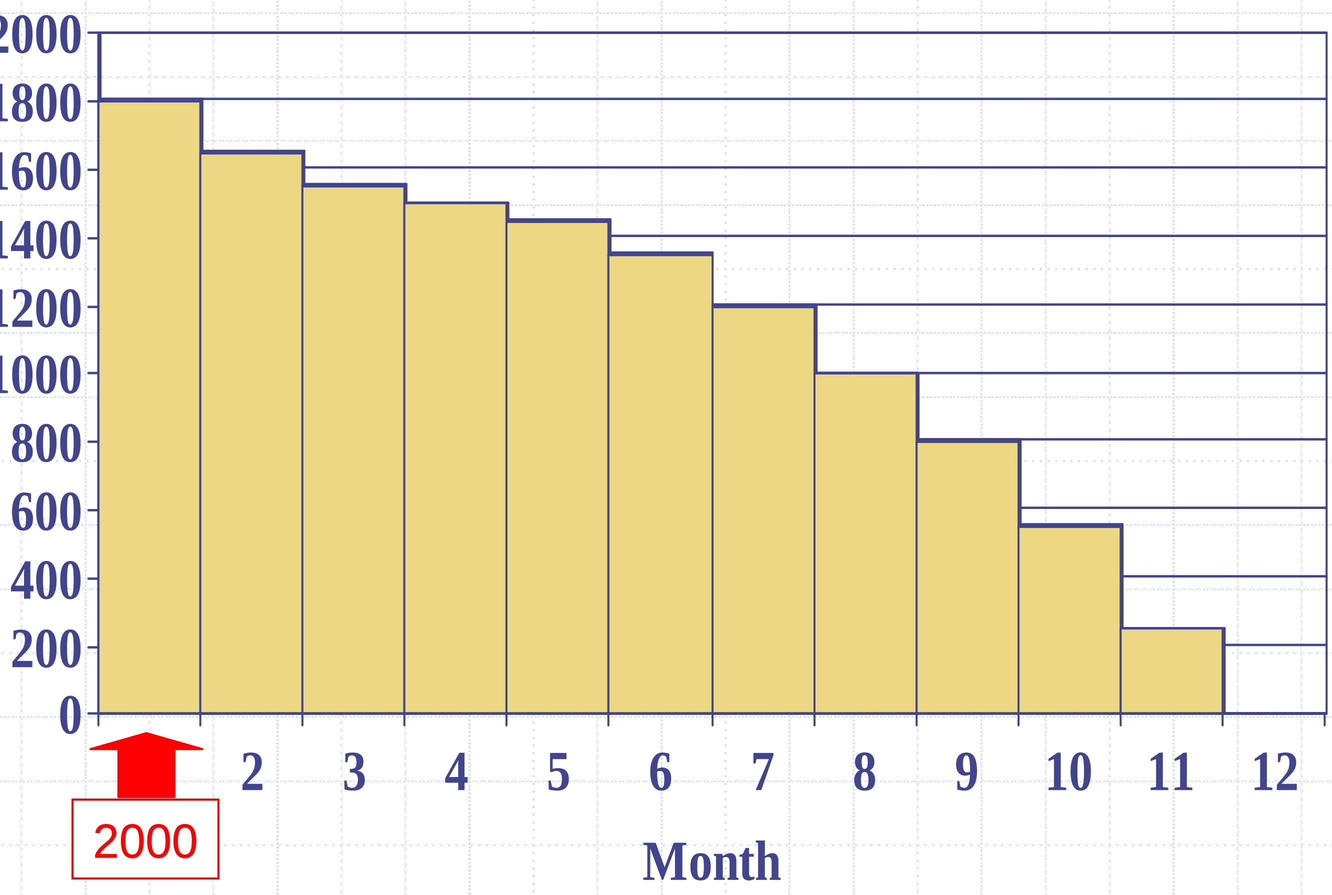
1. Simple Heuristics
   - The One-Time Buy
   - Lot For Lot
   - Fixed Order Quantity (FOQ)
   - Periodic Order Quantity (POQ)
2. Optimal Procedures
   - Wagner-Whitin (Dynamic Programming)
   - Mixed Integer Programming
3. Specialty Heuristics
   - The Silver Meal Algorithm
   - Least Unit Cost (LUC)
   - Part-Period Balancing (PPB)

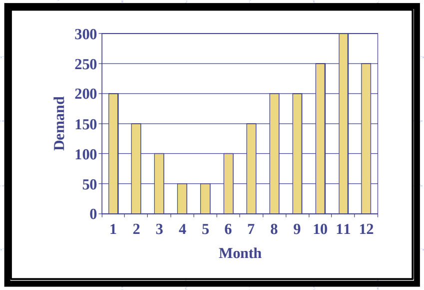
Simple Heuristics

- One Time Buy
- Lot for Lot
- Fixed Economic Order Quantity
- Periodic Order Quantity

Approach: One-Time Buy

Policy
Buy D at time 0



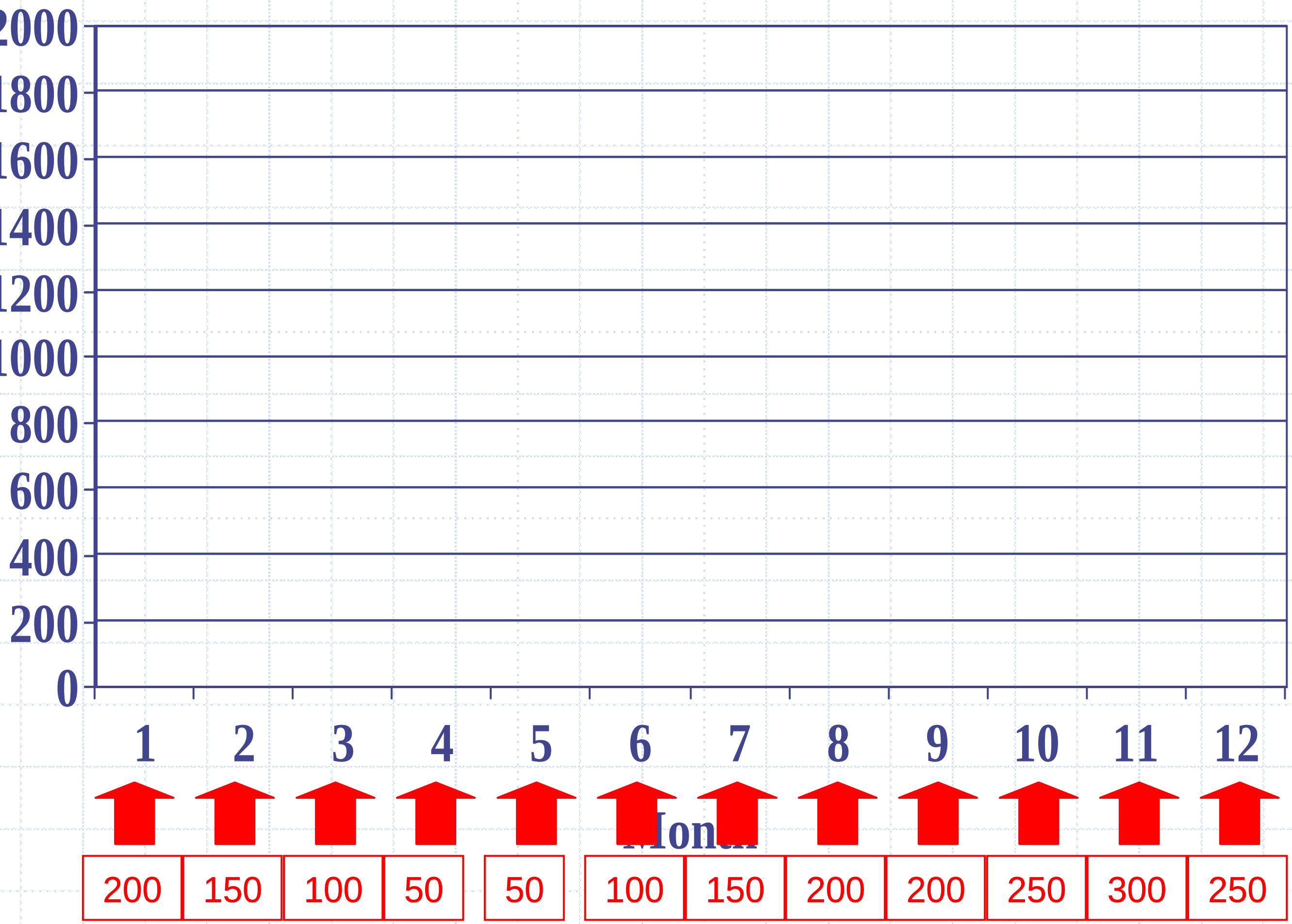


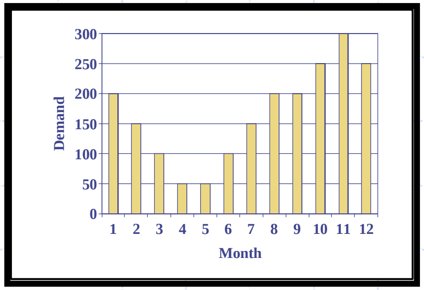
Approach: One-Time Buy

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	2000	$1800	$500	$2300
2	150	0	$1650	$0	$1650
3	100	0	$1550	$0	$1550
4	50	0	$1500	$0	$1500
5	50	0	$1450	$0	$1450
6	100	0	$1300	$0	$1300
7	150	0	$1200	$0	$1200
8	200	0	$1000	$0	$1000
9	200	0	$800	$0	$800
10	250	0	$550	$0	$550
11	300	0	$250	$0	$250
12	250	0	$0	$0	$0
Totals:	2000	2000	$13100	$500	$13600

Approach: Lot for Lot

Policy
Buy D(t) at time t



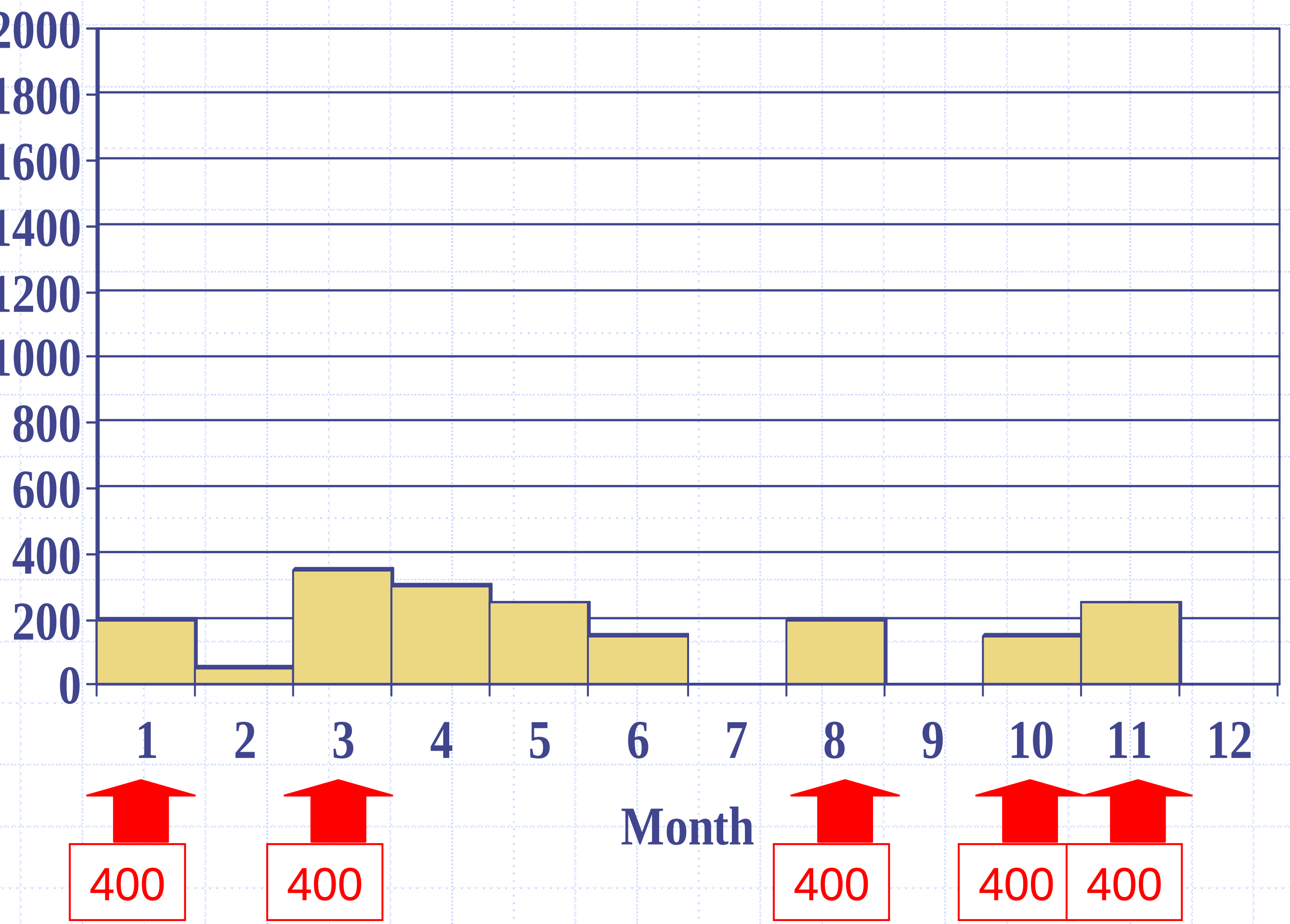


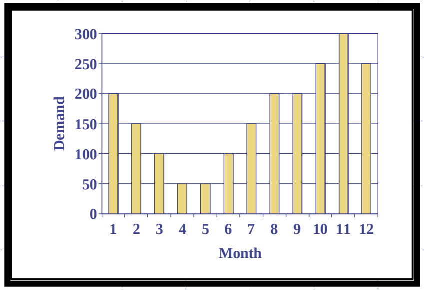
Approach: Lot for Lot

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	200	$0	$500	$500
2	150	150	$0	$500	$500
3	100	100	$0	$500	$500
4	50	50	$0	$500	$500
5	50	50	$0	$500	$500
6	100	100	$0	$500	$500
7	150	150	$0	$500	$500
8	200	200	$0	$500	$500
9	200	200	$0	$500	$500
10	250	250	$0	$500	$500
11	300	300	$0	$500	$500
12	250	250	$0	$500	$500
Totals:	2000	2000	$0	$6000	$6000

Approach: EOQ

Policy
Order Q* if D(t)>IOH





Approach: EOQ

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	400	$200	$500	$700
2	150	0	$50	$0	$50
3	100	400	$350	$500	$850
4	50	0	$300	$0	$300
5	50	0	$250	$0	$250
6	100	0	$150	$0	$150
7	150	0	$0	$0	$0
8	200	400	$200	$500	$700
9	200	0	$0	$0	$0
10	250	400	$150	$500	$650
11	300	400	$250	$500	$750
12	250	0	$0	$0	$0
Totals:	2000	2000	$1900	$2500	$4400

Approach: Periodic Order Quantity

- Similar to EOQ
  - Find the optimal order cycle time, T*, for EOQ using annual demand
  - Set POQ = Round up of T* to nearest integer
  - Every POQ time periods, order enough to satisfy demand for that POQ periods in the future
- Example
  - T*= 0.204 years = 2.45 months
  - POQ = 3 months

Approach: POQ

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	450	$ 250	$ 500	$ 750
2	150	0	$ 100	$ -	$ 100
3	100	0	$ -	$ -	$ -
4	50	200	$ 150	$ 500	$ 650
5	50	0	$ 100	$ -	$ 100
6	100	0	$ -	$ -	$ -
7	150	550	$ 400	$ 500	$ 900
8	200	0	$ 200	$ -	$ 200
9	200	0	$ -	$ -	$ -
10	250	800	$ 550	$ 500	$ 1,050
11	300	0	$ 250	$ -	$ 250
12	250	0	$ -	$ -	$ -
Totals:	2000	2000	$ 2,000	$ 2,000	$ 4,000

Policy

Order Sum(D) every POQ time periods

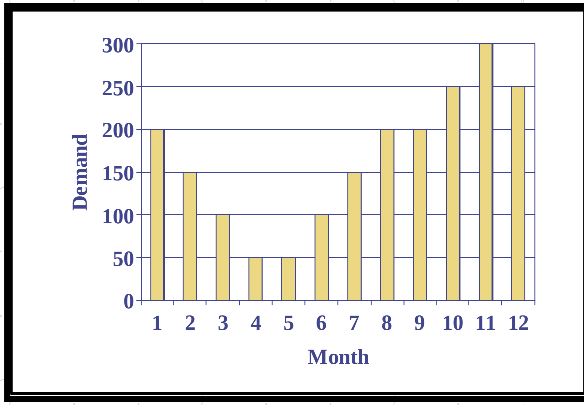
Optimal Methods

- Wagner Whitin
- Mixed Integer Linear Programming

Approach: Wagner-Whitin

- Relies on 2 Key Properties
  - Zero Inventory Ordering Property exists
  - Upper limit on holding time for demand
- Algorithm
  - Start at t=1,
  - Find cost for ordering just enough for D(t)
  - Look at past orders (until t=1)
    - Find cost for ordering enough for D(t) by adding it into the previous order for D(t-1)
  - Pick lowest cost of Options – Go to next t
  - At t=N – find lowest cost option and work backwards

Approach: Wagner-Whitin



Example:

- Period 1:
  - Order 200 at a cost of A=$500
- Period 2:
  - Option 1: Best Period 1 Plan plus new order in period 2
    Cost = F(1) + A = $1000
  - Option 2: Order enough in period 1 to cover demand up to period 2
    Cost = $A+C_{hm}D(2)$ = $500 + (1$)(150) = $650
- Period 3:
  - Option 1: Best Period 2 Plan plus new order in period 3
    Cost = F(2) + A = $650 + $500 = $1,150
  - Option 2: Best Period 1 Plan plus Period 2 Order to cover demand up to period 3
    Cost = $F(1) + A + C_{hm}D(3)$ = $500 + $500 + ($1)(100) = $1,100
  - Option 3: Order enough in period 1 to cover demand up to period 3
    Cost = $A+C_{hm}D(2) + 2C_{hm}D(3)$ = $500+(1$)(150)+2(1)(100) = $850

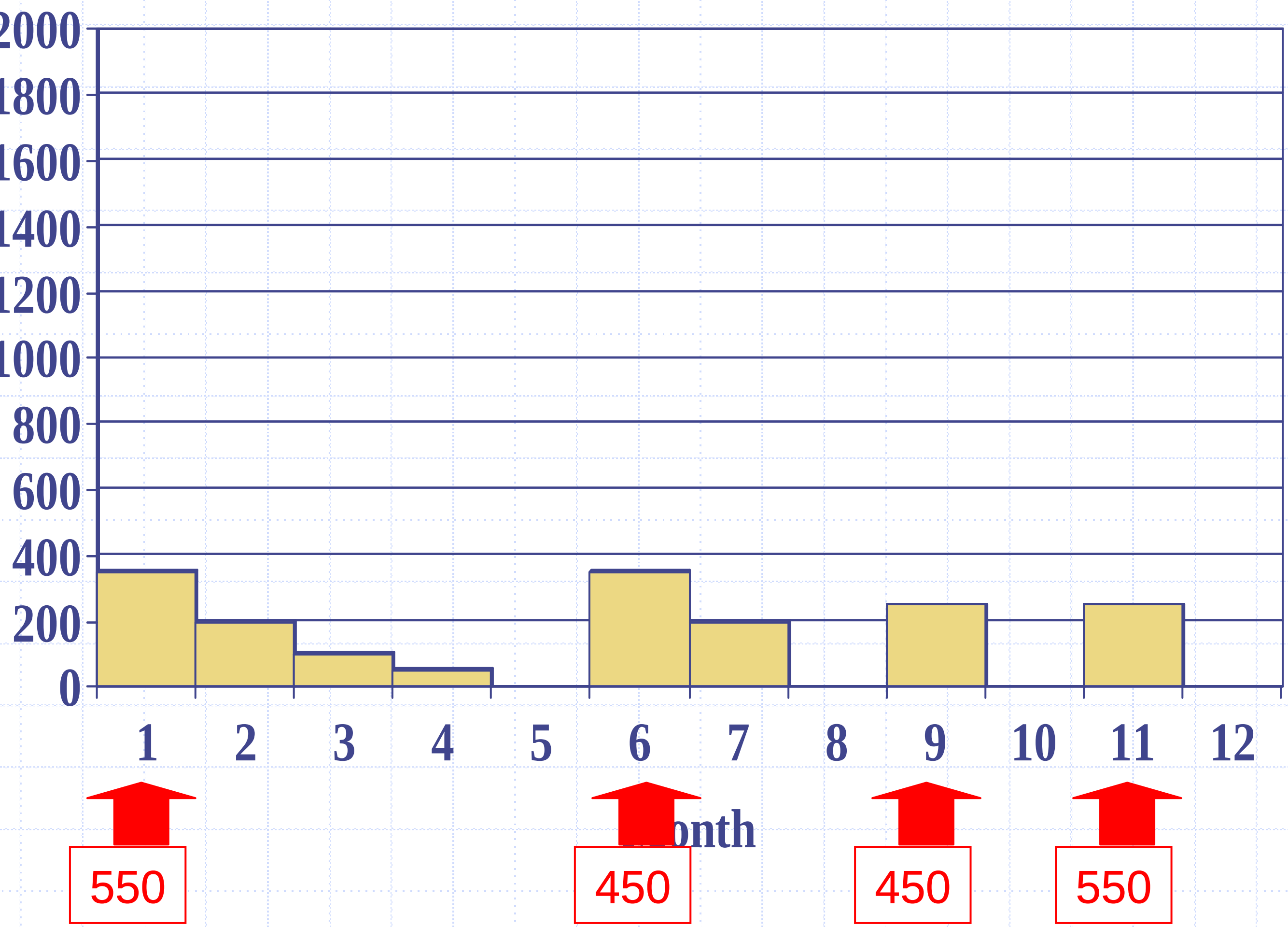
- Easy to build a Spreadsheet model
- Note that if Demand of any period, j, is greater than A/C_{hm} then we know that it is best to order in that period. Why?

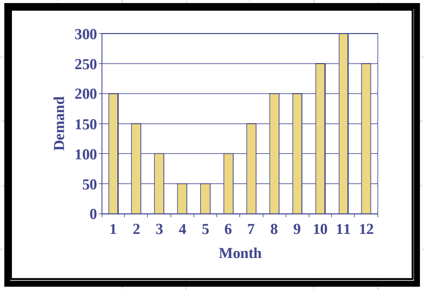
Approach: Wagner-Whitin

Period	1	2	3	4	5	6	7	8	9	10	11	12
Demand	**200**	**150**	**100**	**50**	**50**	**100**	**150**	**200**	**200**	**250**	**300**	**250**
Order 1	500	650	850	1,000	**1,200**	1,700	2,600	4,000	5,600	7,850	10,850	13,600
Order 2		1,000	1,100	1,200	1,350	1,750	2,500	3,700	5,100	7,100	9,800	12,300
Order 3			1,150	1,200	1,300	1,600	2,200	3,200	4,400	6,150	8,550	10,800
Order 4				1,350	1,400	1,600	2,050	2,850	3,850	5,350	7,450	9,450
Order 5					1,500	1,600	1,900	2,500	3,300	4,550	6,350	8,100
Order 6						1,700	1,850	**2,250**	2,850	3,850	5,350	6,850
Order 7							2,100	2,300	2,700	3,450	4,650	5,900
Order 8								2,350	2,550	3,050	3,950	4,950
Order 9									2,750	**3,000**	3,600	4,350
Order 10										3,050	3,350	3,850
Order 11											3,500	**3,750**
Order 12												3,850

Optimal Order Policy:
Order 550 in period 1, 450 in period 6, 450 in period 9, and 550 in period 11

Approach: Wagner-Whitin





Approach: Optimization (MILP)

Decision Variables:

- Qi = Quantity purchased in period i
- Zi = Buy variable = 1 if Q_i>0, =0 o.w.
- Bi = Beginning inventory for period I
- Ei = Ending inventory for period I

Data:

- D_i = Demand per period, i = 1,,n
- C_o = Ordering Cost
- C_{hp} = Cost to Hold, $/unit/period
- M = a very large number….

MILP Model

Objective Function:

- Minimize total relevant costs

Subject To:

- Beginning inventory for period 1 = 0
- Beginning and ending inventories must match
- Conservation of inventory within each period
- Nonnegativity for Q, B, E
- Binary for Z

Approach: Optimization (MILP)

$$Min\ TC = \sum_{i=1}^{n} C_O Z_i + \sum_{i=1}^{n} C_{HP} E_i$$

Objective Function

$s.t.$

$$B_1 = 0$$
$$B_i - E_{i-1} = 0 \qquad \forall i = 2,3,\ldots n$$

Beginning & Ending Inventory Constraints

$$E_i - B_i - Q_i = -D_i \qquad \forall i = 1,2,\ldots n$$

Conservation of Inventory Constraints

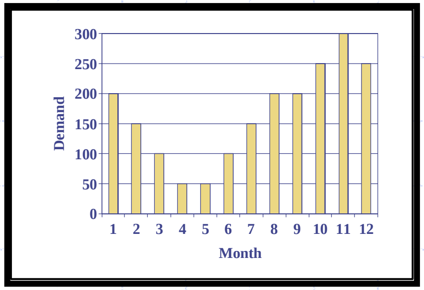
$$MZ_i - Q_i \geq 0 \qquad \forall i = 1,2,\ldots n$$

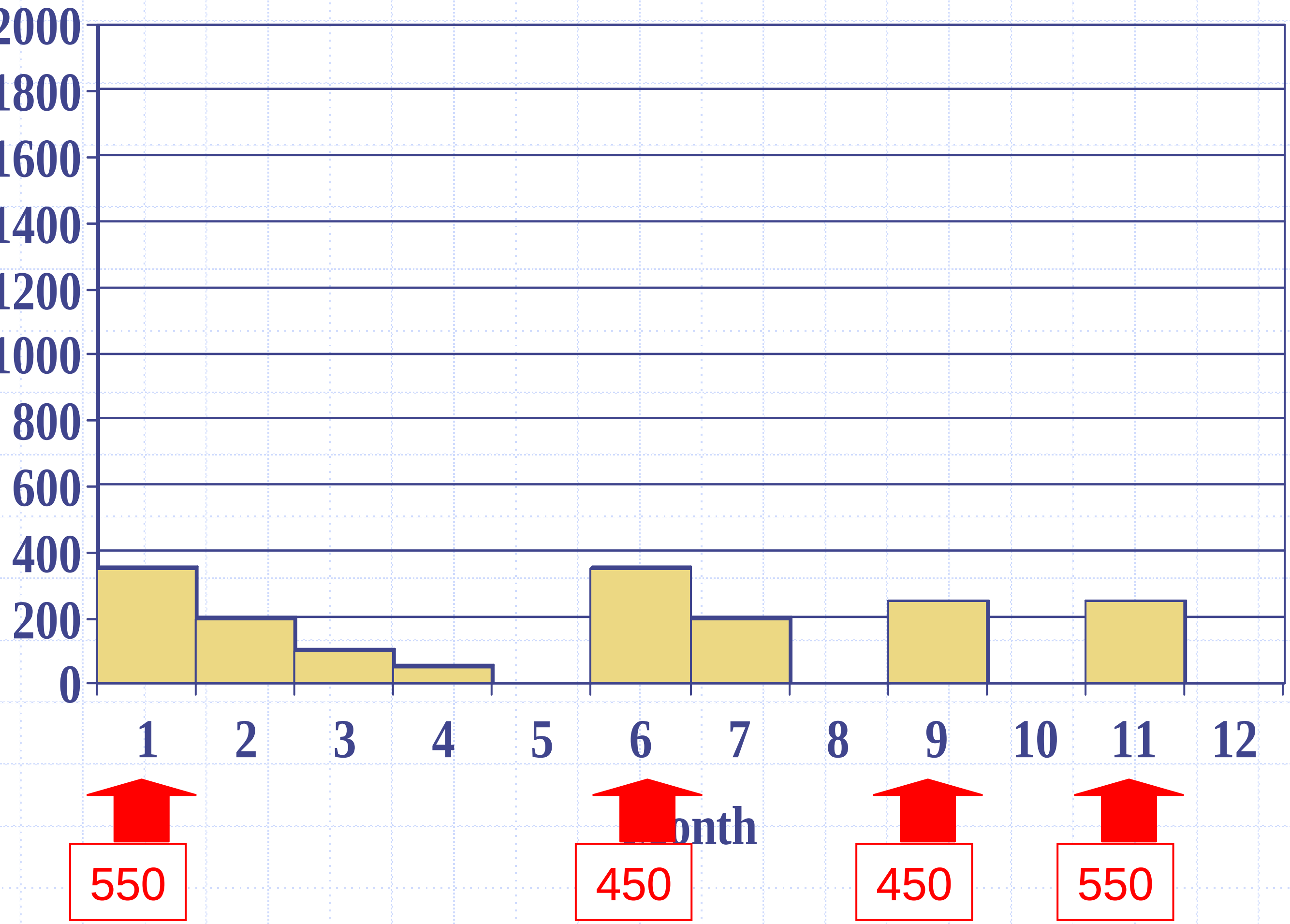
Ensures buys occur only if Q>0

$$B_i \geq 0 \qquad \forall i = 1,2,\ldots n$$
$$E_i \geq 0 \qquad \forall i = 1,2,\ldots n$$
$$Q_i \geq 0 \qquad \forall i = 1,2,\ldots n$$
$$Z_i = \{0,1\} \qquad \forall i = 1,2,\ldots n$$

Non-Negativity & Binary Constraints

Approach: Optimization (MILP)





Special Heuristics

- Silver-Meal (Least Period Cost)
- Least Unit Cost
- Part-Period Balancing

Approach: Silver-Meal Algorithm

- Objective
  - Minimize total relevant cost per unit time (TRCUT)
  - TRCUT(T) = TRC(T)/T = (Order + Carrying)/T
- Decision Rule:
  - Add next period's demand to the order if the average cost per period is reduced
- Algorithm
  1. Start at first period
  2. Set T=1
  3. If TRCUT(T) > TRCUT(T-1) then
     - Previous order goes for T-1 periods with Q=sum(D) for T,
     - Start new order and go to 2
  4. Else, T=T+1 and go to 3

Approach: Silver-Meal Algorithm

Mon	Dmd	Lot Qty	Order Cost	Holding Cost	Lot Cost	TRCUT
1st	Buy:					
1	200	200	$500	$0	$500	$500
2	150	350	$500	$150	$650	$325
3	100	450	$500	$150+$200	$850	$283
4	50	500	$500	$150+$200+$150	$1000	$250
5	50	550	$500	$150+$200+$150+$200	$1200	$240
6	100	650	$500	$150+$200+$150+ $200+$500	$1700	$283
2nd	Buy:					
6	100	100	$500	$0	$500	$500
7	150	250	$500	$150	$650	$325
8	200	450	$500	$150+$400	$1050	$350

Approach: Silver-Meal Algorithm

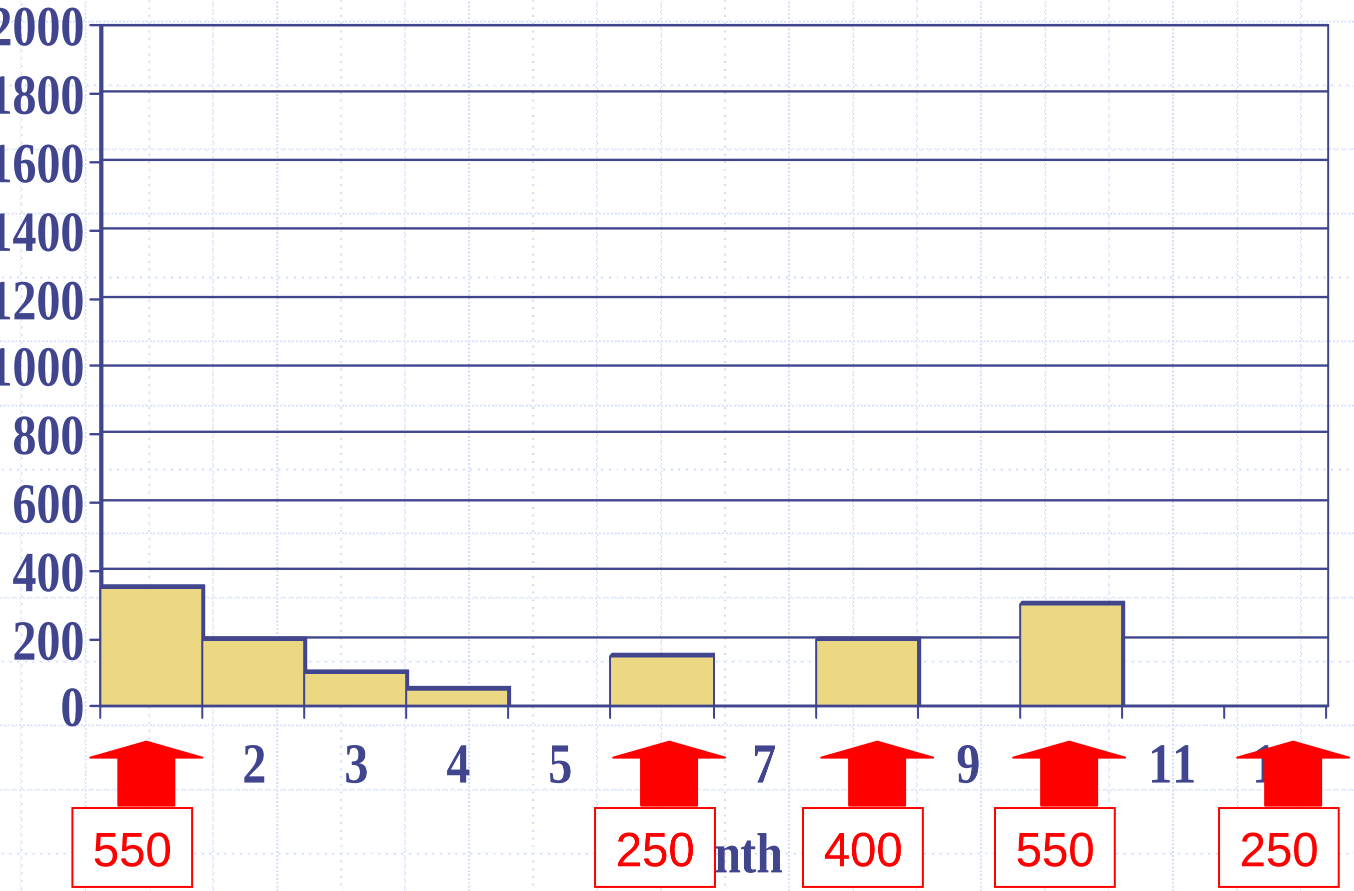
Mon	Dmd	Lot Qty	Order Cost	Holding Cost	Lot Cost	TRCUT
3rd	Buy:					
8	200	200	$500	$0	$500	$500
9	200	400	$500	$200	$700	$350
10	250	650	$500	$200+$500	$1200	$400
4th	Buy:					
10	250	250	$500	$0	$500	$500
11	300	550	$500	$300	$800	$400
12	250	800	$500	$300+$500	$1300	$433
5th	Buy:					
12	250	250	$500	$0	$500	$500

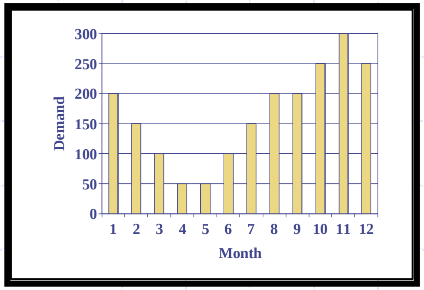
Approach: Silver-Meal Algorithm

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	550	$350	$500	$850
2	150	0	$200	$0	$200
3	100	0	$100	$0	$100
4	50	0	$50	$0	$50
5	50	0	$0	$0	$0
6	100	250	$150	$500	$650
7	150	0	$0	$0	$0
8	200	400	$200	$500	$700
9	200	0	$0	$0	$0
10	250	550	$300	$500	$800
11	300	0	$0	$0	$0
12	250	250	$0	$500	$500
Totals:	2000	2000	$1350	$2500	$3850

Approach: Silver-Meal Algorithm

Policy: Order 550 in period 1, 250 in period 6, 400 in period 8, 550 in period 10, and 250 in period 12





Approach: Least Unit Cost

- Objective
  - Minimize total relevant cost per item (TRCI)
  - TRCI(T) = TRC(T)/Sum(D)
    = (Order + Carrying)/(Lot Size)
- Decision Rule:
  - Add next period's demand to the order if the average cost per item is reduced
- Algorithm
  1. Start at first period
  2. Set T=1
  3. If TRCI(T) > TRCI(T-1) then
     - Previous order goes for T-1 periods with Q=sum(D) for T,
     - Start new order and go to 2
  4. Else, T=T+1 and go to 3

Approach: Least Unit Cost

PER	Demand	Lot Size	Order Cost	Hold Cost	Lot Cost	Cost Per Item	Next CPI	CNT	BUY	ORDER
1	200	200	$500	$0	$500	$ 2.50	$ 1.86	1	1	|
2	150	350	$500	$150	$650	$ 1.86	$ 1.89	2	1	**350**
3	100	100	$500	$0	$500	$ 5.00	$ 3.67	1	2	|
4	50	150	$500	$50	$550	$ 3.67	$ 3.25	2	2	|
5	50	200	$500	$150	$650	$ 3.25	$ 3.17	3	2	|
6	100	300	$500	$450	$950	$ 3.17	$ 3.44	4	2	**300**
7	150	150	$500	$0	$500	$ 3.33	$ 2.00	1	3	|
8	200	350	$500	$200	$700	$ 2.00	$ 2.00	2	3	**350**
9	200	200	$500	$0	$500	$ 2.50	$ 1.67	1	4	|
10	250	450	$500	$250	$750	$ 1.67	$ 1.80	2	4	**450**
11	300	300	$500	$0	$500	$ 1.67	$ 1.36	1	5	|
12	250	550	$500	$250	$750	$ 1.36	$ 1.36	2	5	**550**

Policy:
Order 350 in period 1, 300 in period 3,
350 in period 7, 450 in period 9, and 550 in period 11

Approach: Part Period Balancing

- Objective
  - Balancing holding and order costs for each replenishment
- Decision Rule:
  - Select number of periods to cover so that carrying costs is close to order (set up) costs
- Algorithm
  - Starting with first period, find holding cost
  - Add period to order until the holding cost is "close" to A
  - Start new order

Approach: Part Period Balancing

Month	Demand	Order Quantity	Holding Cost	Ordering Cost
1	200	500	$ 300	$ 500
2	150	0	$ 150	$ -
3	100	0	$ 50	$ -
4	50	0	$ -	$ -
5	50	300	$ 250	$ 500
6	100	0	$ 150	$ -
7	150	0	$ -	$ -
8	200	650	$ 450	$ 500
9	200	0	$ 250	$ -
10	250	0	$ -	$ -
11	300	550	$ 250	$ 500
12	250	0	$ -	$ -
Totals:	2000	2000	$ 1,850	$ 2,000

Policy:
Order 500 in period 1, 300 in period 5, 650 in period 8, and 550 in period 11

Comparison of Approaches

Month	Demand	1TB	L4L	EOQ	POQ	Optimal	SM	LUC	PBB
1	200	2000	200	400	450	550	550	350	500
2	150	0	150	0	0	0	0	0	0
3	100	0	100	400	0	0	0	300	0
4	50	0	50	0	200	0	0	0	0
5	50	0	50	0	0	0	0	0	300
6	100	0	100	0	0	450	250	0	0
7	150	0	150	0	550	0	0	350	0
8	200	0	200	400	0	0	400	0	650
9	200	0	200	0	0	450	0	450	0
10	250	0	250	400	800	0	550	0	0
11	300	0	300	400	0	550	0	550	550
12	250	0	250	0	0	0	250	0	0
Holding Cost		$ 13,100	$ -	$ 1,900	$ 2,000	$ 1,750	$ 1,350	$ 1,850	$ 1,850
Order Cost		$ 500	$ 6,000	$ 2,500	$ 2,000	$ 2,000	$ 2,500	$ 2,500	$ 2,000
Total Cost		$ 13,600	$ 6,000	$ 4,400	$ 4,000	$ 3,750	$ 3,850	$ 4,350	$ 3,850
Inv Turn Over		1.83	Inf	12.60	12.00	13.70	17.80	13.00	13.00
Pct > Optimal		263%	60%	17%	7%	0%	3%	16%	3%

Take Aways from FPH

- Many ways to solve the problem with implicit trade-offs
  - Heuristics – Fast, simple, not always good
  - Optimal Methods – Requires more time and data
  - Specialty Heuristics – More Focused, harder to set up, better ‘real-world’ results
- An “optimal” solution might not be optimal in the real-world
- Best solution to the problem . . . depends

Questions?
Comments
Suggestions?