Practical multitone architectures

Lecture 4

Vladimir Stojanović

6.973 Communication System Design – Spring 2006

Massachusetts Institute of Technology

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

Loading with discrete information units

- Chow's algorithm (rounding of waterfilling results)
- Levin-Campello algorithm (greedy optimization)
  - Each additional bit placed on subchannel that requires the least incremental energy for its transport
  - Incremental energy $e_n(b_n) \triangleq \varepsilon_n(b_n) - \varepsilon_n(b_n - \beta)$

bit increment

$$b_n = \log_2\left(1 + \frac{e_n(b_n)g_n}{\Gamma}\right) \longrightarrow \begin{aligned} e_n(b_n) &= \frac{\Gamma}{g_n}\left(2 \cdot 2^{b_n} - 2^{b_n}\right) \\ &= \frac{\Gamma}{g_n} \cdot 2^{b_n}(2-1) \\ &= \frac{\Gamma}{g_n} 2^{b_n} \\ &= 2 \cdot e_n(b_n - 1) \quad , \end{aligned}$$

  - Efficiency of bit distribution $\max_n \left[e_n(b_n)\right] \le \min_m \left[e_m(b_m + \beta)\right]$
    - Can't move a bit from one channel to another and reduce the total energy

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

Levin-Campello (LC) algorithm- components

- Efficientizing (EF) – finding energy-efficient bit distribution
  - Always replace a bit distribution with a more efficient – exhaustively search all single information unit changes at each step

```
1. $m \leftarrow \arg\{\min_{1\le i\le N}[e_i(b_i+\beta)]\}$ ;
2. $n \leftarrow \arg\{\max_{1\le j\le N}[e_j(b_j)]\}$ ;
3. While $e_m(b_m+\beta) < e_n(b_n)$ do
    (a) $b_m \leftarrow b_m+\beta$
    (b) $b_n \leftarrow b_n-\beta$
    (c) $m \leftarrow \arg\{\min_{1\le i\le N}[e_i(b_i+\beta)]\}$ ;
    (d) $n \leftarrow \arg\{\max_{1\le j\le N}[e_j(b_j)]\}$ ;
```

Smallest energy to add a new bit

Largest energy to subtract the bit

Subtract bits from costly sub-channels and add to least costly sub-channels

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

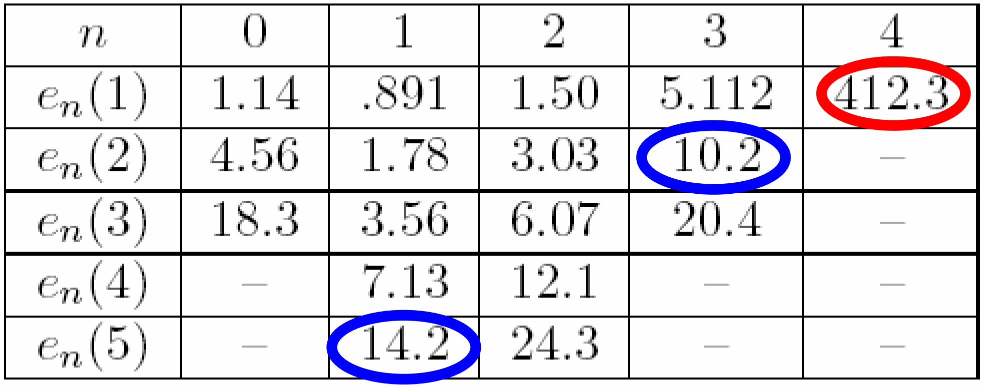
Efficientizing example

- $1+0.9D^{-1}$ channel ($P_e=10^{-6}$, gap=8.8dB, PAM/QAM)
  - PAM and single-sideband
  - QAM

$$\mathcal{E}_n(b_n) = \frac{10^{.88}}{g_n}\left(2^{2b_n}-1\right)$$

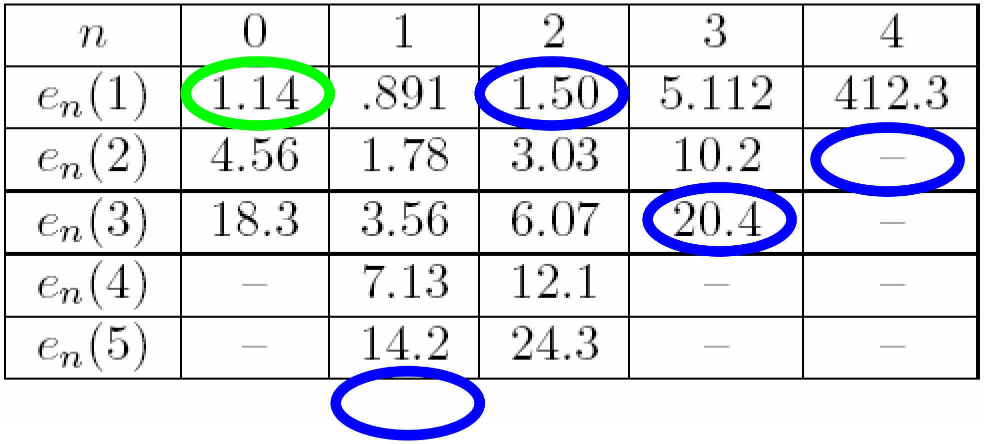
$$\mathcal{E}_n(b_n) = 2\cdot\frac{10^{.88}}{g_n}\left(2^{b_n}-1\right)$$

b_n

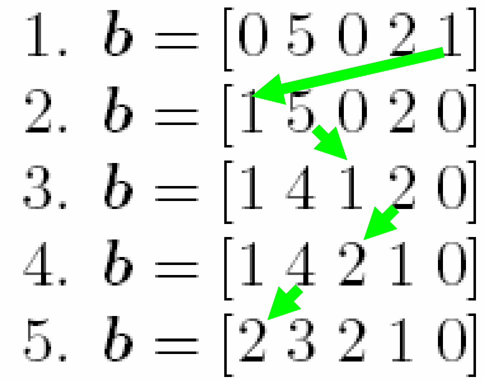


n	0	1	2	3	4
$e_n(1)$	1.14	.891	1.50	5.112	412.3
$e_n(2)$	4.56	1.78	3.03	10.2	–
$e_n(3)$	18.3	3.56	6.07	20.4	–
$e_n(4)$	–	7.13	12.1	–	–
$e_n(5)$	–	14.2	24.3	–	–

$b_n + 1$



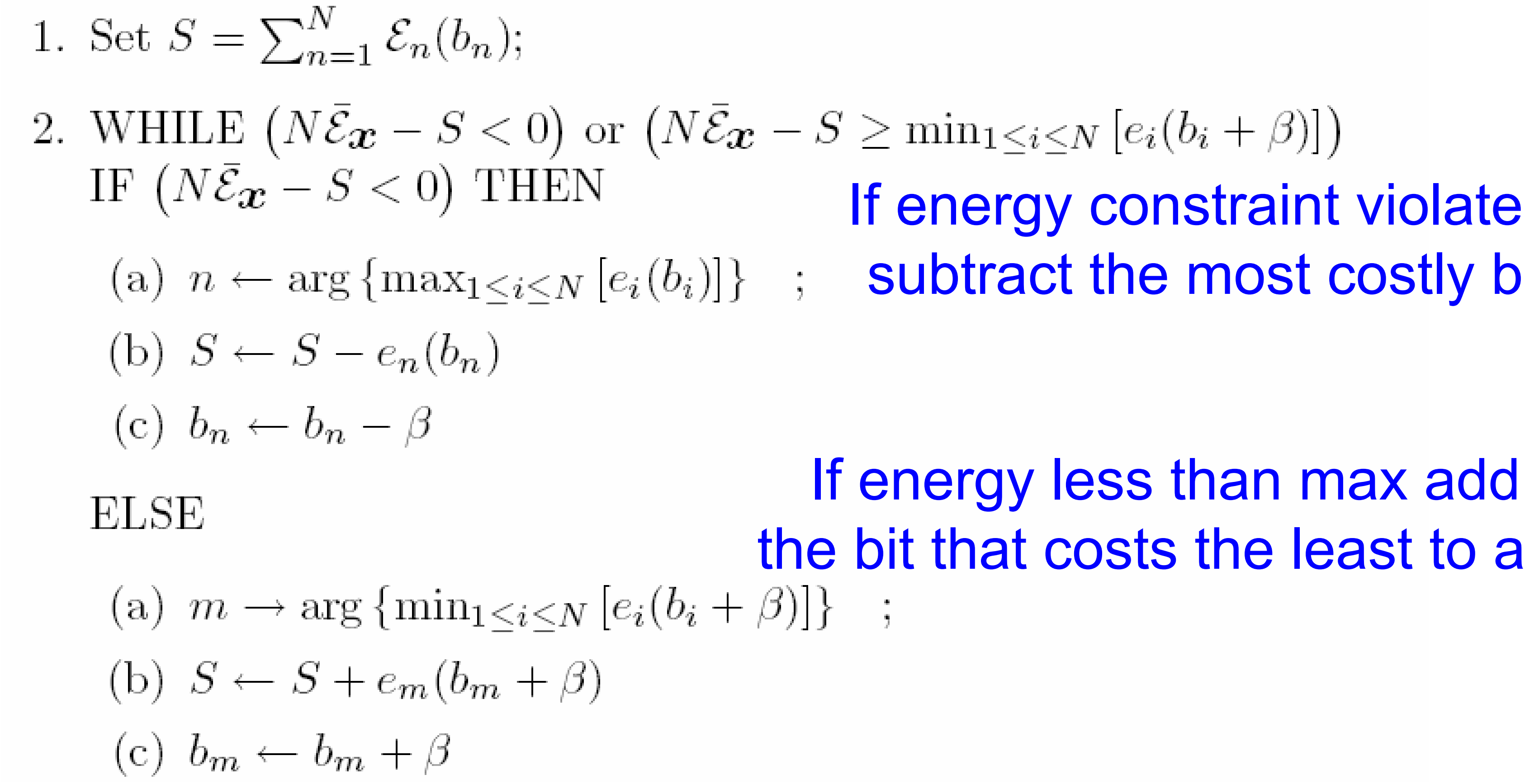
n	0	1	2	3	4
$e_n(1)$	1.14	.891	1.50	5.112	412.3
$e_n(2)$	4.56	1.78	3.03	10.2	–
$e_n(3)$	18.3	3.56	6.07	20.4	–
$e_n(4)$	–	7.13	12.1	–	–
$e_n(5)$	–	14.2	24.3	–	–



1. $\boldsymbol{b} = [0\ 5\ 0\ 2\ 1]$
2. $\boldsymbol{b} = [1\ 5\ 0\ 2\ 0]$
3. $\boldsymbol{b} = [1\ 4\ 1\ 2\ 0]$
4. $\boldsymbol{b} = [1\ 4\ 2\ 1\ 0]$
5. $\boldsymbol{b} = [2\ 3\ 2\ 1\ 0]$

Rate adaptive solution

- Need also to satisfy the energy constraint
- E-tightness (ET) $$0 \leq N\bar{\mathcal{E}}_{\boldsymbol{x}} - \sum_{n=1}^{N} E_n(b_n) \leq \min_{1\leq i\leq N} [e_i(b_i+\beta)]$$
  - Can't add another bit without violating the E constraint



```
1. Set $S = \sum_{n=1}^{N} \mathcal{E}_n(b_n)$;
2. WHILE $(N\bar{\mathcal{E}}_{\boldsymbol{x}} - S < 0)$ or $(N\bar{\mathcal{E}}_{\boldsymbol{x}} - S \geq \min_{1\leq i\leq N} [e_i(b_i+\beta)])$
   IF $(N\bar{\mathcal{E}}_{\boldsymbol{x}} - S < 0)$ THEN
      (a) $n \leftarrow \arg\{\max_{1\leq i\leq N} [e_i(b_i)]\}$ ;
      (b) $S \leftarrow S - e_n(b_n)$
      (c) $b_n \leftarrow b_n - \beta$
   ELSE
      (a) $m \rightarrow \arg\{\min_{1\leq i\leq N} [e_i(b_i+\beta)]\}$ ;
      (b) $S \leftarrow S + e_m(b_m+\beta)$
      (c) $b_m \leftarrow b_m + \beta$
```

If energy constraint violated subtract the most costly bit

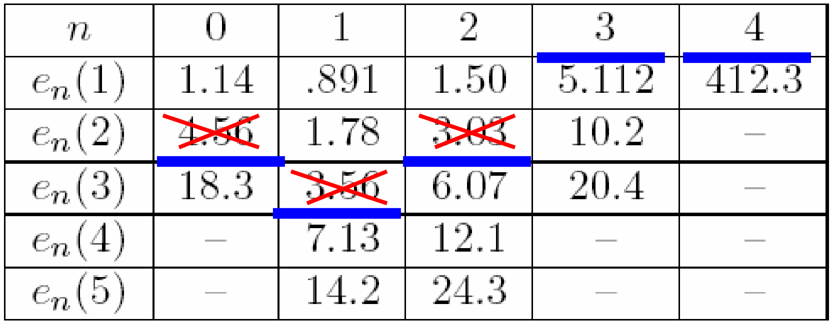
If energy less than max add the bit that costs the least to add

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

LC - Rate adaptive algorithm

- ET example (continue from EF example)
  - Start from efficient distribution that blows up the energy constraint $b = [2\ 3\ 2\ 1\ 0]$ $N\bar{\mathcal{E}}_x = 8 \cdot 1 = 8$

1. $b = [2\ 3\ 2\ 0\ 0]$ with $\mathcal{E} \to 16.49$
2. $b = [1\ 3\ 2\ 0\ 0]$ with $\mathcal{E} \to 11.92$
3. $b = [1\ 2\ 2\ 0\ 0]$ with $\mathcal{E} \to 8.36$
4. $b = [1\ 2\ 1\ 0\ 0]$ with $\mathcal{E} \to 5.32$



n	0	1	2	3	4
$e_n(1)$	1.14	.891	1.50	5.112	412.3
$e_n(2)$	~~4.56~~	1.78	~~3.03~~	10.2	–
$e_n(3)$	18.3	~~3.56~~	6.07	20.4	–
$e_n(4)$	–	7.13	12.1	–	–
$e_n(5)$	–	14.2	24.3	–	–

  - Margin

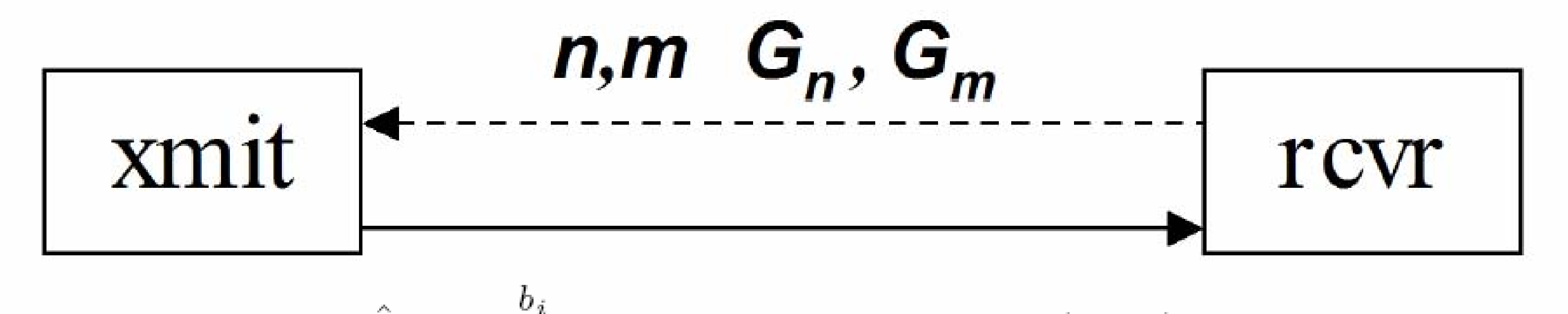
$$\frac{N\bar{\mathcal{E}}_x}{\varepsilon} = \frac{8}{5.32} = 1.8\,dB$$

- **Levin-Campello Rate Adaptive algorithm**
  - Choose any bit distribution
  - Make it efficient using (EF algorithm)
  - Make it energy tight using (ET algorithm)

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

Dynamic rate adaptation

- Change the loading when channel changes
- LC is a natural candidate
- Keep the ET bit distribution and perturb based on channel changes
  - Bit is moved from channel n to m



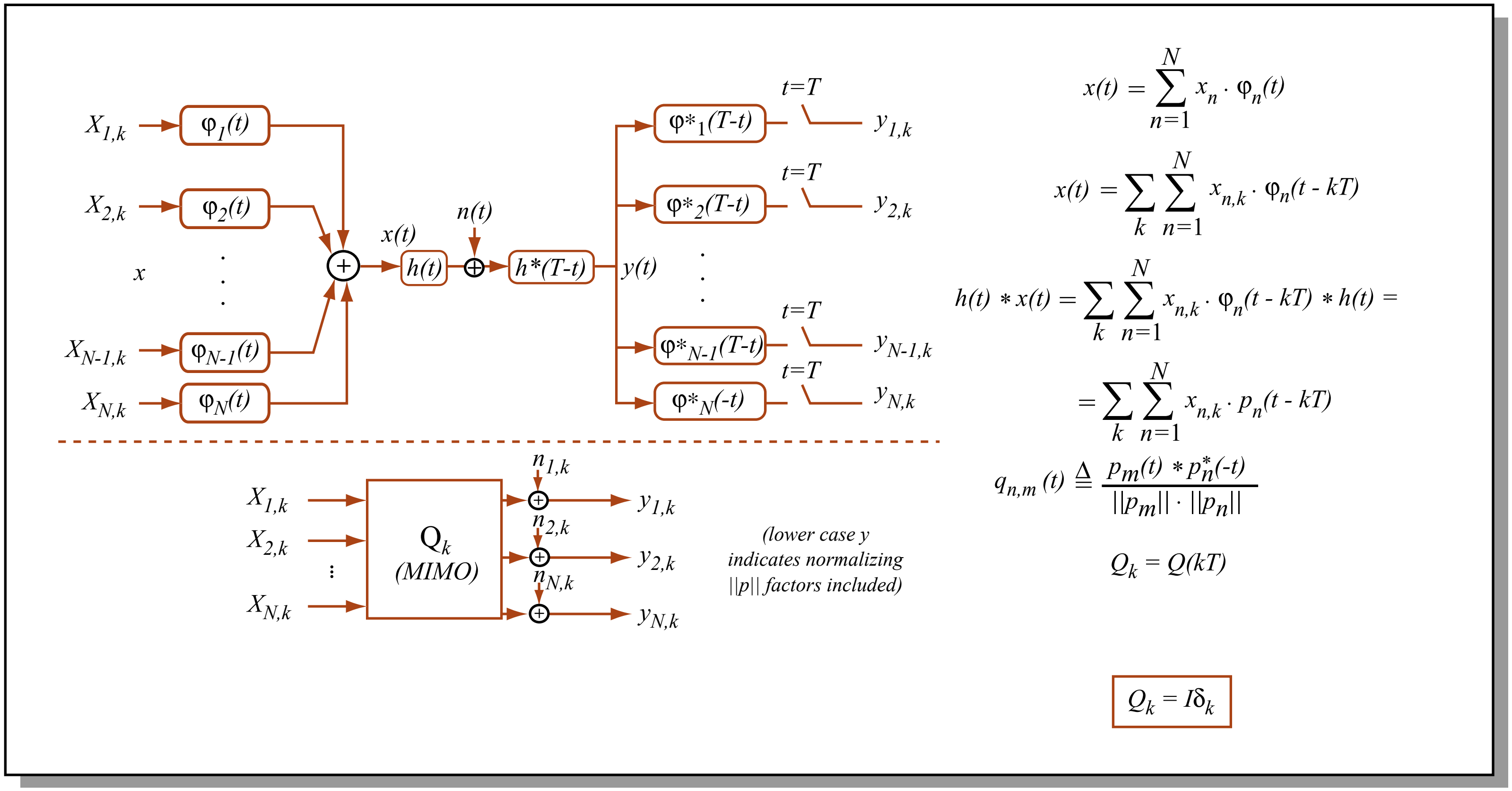
$$\mathcal{E}_i(new) = \frac{\hat{g}_{i,old}}{\hat{g}_{i,new}} \sum_{j=1}^{b_i} e_{i,old}(j) \qquad G_i = \frac{\mathcal{E}_i(new)}{\mathcal{E}_i(old)} \quad \text{for } i = m, n$$

- Tightly coupled with channel and noise estimation
  - Will cover in later lectures

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

Channel partitioning

- Divide the channel into a set of parallel, independent channels



$$x(t) = \sum_{n=1}^{N} x_n \cdot \varphi_n(t)$$

$$x(t) = \sum_k \sum_{n=1}^{N} x_{n,k} \cdot \varphi_n(t-kT)$$

$$h(t) * x(t) = \sum_k \sum_{n=1}^{N} x_{n,k} \cdot \varphi_n(t-kT) * h(t) =$$

$$= \sum_k \sum_{n=1}^{N} x_{n,k} \cdot p_n(t-kT)$$

$$q_{n,m}(t) \triangleq \frac{p_m(t) * p_n^*(-t)}{||p_m|| \cdot ||p_n||}$$

(lower case y indicates normalizing ||p|| factors included)

$$Q_k = Q(kT)$$

$$\boxed{Q_k = I\delta_k}$$

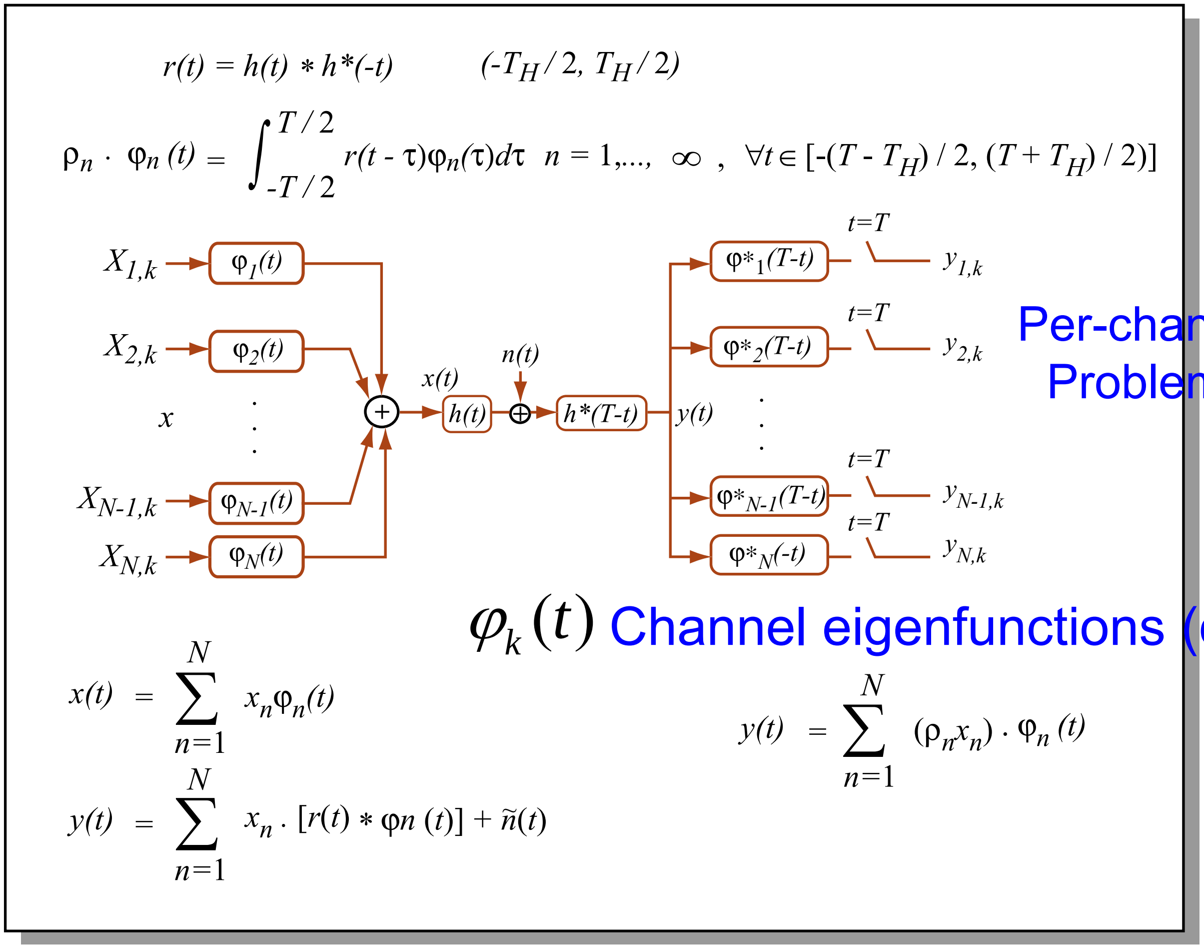
Figure by MIT OpenCourseWare.

- Generalized Nyquist criterion
  - No interference between symbols
  - No interference between sub-channels

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

Modal modulation

- Transmission with eigen-functions



$$r(t) = h(t) * h^*(-t) \qquad (-T_H/2,\ T_H/2)$$

$$\rho_n \cdot \varphi_n(t) = \int_{-T/2}^{T/2} r(t-\tau)\varphi_n(\tau)d\tau \quad n = 1,\dots,\ \infty\ ,\ \forall t \in [-(T - T_H)/2,\ (T + T_H)/2)]$$

Per-channel MAP is optimal
Problem is vector AWGN

$\varphi_k(t)$ Channel eigenfunctions (channel dependent)

$$x(t) = \sum_{n=1}^{N} x_n \varphi_n(t)$$

$$y(t) = \sum_{n=1}^{N} (\rho_n x_n) \cdot \varphi_n(t)$$

$$y(t) = \sum_{n=1}^{N} x_n \cdot [r(t) * \varphi n\ (t)] + \tilde{n}(t)$$

Figure by MIT OpenCourseWare.

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

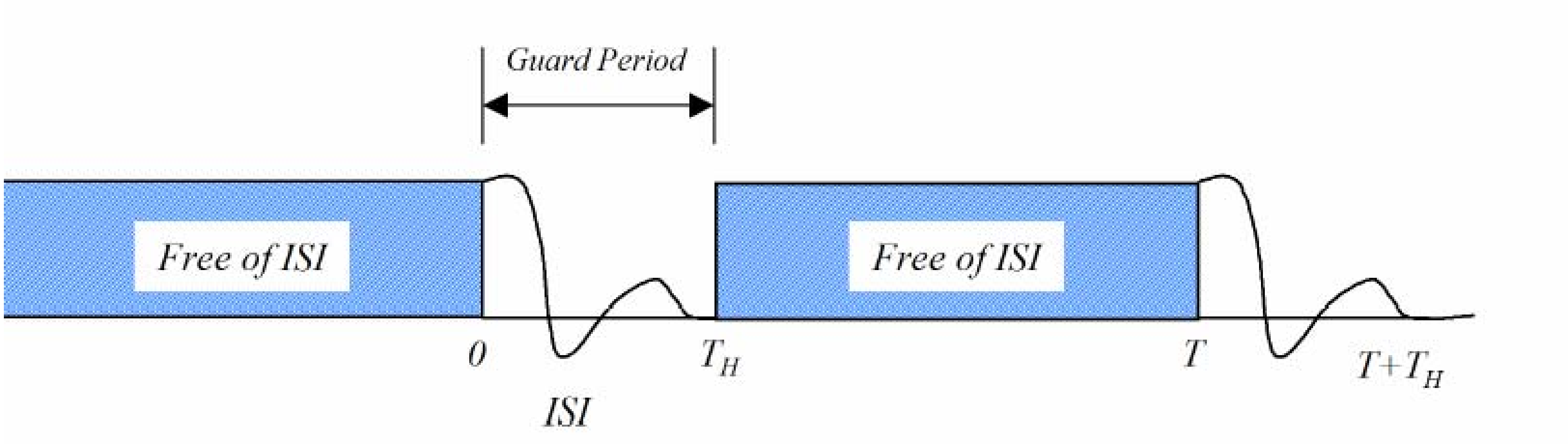
Convergence of multitone to modal modulation

- ❑ Modal modulation is optimal for finite symbol time
  - Eigenfunctions hard to compute and channel dependent
- ❑ Multitone converges to Modal modulation
  - As both N->*inf* and [-T/2, T/2]->(-*inf*,+*inf*)
    - Set of eigenvalues of any autocorrelation function is unique
      - This set determines the performance of MM through SNR
      - Eigen-functions are not unique
    - $\varphi_n(t) = e^{-\jmath \frac{2\pi n}{T} t}$ is also a valid eigen-function for *inf* symbol period
      - Corresponding eigenvalues are R(2πn/T)
      - No ISI on any tone since symbol period is infinite
        - Each tone is AWGN channel
        - SBS detector is MAP optimal
  - If channel is periodic (does not exist in practice)
    - $\varphi_n(t) = e^{-\jmath \frac{2\pi n}{T} t}$ are then eigen-functions even on finite [-T/2,T/2]
    - Can use extra bandwidth in the design to make the channel “look” periodic

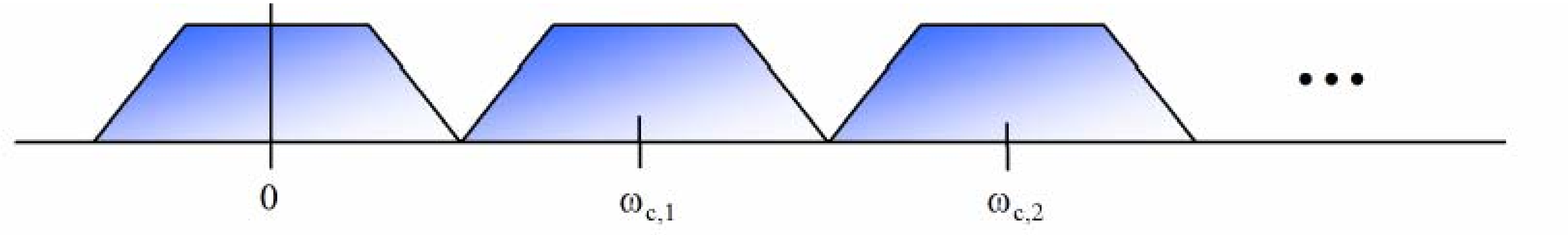
Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.

Finite symbol duration effects

- ISI corrupts the neighboring symbol
  - Need to leave the guardband



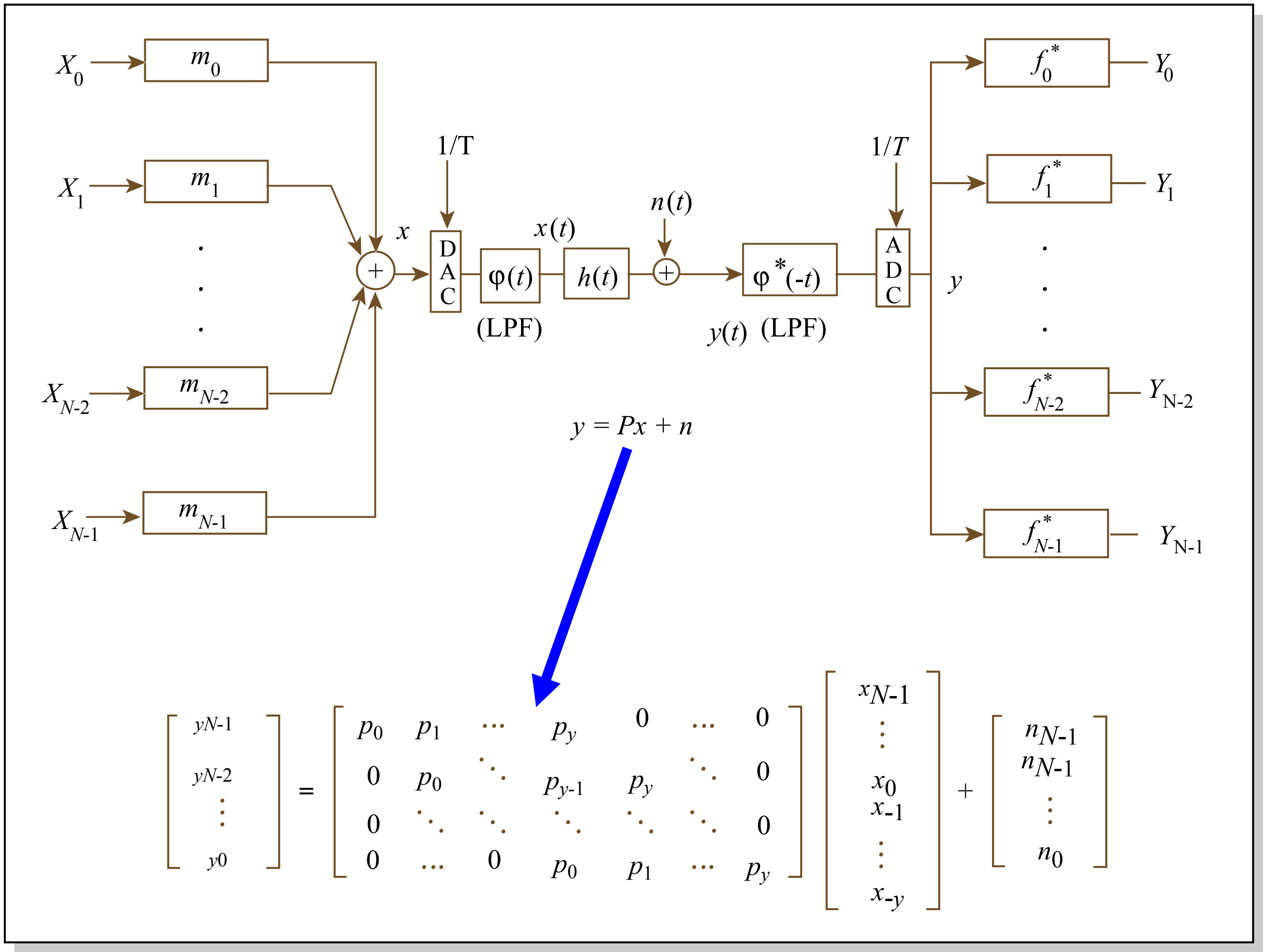
  - Can try to create a more sinc-looking symbols in time by filtering the tones in frequency domain
    - Need excess bandwidth for filter roll-off



Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006. MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].

Discrete time channel partitioning

- Digital realization



$$\begin{bmatrix} y_{N-1} \\ y_{N-2} \\ \vdots \\ y_0 \end{bmatrix} = \begin{bmatrix} p_0 & p_1 & \cdots & p_y & 0 & \cdots & 0 \\ 0 & p_0 & \ddots & p_{y-1} & p_y & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & p_0 & p_1 & \cdots & p_y \end{bmatrix} \begin{bmatrix} x_{N-1} \\ \vdots \\ x_0 \\ x_{-1} \\ \vdots \\ x_{-y} \end{bmatrix} + \begin{bmatrix} n_{N-1} \\ n_{N-1} \\ \vdots \\ n_0 \end{bmatrix}$$

Figure by MIT OpenCourseWare.

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006. MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].

Vector coding

- Creates a set of parallel channels by using SVD

$$y = Px + n$$

$$P = F\left[\Lambda \vdots \mathbf{0}_{N,\nu}\right] M^*$$

singular-value decomposition of P

discrete transmitter waveforms

discrete matched filters

$$Y = F^* y = \begin{bmatrix} f^*_{N-1} y \\ \cdots \\ f^*_0 y \end{bmatrix}$$

$$x = M \begin{bmatrix} X \\ 0 \\ \vdots \\ 0 \end{bmatrix} = [m_{N-1}\ m_{N-2} \dots m_1 m_0 \dots m_{-\nu}] \begin{bmatrix} X_{N-1} \\ X_{N-2} \\ \vdots \\ X_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \sum_{n=0}^{N-1} X_n m_n$$

$$Y = \Lambda X + N$$

since F is unitary N is also AWGN

$$Y_n = \lambda_n \cdot X_n + N_n$$

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

VC example

- $1+0.9D^{-1}$, N=8
  - Need one sample guardband
    - Etot=(N+1)*E_dim=(8+1)*1=9
  - SVD on $$p = \begin{bmatrix} .9 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .9 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .9 & 1 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & .9 & 1 \end{bmatrix}$$
  - Gives singular values $[1.87\ 1.78\ 1.64\ 1.45\ 1.22\ .95\ .66\ .34]$
  - Sub-channel SNRs $g_n = \frac{\lambda_n^2}{\sigma^2} = [19.3\ 17.6\ 15.0\ 11.7\ 8.3\ 5.0\ 2.4\ .66]$
  - Waterfilling shows only 7 dimensions can be used
  - Sub-channel energies $[1.38\ 1.37\ 1.36\ 1.34\ 1.30\ 1.23\ 1.01\ 0]$ $K = \frac{1}{7}\left(9 + \sum_{n=0}^{6} \frac{1}{g_n}\right) = 1.43$
  - SNRs are then $[26.6\ 24.2\ 20.4\ 15.8\ 10.8\ 6.2\ 2.4\ 0]$
  - Total SNR $$\mathrm{SNR}_{VC} = \left[\prod_{n=0}^{6} \mathrm{SNR}_n + 1\right]^{1/9} - 1 = 6.46 = 8.1\ \mathrm{dB}$$
  - VC capacity $$\bar{c} = \frac{1}{9}\sum_{n=0}^{6} \frac{1}{2}\log_2(1+\mathrm{SNR}_n) = \boxed{1.45\mathrm{bits/dim}}$$
    - Would get 1.55bits/dim if N-> inf

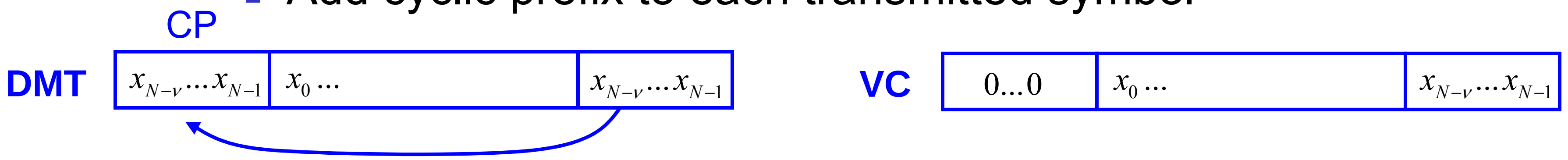
DMT and OFDM

- Discrete multitone (DMT) and Orthogonal frequency Division Multiplexing (OFDM)
- DMT used on slowly time-varying channels
  - Optimize bn and En per sub-channel
- OFDM uses same channel partitioning as DMT
  - But uses same bn and En on all channels
  - Used on one-way broadcast channels
- Forms of vector coding with added restrictions
  - In vector coding, M,F channel dependent
  - Make the channel circular and make M,F channel independent - simplify hardware implementation

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.

DMT/OFDM channel partitioning

- Make channel “look” circular
  - Repeat the tail of the symbol at its beginning
    - Add cyclic prefix to each transmitted symbol



$$\begin{bmatrix} y_{N-1} \\ y_{N-2} \\ \vdots \\ y_0 \end{bmatrix} = \underbrace{\begin{bmatrix} p_0 & p_1 & \dots & p_\nu & 0 & \dots & 0 \\ 0 & p_0 & \ddots & p_{\nu-1} & p_\nu & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & p_0 & p_1 & \dots & p_\nu \\ p_\nu & 0 & \dots & 0 & p_0 & \dots & p_{\nu-1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ p_1 & \dots & p_\nu & 0 & \dots & 0 & p_0 \end{bmatrix}}_{N \times N} \begin{bmatrix} x_{N-1} \\ \vdots \\ x_0 \end{bmatrix} + \begin{bmatrix} n_{N-1} \\ n_{N-2} \\ \vdots \\ n_0 \end{bmatrix}$$

$$y = Px + n \, .$$

$$P = M \Lambda M^*$$

- SVD can be replaced by eigen-decomposition (spectral factorization)
  - A discrete form of modal modulation
  - While SNRs are unique, many choices for M and F

IDFT and DFT as orthogonal transformations

- DMT and OFDM use IDFT and DFT as eigen-vectors

$$\boldsymbol{X} \triangleq \begin{bmatrix} X_{N-1} \\ X_{N-2} \\ \vdots \\ X_0 \end{bmatrix} \qquad X_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k e^{-\jmath \frac{2\pi}{N} kn} \ \forall \ n \in [0, N-1] \qquad \boldsymbol{x} = \begin{bmatrix} x_{N-1} \\ x_{N-2} \\ \vdots \\ x_0 \end{bmatrix}$$

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{\jmath \frac{2\pi}{N} kn} \ \forall \ k \in [0, N-1]$$

$$\begin{aligned} \boldsymbol{X} &= Q\boldsymbol{x} \\ \boldsymbol{x} &= Q^{*}\boldsymbol{X} \end{aligned} \qquad Q = \frac{1}{\sqrt{N}} \begin{bmatrix} e^{-\jmath \frac{2\pi}{N}(N-1)(N-1)} & \ldots & e^{-\jmath \frac{2\pi}{N} 2(N-1)} & e^{-\jmath \frac{2\pi}{N}(N-1)} & 1 \\ e^{-\jmath \frac{2\pi}{N}(N-1)\cdot(N-2)} & \ldots & e^{-\jmath \frac{2\pi}{N} 2(N-2)} & e^{-\jmath \frac{2\pi}{N}(N-2)} & 1 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ e^{-\jmath \frac{2\pi}{N}(N-1)} & \ldots & e^{-\jmath \frac{2\pi}{N} 2} & e^{-\jmath \frac{2\pi}{N}} & 1 \\ 1 & \ldots & 1 & 1 & 1 \end{bmatrix} \qquad Q^{*} = [\boldsymbol{q}_{N-1}, \ldots, \boldsymbol{q}_0]$$

DFT

IDFT

- Proof

$$\lambda_n \boldsymbol{q}_n = P\boldsymbol{q}_n \qquad Q^{*}\Lambda = PQ^{*}$$

channel gain at tone n

$$\frac{1}{\sqrt{N}} \cdot \left(p_0 + p_1 \cdot e^{-\jmath \frac{2\pi}{N} n} + \ldots + p_{N-1} \cdot e^{-\jmath \frac{2\pi(N-1)}{N} n} \right) = P_n$$

$$\frac{1}{\sqrt{N}} \cdot \left(p_0 \cdot e^{\jmath \frac{2\pi}{N} n} + p_1 + \ldots + p_{N-1} \cdot e^{-\jmath \frac{2\pi(N-2)}{N} n} \right) = P_n \cdot \cdot e^{+\jmath \frac{2\pi}{N} n}$$

$$\vdots$$

$$P\boldsymbol{q}_n = P_n \boldsymbol{q}_n \longrightarrow$$ q_n is eigen-vector of P

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

DMT/OFDM implementation

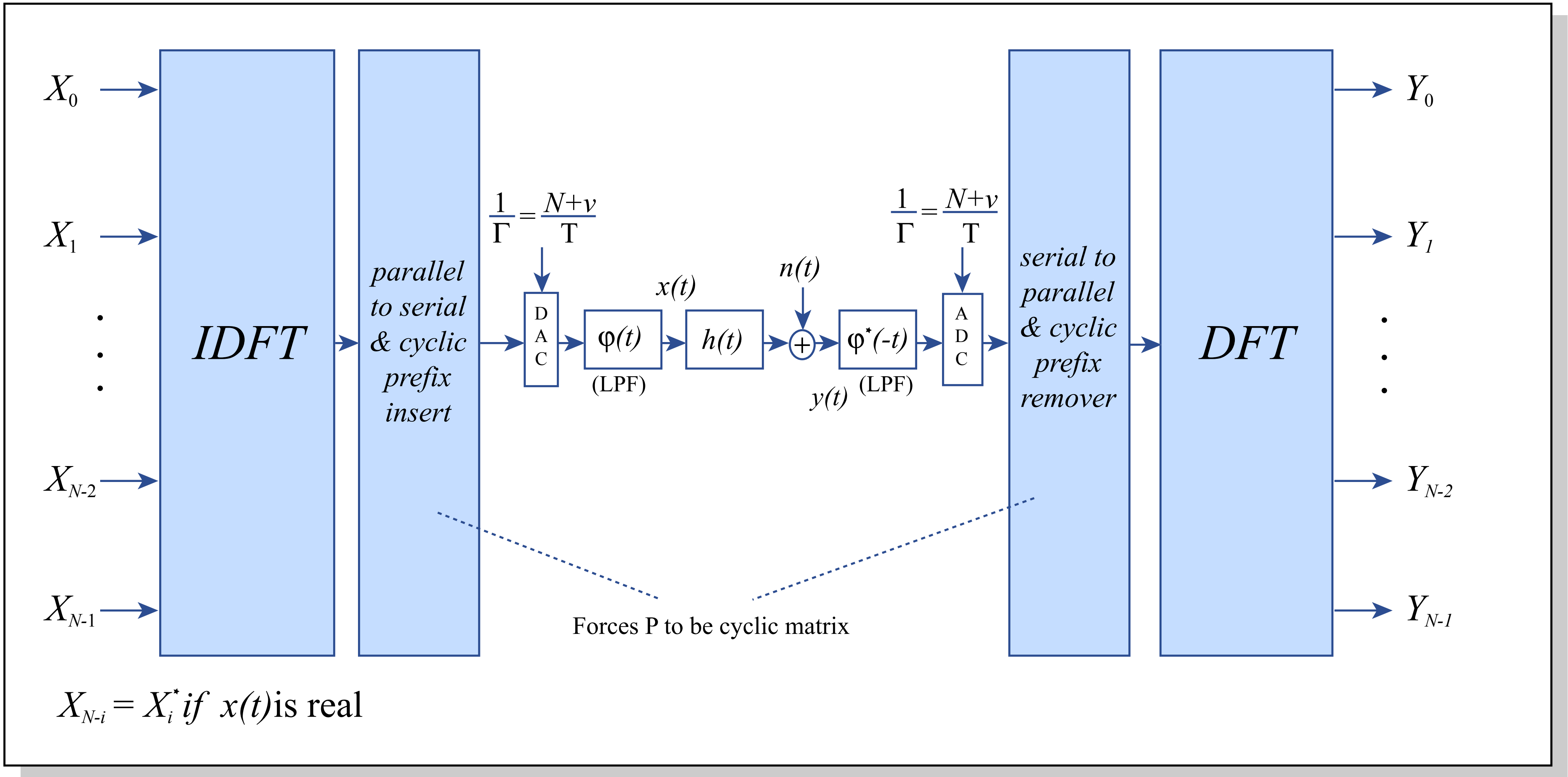


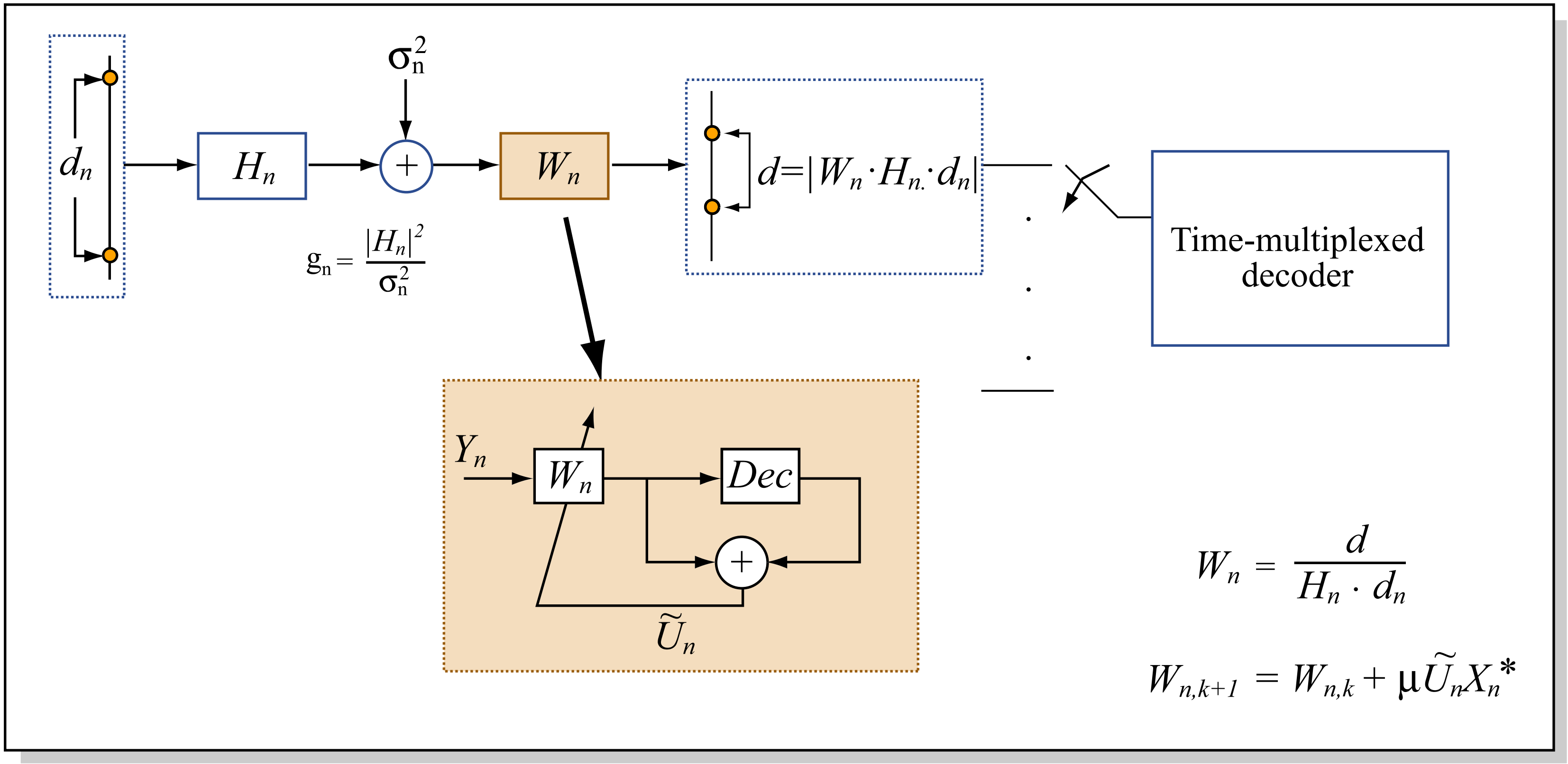
Figure by MIT OpenCourseWare.

- Data rate penalty $N/(N+v)$

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

One-tap frequency equalizer

- Need to compensate for channel attenuation
  - To recover the original constellation distance



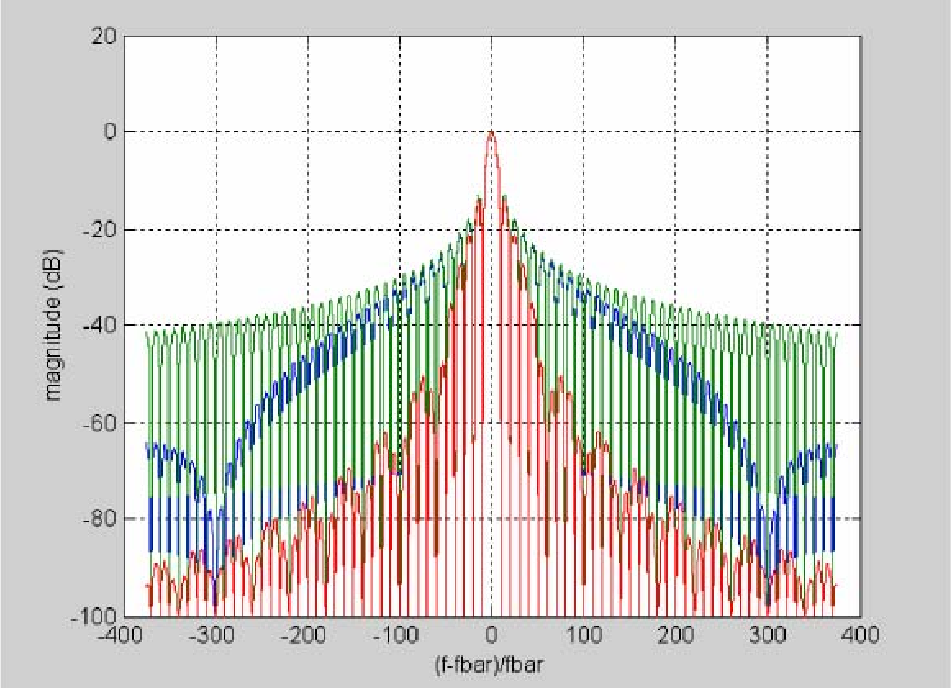
$$W_n = \frac{d}{H_n \cdot d_n}$$

$$W_{n,k+1} = W_{n,k} + \mu \tilde{U}_n X_n^*$$

Figure by MIT OpenCourseWare.

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

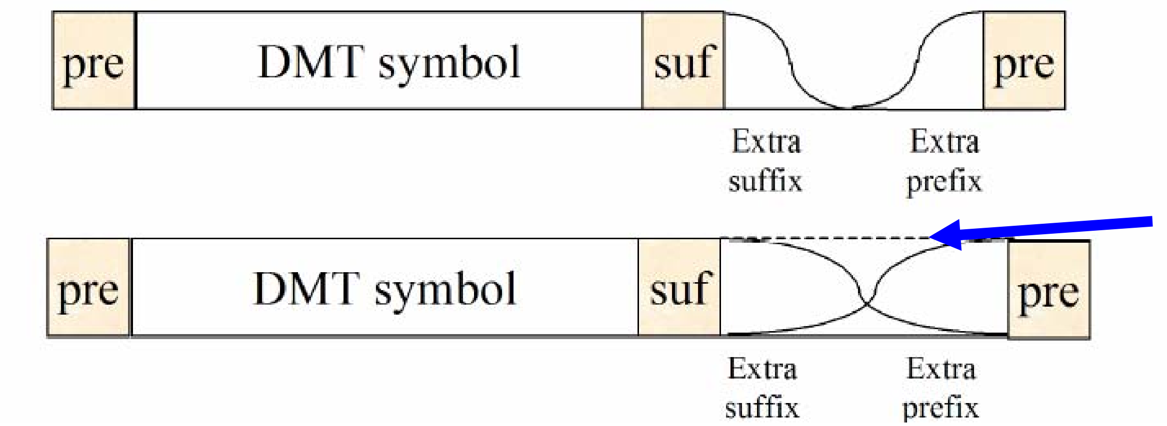
Rx/Tx Windowing



Rectangular window

Raised cosine 5%

Raised cosine 25%



Can overlap as long as sum is constant

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.

$1{+}0.9D^{-1}$ DMT example

- N=8

$$P = \begin{bmatrix} .9 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .9 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .9 & 1 & 0 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & .9 \end{bmatrix}$$

n	$\lambda_n = \lvert P_n \rvert$	$g_n = \frac{\lvert P_n \rvert^2}{.181}$	$\mathcal{E}_n$	SNR_n
0	1.90	20	1.24	24.8
1	1.76	17	1.23	20.9
2	1.76	17	1.23	20.9
3	1.35	9.8	1.19	11.7
4	1.35	9.8	1.19	11.7
5	.733	3	.96	2.9
6	.733	3	.96	2.9
7	.100	.05525	0	0

- Waterfilling with Etot=8
  - Waste one unit on CP

$$\mathrm{SNR}_{DMT} = \left[\prod_{n=0}^{6}(1+\mathrm{SNR}_n)\right]^{1/9} - 1 = 7.6\text{ dB}$$

lower than VC (8.1dB)

  - For N=16 quickly reaches max of 8.8dB

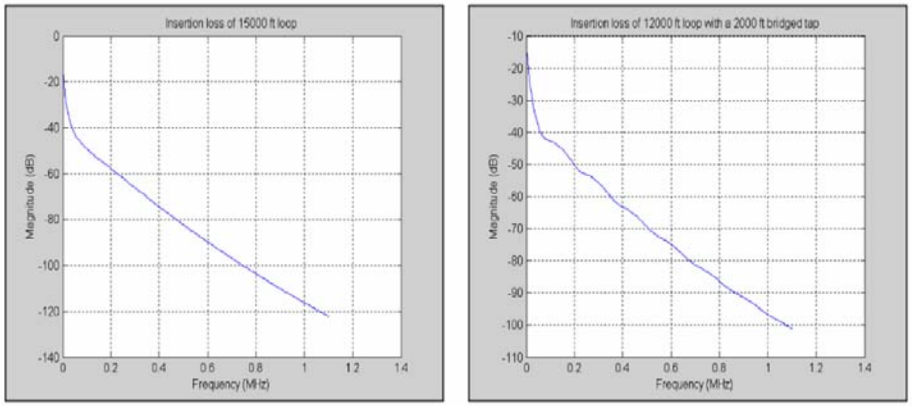
Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.

Complexity of DFE, VC, DMT

- Example $1+0.9D^{-1}$ channel
- MMSE-DFE, SNR=7.6 dB, 3ff, 1fb tap, 4 mac/sample
- VC, N=8, SNR=8.1 dB, 7*8/9=6.2MAC/sample
- DMT, N=8, SNR=7.6 dB, 8pt FFT/IFFT, 2.7MAC/sample
  - N=16, 3.8MAC/sample, SNR=8.8 dB
  - DFE needs 10FF taps, 1FB tap, SNR=8.4 dB, 11MAC/sample

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.

Asymmetric digital subscriber line (ADSL)



5 km loop

4 km loop with bridged tap

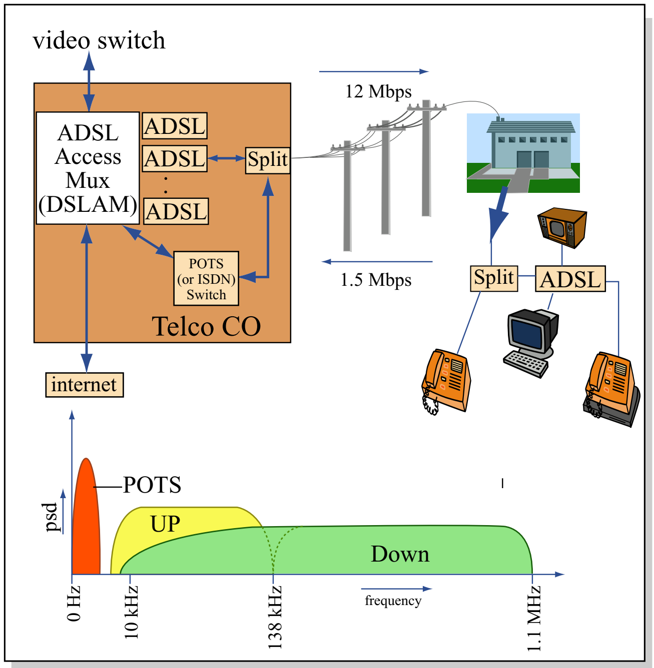


Figure by MIT OpenCourseWare.

- Symbol rate T=250us
- Nd=256, 4.3125kHz wide
  - 1/T'=2.208 MHz (CP = 40 samples)
  - Each time domain symbol 2*256+40=552 samples
    - Hermitian symmetry creates real signal transmitted from 0-1.1MHz
    - First 2-3 tones near DC not used – avoid interference with voice
    - Tone 256 also not used, 64 reserved for pilot
- Nup=32, CP=5, each symbol 2*32+5=69 samples
  - Exactly 1/8 of downstream

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

802.11a Wireless LAN example

- Up to 54Mbps symmetrically (<100m)
- 1-3 Tx power levels
- Complex baseband
  - unlike ADSL which is real baseband
- N=64 (-31 … 31) (so 128 dimensions)
  - Symbol length = 80 samples, CP=16
  - Symbol rate 250kHz (T=4uS, T'=50ns), CPguard=0.8us

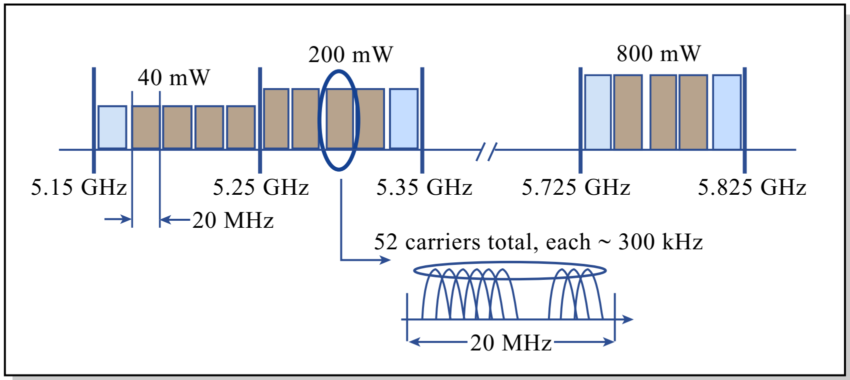
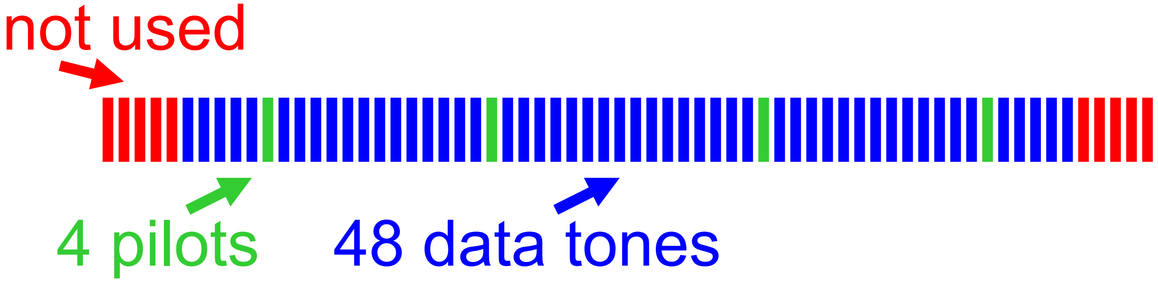


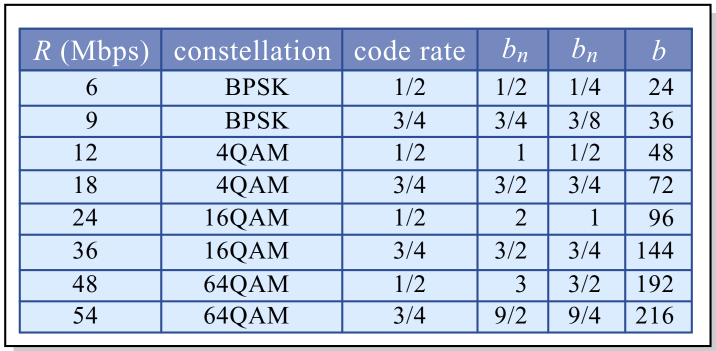
Figure by MIT OpenCourseWare.



not used

4 pilots

48 data tones



R (Mbps)	constellation	code rate	b_n	b_n	b
6	BPSK	1/2	1/2	1/4	24
9	BPSK	3/4	3/4	3/8	36
12	4QAM	1/2	1	1/2	48
18	4QAM	3/4	3/2	3/4	72
24	16QAM	1/2	2	1	96
36	16QAM	3/4	3/2	3/4	144
48	64QAM	1/2	3	3/2	192
54	64QAM	3/4	9/2	9/4	216

Figure by MIT OpenCourseWare.

- Broadcast channel – can't optimize bit allocation
  - FCC demands flat spectrum so no energy-allocation
  - The only knob is data rate selection

$$R = k(1\ \text{bit}\ /2\ \text{dimensions}) \cdot (48\ \text{tones}) \cdot 250kHz\ \text{or}\ 6 \cdot kMbps$$

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

High-level system view

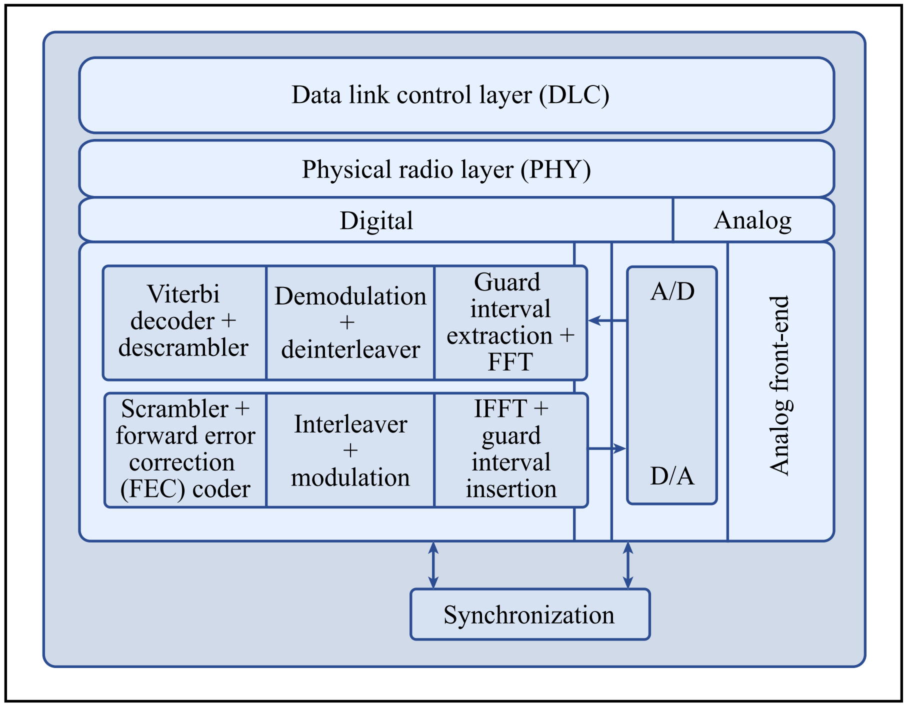
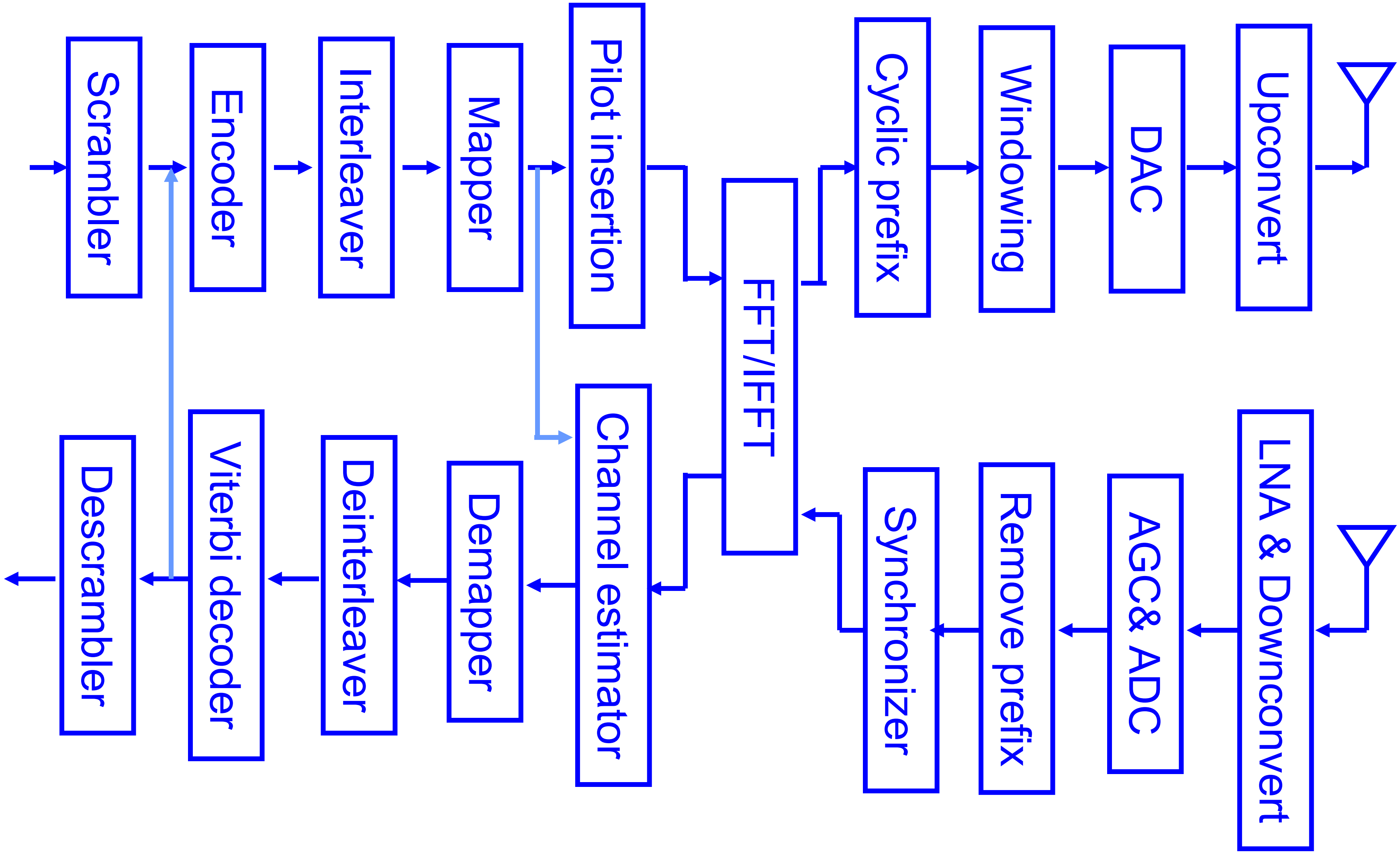


Figure by MIT OpenCourseWare.

Transceiver architecture



[2]

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

Transmitter architecture detail

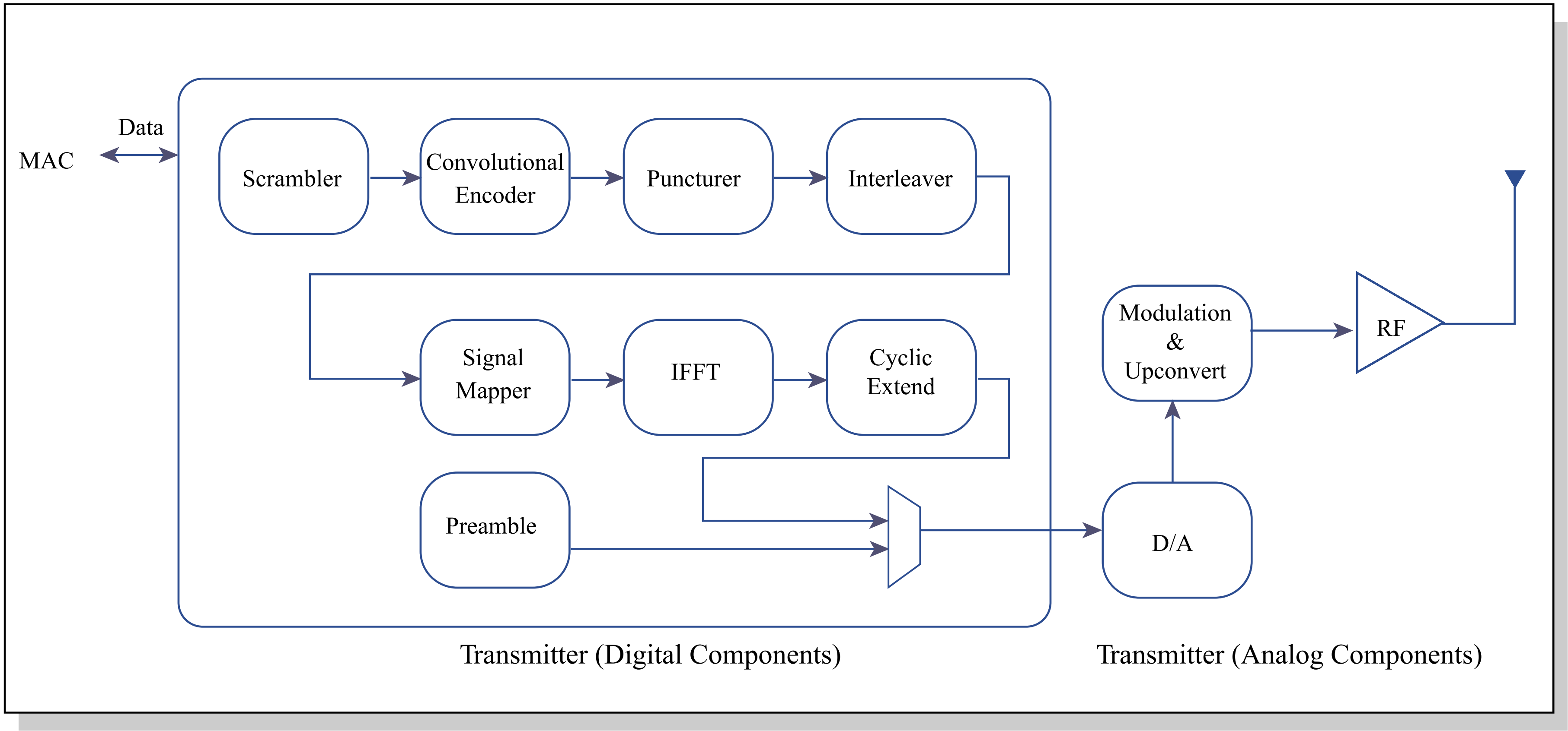
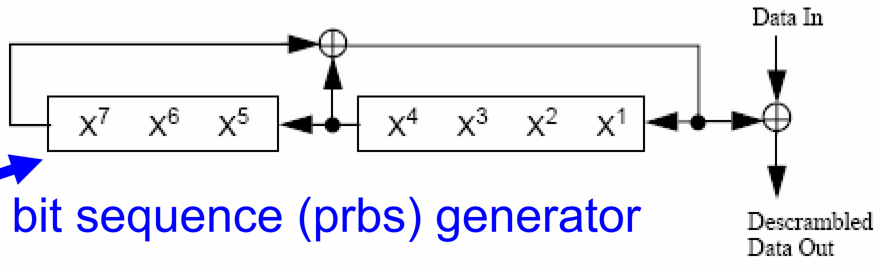


Figure by MIT OpenCourseWare.

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

Scrambling

- Need to randomize incoming data
- Enables a number of tracking algorithms in the receiver
- Provides flat spectrum in the given band



pseudo-random bit sequence (prbs) generator

What is the period of this pseudo-random sequence?

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

Interleaver

- Protects the code from overload by burst errors
- Block interleaver
  - Block size is the #of coded bits in OFDM symbol (N_{CBPS})
  - Two-step permutation
  - Adjacent coded bits mapped
    - Onto nonadjacent sub-carriers

$$i = (N_{CBPS}/16)\,(k \bmod 16) + \mathrm{floor}(k/16) \qquad k = 0,1,\ldots,N_{CBPS}-1$$

    - Alternate between less and more significant bits in the constellation – avoid long runs of low reliability LSBs

$$j = s \times \mathrm{floor}(i/s) + (i + N_{CBPS} - \mathrm{floor}(16 \times i/N_{CBPS})) \bmod s \qquad i = 0,1,\ldots N_{CBPS}-1$$

$$s = \max(N_{BPSC}/2,1)$$

Convolutional Encoder

- Rate 1/2 convolutional encoder
  - Punctured to obtain 2/3 and 3/4 rate

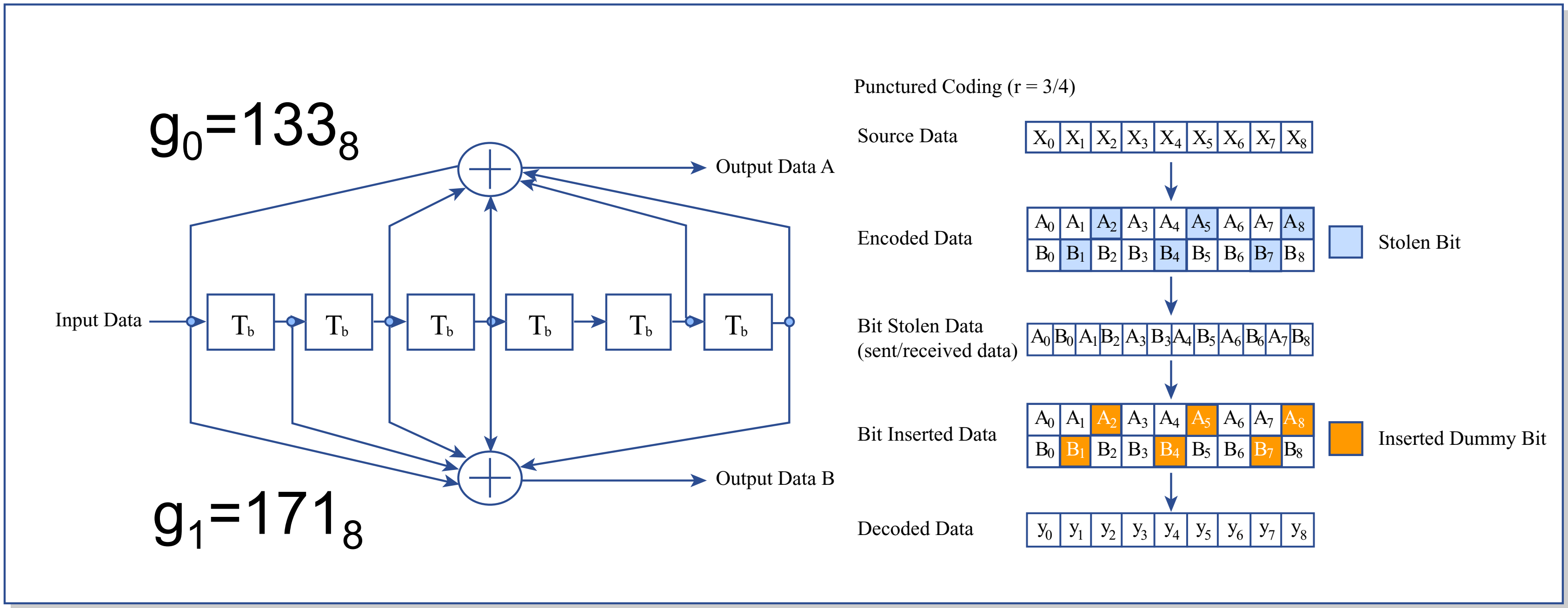


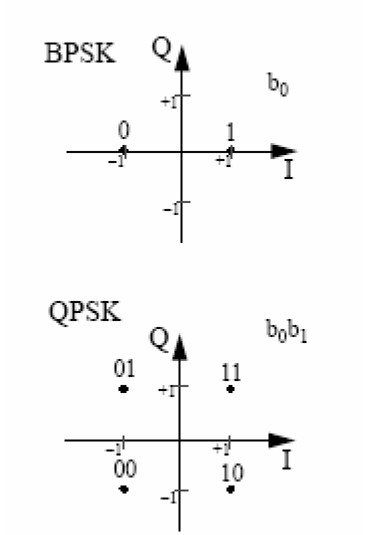
Figure by MIT OpenCourseWare.

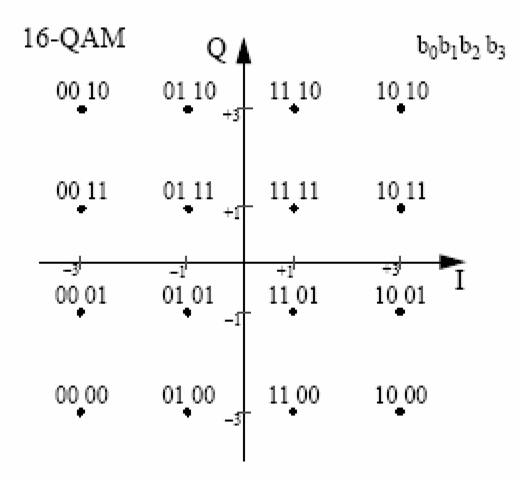
- 64-state (constraint length K=7) code
- Viterbi algorithm applied in the decoder

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

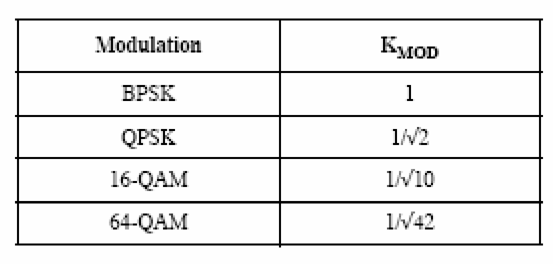
Signal mapper

- BPSK, QPSK, 16-QAM, 64-QAM
  - Data divided into groups of (1,2,4,6) bits and mapped to a constellation point (i.e. a complex number)
  - Gray-coded constellation mapings





$$d = (I + jQ) \times K_{MOD}$$



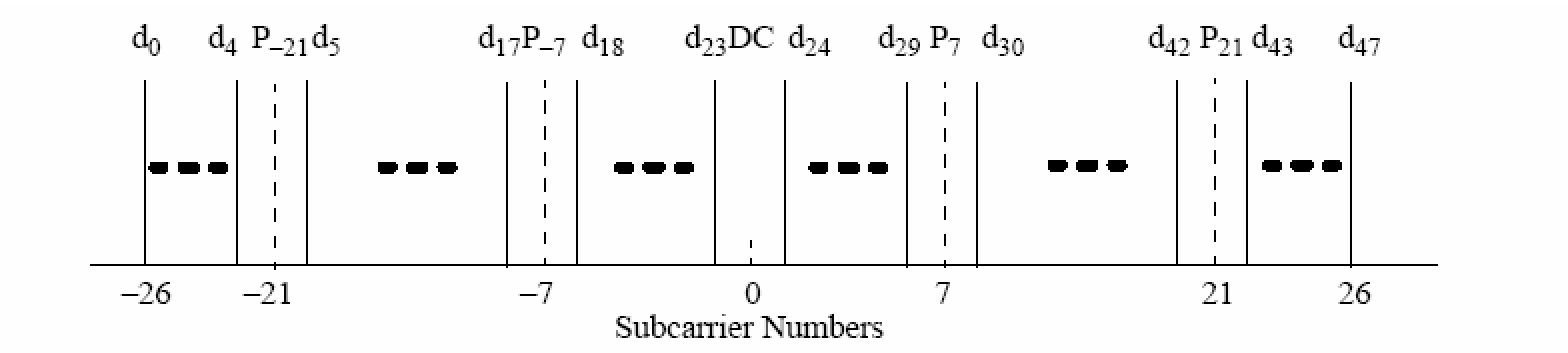
Modulation	K_{MOD}
BPSK	1
QPSK	$1/\sqrt{2}$
16-QAM	$1/\sqrt{10}$
64-QAM	$1/\sqrt{42}$

- Need the same average power for all mappings
  - Scale the output by K_{MOD}

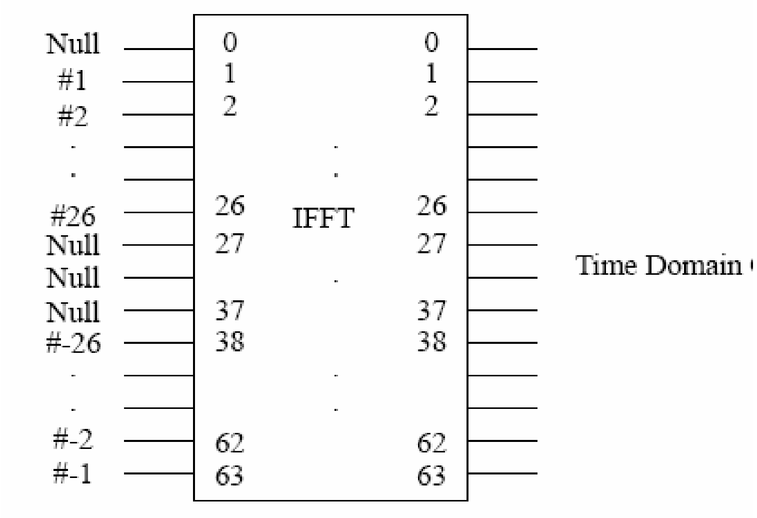
Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006. MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].

Pilot insertion and FFT/IFFT

- Pilot insertion
  - Pilots BPSK, prbs modulated



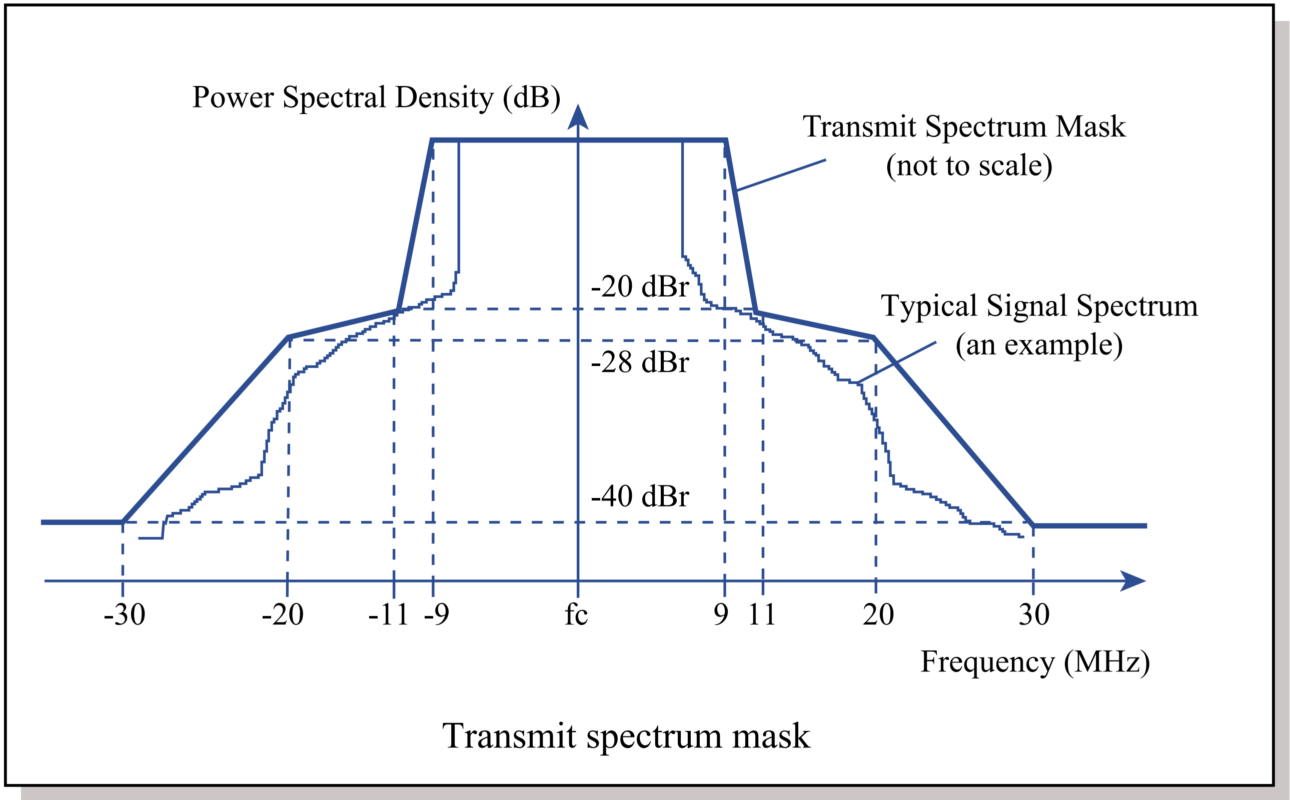
- FFT and IFFT shared
  - Just flip the Re and Im inputs



Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

Spectral mask

- Cannot use last 5 tones on each side
- Does not use extra windowing



Transmit spectrum mask

Figure by MIT OpenCourseWare.

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

Receiver architecture

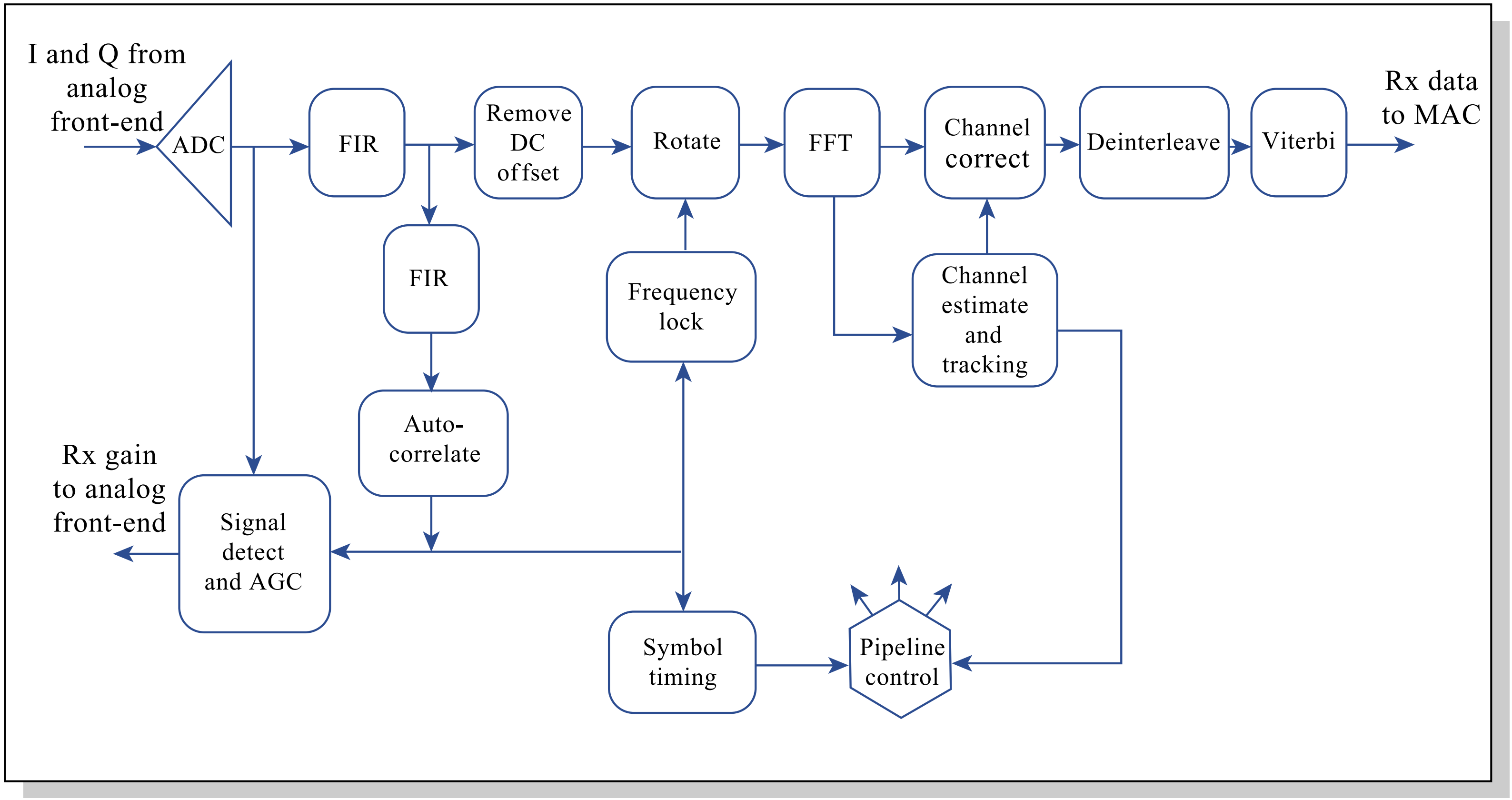


Figure by MIT OpenCourseWare.

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

Pilot tracking and channel correction

- OFDM packet structure

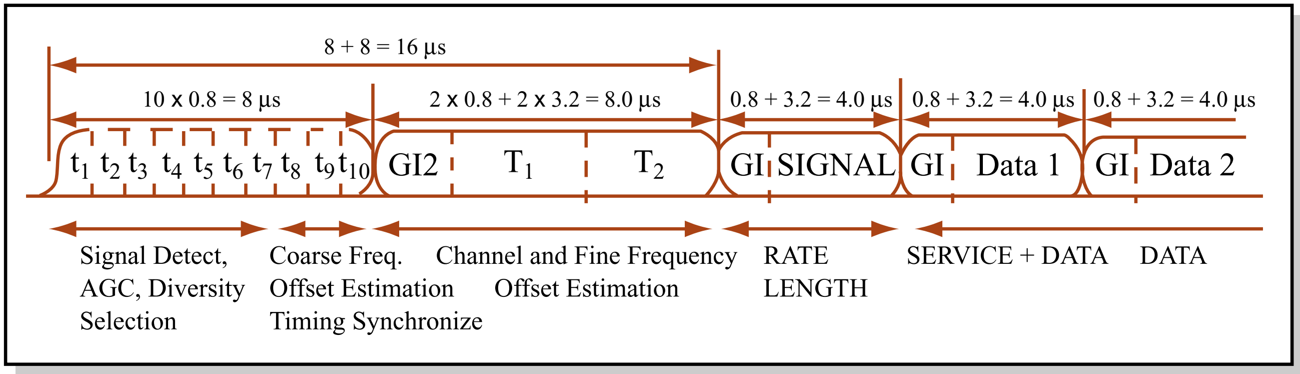


Figure by MIT OpenCourseWare.

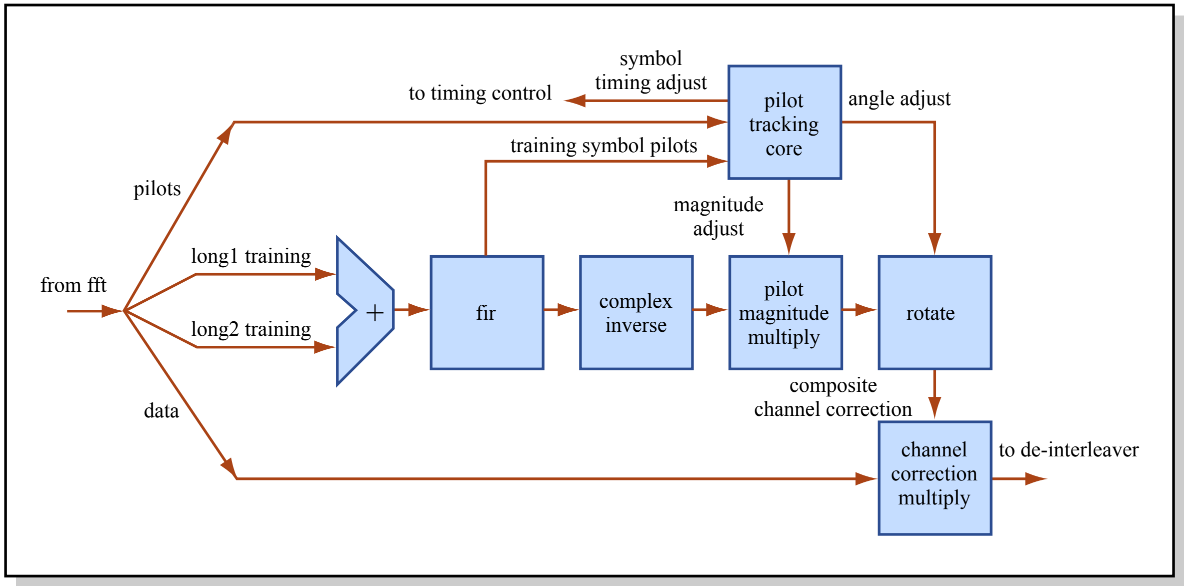


Figure by MIT OpenCourseWare.

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

Synchronizer

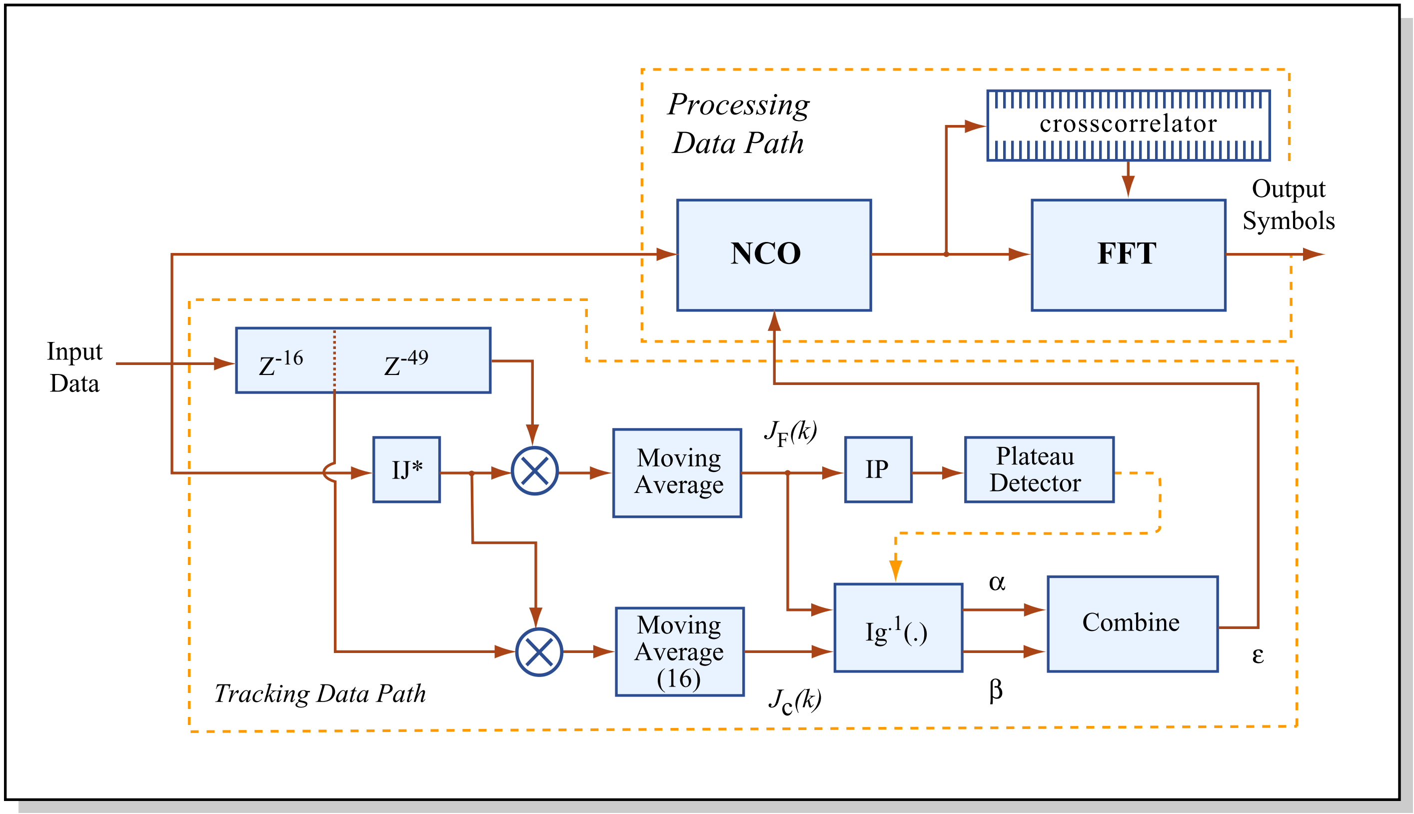


Figure by MIT OpenCourseWare.

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

Channel estimator

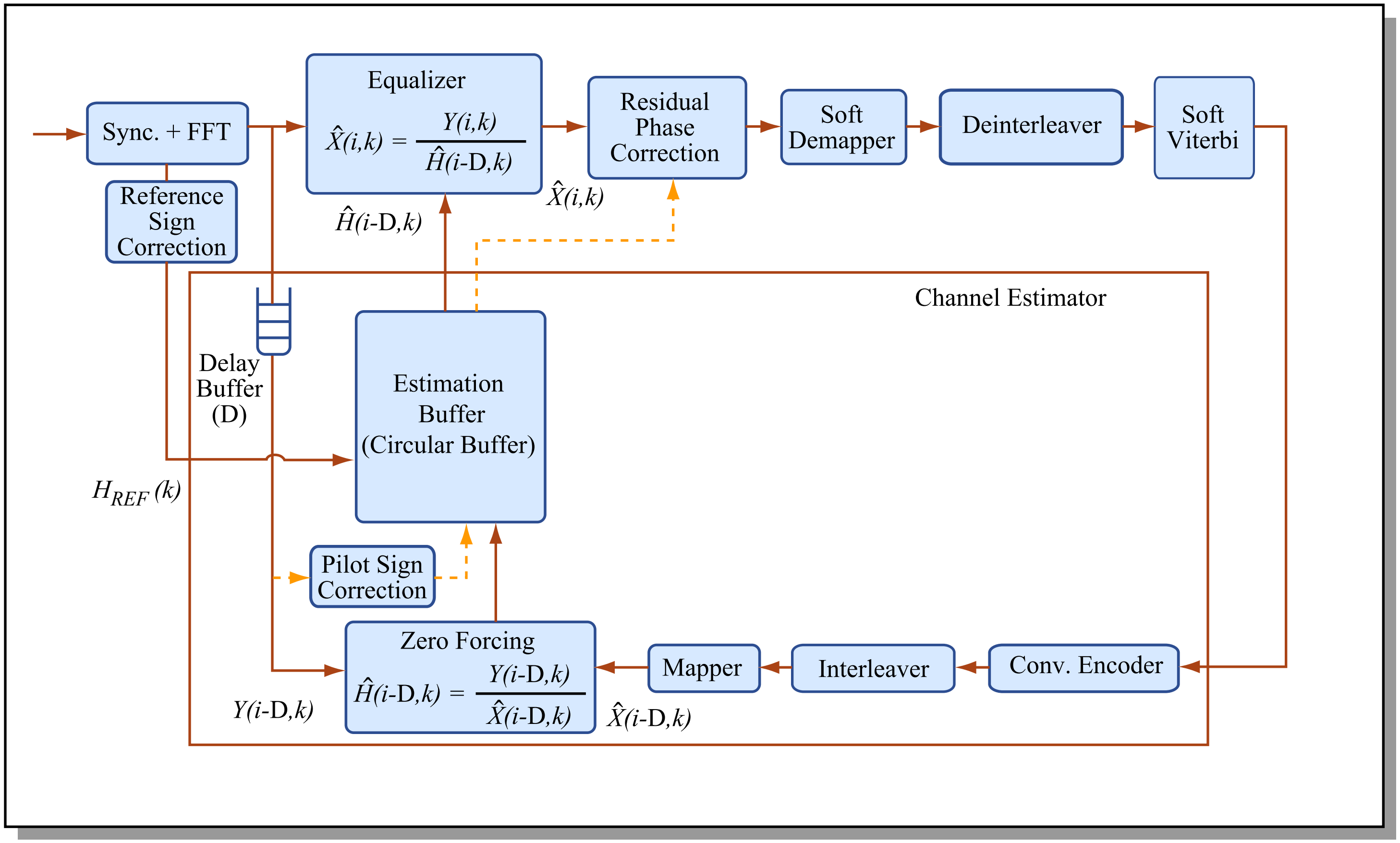


Figure by MIT OpenCourseWare.

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].

First 802.11a chip

Image removed due to copyright restrictions.

[1]

References

- [1] T.H. Meng, B. McFarland, D. Su and J. Thomson "Design and implementation of an all-CMOS 802.11a wireless LAN chipset," *Communications Magazine, IEEE* vol. 41, no. 8 SN - 0163-6804, pp. 160-168, 2003
- [2] M. Krstic, K. Maharatna, A. Troya, E. Grass and U. Jagdhold "Implementation of an IEEE 802.11a compliant low-power baseband processor," *Telecommunications in Modern Satellite, Cable and Broadcasting Service, 2003. TELSIKS 2003. 6th International Conference on* vol. 1, no. SN -, pp. 97-100 vol.1, 2003.
- [3] J. Thomson, B. Baas, E.M. Cooper, J.M. Gilbert, G. Hsieh, P. Husted, A. Lokanathan, J.S. Kuskin, D. McCracken, B. McFarland, T.H. Meng, D. Nakahira, S. Ng, M. Rattehalli, J.L. Smith, R. Subramanian, L. Thon, Y.-H. Wang and R. Yu "An integrated 802.11a baseband and MAC processor," *Solid-State Circuits Conference, 2002. Digest of Technical Papers. ISSCC. 2002 IEEE International* vol. 1, no. SN -, pp. 126-451 vol.1, 2002.
- [4] E. Grass, K. Tittelbach-Helmrich, U. Jagdhold, A. Troya, G. Lippert, O. Kruger, J. Lehmann, K. Maharatna, K.F. Dombrowski, N. Fiebig, R. Kraemer and P. Mahonen "On the single-chip implementation of a Hiperlan/2 and IEEE 802.11a capable modem," *Personal Communications, IEEE [see also IEEE Wireless Communications]* vol. 8, no. 6 SN - 1070-9916, pp. 48-57, 2001.

Cite as: Vladimir Stojanovic, course materials for 6.973 Communication System Design, Spring 2006.
MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.
Downloaded on [DD Month YYYY].