6.852: Distributed Algorithms
Fall, 2009

Class 25

Today’s plan

- Partially synchronous (timed) distributed systems
- Modeling timed systems
- Proof methods
- Mutual exclusion in timed systems
- Consensus in timed systems
- Clock synchronization
- Reading:
  - Chapters 23, 24, 25
  - [Attiya, Welch], Section 6.3, Chapter 13

Partially synchronous system models

- We've studied distributed algorithms in synchronous and asynchronous distributed models.
- Now, intermediate, partially synchronous models.
  - Involve some knowledge of time, but not synchronized rounds:
    - Bounds on relative speed of processes,
    - Upper and lower bounds for message delivery,
    - Local clocks, proceeding at approximately-predictable rates.
- Useful for studying:
  - Distributed algorithms whose behavior depends on time.
  - Practical communication protocols.
  - (Newer) Mobile networks, embedded systems, robot control,…
- Needs new models, new proof methods.
- Leads to new distributed algorithms, impossibility results.

Modeling Timed Systems

Modeling timed systems

MMT automata [Merritt, Modugno, Tuttle]

- Simple, special-cased timed model
- Immediate extension of I/O automata

GTA, more general timed automata

Timed I/O Automata

- Still more general
- [Kaynar, Lynch, Segala, Vaandrager] monograph
- Mathematical foundation for Tempo.

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Kaynar, Dilsun, Nancy Lynch, Roberto Segala, and Frits Vaandrager. *The Theory of Timed I/O Automata (Synthesis Lectures on Distributed Computing Theory).* 2nd ed. San Rafael, CA: Morgan & Claypool, 2010. ISBN: 978-1608450022.

MMT Automata

- Definition: An MMT automaton is an I/O automaton with finitely many tasks, plus a boundmap (lower, upper), where:
  - lower maps each task T to a lower bound lower(T), $0 \leq$ lower(T) $< \infty$ (can be 0, cannot be infinite),
  - upper maps each task T to an upper bound upper(T), $0 <$ upper(T) $\leq \infty$ (cannot be 0, can be infinite),
  - For every T, lower(T) $\leq$ upper(T).
- Timed executions:
  - Like ordinary executions, but with times attached to events.
  - $\alpha = s_0, (\pi_1, t_1), s_1, (\pi_2, t_2), s_2, \ldots$
  - Subject to the upper and lower bounds.
    - Task T can't be continuously enabled for more than time upper(T) without an action of T occurring.
    - If an action of T occurs, then T must have been continuously enabled for time at least lower(T).
  - Restricts the set of executions (unlike having just upper bounds):
  - No fairness anymore, just time bounds.

MMT Automata, cont'd

- Timed traces:
  - Suppress states and internal actions.
  - Keep info about external actions and their times of occurrence.
- Admissible timed executions:
  - Infinite timed executions with times approaching ∞, or
  - Finite timed executions such that upper(T) = ∞ for every task enabled in the final state.
- Rules out:
  - Infinitely many actions in finite time ("Zeno behavior").
  - Stopping when some tasks still have work to do and upper bounds by which they should do it.

- Simple model, not very general, but good enough to describe some interesting examples:

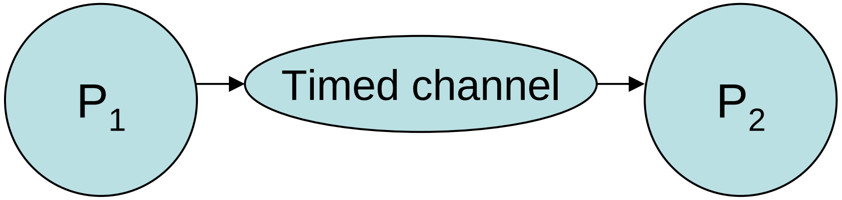
Example: Timed FIFO channel

- Consider our usual FIFO channel automaton.
  - State: queue
  - Actions:
    - Inputs: send(m), m in M
    - Outputs: receive(m), m in M
  - Tasks: receive = { receive(m) : m in M }
- Boundmap:
  - Associate lower bound 0, upper bound d, with the receive task.
- Guarantees delivery of oldest message in channel (head of queue), within time d.

Composition of MMT automata

- Compose MMT automata by
  - Composing the underlying I/O automata,
  - Combining all the boundmaps.
  - Composed automaton satisfies all timing constraints, of all components.
- Satisfies pasting, projection, as before:
  - Project timed execution (or timed trace) of composition to get timed executions (timed traces) of components.
  - Paste timed executions (or timed traces) that match up at boundaries to obtained timed executions (timed traces) of the composition.
- Also, a hiding operation, which makes some output actions internal.

Example: Timeout system



- P_1: Sender process
  - Sends "alive" messages at least every time l, unless it has failed.
  - Express using one send task, bounds [0,l].
- P_2: Timeout process
  - Decrements a count from k; if reaches 0 without a message arriving, output timeout.
  - Express with 2 tasks, decrement with bounds [l1, l2], and timeout with bounds [0,l].
  - Need non-zero lower bound for decrement, so that steps can be used to measure elapsed time.
- Compose P_1, P_2, and timed channel with bound d.
- Guarantees (assuming that $k\, l_1 > l + d$):
  - If P_2 times out P_1 then P_1 has in fact failed.
    - Even if P_2 takes steps as fast as possible, enough time has passed when it does a timeout.
  - If P_1 fails then P_2 times out P_1, and does so by time $k\, l_2 + l$.
    - P_2 could actually take steps slowly.

Example: Two-task race

- One automaton, two tasks:
  - Main = { increment, decrement, report }
    - Bounds [l_1, l_2].
  - Interrupt = { set }
    - Bounds [0,l].
- Increment count as long as flag = false, then decrement.
- When count returns to 0, output report.
- Set action sets flag true.
- Q: What is a good upper bound on the latest time at which a report may occur?
- $l + l_2 + (l_2 / l_1) l$
- Obtained by incrementing as fast as possible, then decrementing as slowly as possible.

General Timed Automata

- MMT is simple, but can’t express everything we might want:
  - Example: Perform actions “one”, then “two”, in order, so that “one” occurs at an arbitrary time in [0,1] and “two” occurs at time exactly 1.
- GTAs:
  - More general, expressive.
  - No tasks and bounds.
  - Instead, explicit time-passage actions $\upsilon(t)$, in addition to inputs, outputs, internal actions.
  - Time-passage steps $(s, \upsilon(t), s')$, between ordinary discrete steps.

Example: Timed FIFO Channel

- Delivers oldest message within time d
- States:

  queue
  now, a real, initially 0
  last, a real or ∞, initially ∞

  Time-valued variables

- Transitions:

  send(m)

  Effect:

  add m to queue
  if |queue| = 1 then last := now + d

  receive(m)

  Precondition:

  m = head(queue)

  Effect:

  remove head of queue
  if queue is nonempty then last := now + d else last := ∞

  $\upsilon(t)$

  Precondition:

  now + t $\leq$ last

  Effect:

  now := now + t

Another Timed FIFO Channel

- Delivers every message within time d
- States:

  queue, FIFO queue of (message, real) pairs

  now, a real, initially 0
- Transitions:

  send(m)

  Effect:

  add (m, now + d) to queue

  receive(m)

  Precondition:

  (m,t) = head(queue), for some t

  Effect:

  remove head of queue

  $\upsilon(t)$

  Precondition:

  now + t $\leq$ t′, for every (m, t′) in queue

  Effect:

  now := now + t

Transforming MMTAs to GTAs

- Program the timing constraints explicitly.
- Add state components:
  - now, initially 0
  - For each task T, add time-valued variables:
    - first(T), initially lower(T) if T is enabled in initial state, else 0.
    - last(T), initially upper(T) if T is enabled in initial state, else ∞.
- Manipulate the first and last values to express the MMT upper and lower bound requirements, e.g.:
  - Don't perform any task T if now $<$ first(T).
  - Don't let time pass beyond any last() value.
  - When task T becomes enabled, set first(T) to lower(T) and last(T) to upper(T).
  - When task T performs a step and is again enabled, set first(T) to lower(T) and last(T) to upper(T).
  - When task T becomes disabled, set first(T) to 0 and last(T) to ∞.

Two-task race

- New state components:
  now, initially 0
  first(Main), initially l_1
  last(Main), initially l_2
  last(Interrupt), initially l

- Transitions:
  increment:
  Precondition:
  flag = false
  now $\geq$ first(Main)
  Effect:
  count := count + 1
  first(Main) := now + l_1
  last(Main) := now + l_2

decrement:
Precondition:
flag = true
count > 0
now $\geq$ first(Main)
Effect:
count := count - 1
first(Main) := now + l_1
last(Main) := now + l_2

report:
- Precondition:
  flag = true
  count = 0
  reported = false
  now $\geq$ first(Main)
- Effect:
  reported := true
  first(Main) := 0
  last(Main) := ∞

Two-task race

set:

Precondition:

flag = false

Effect:

flag := true

last(Interrupt) := ∞

$\upsilon(t)$:

Precondition:

now + t $\leq$ last(Main)

now + t $\leq$ last(Interrupt)

Effect:

now := now + t

More on GTAs

- Composition operation
  - Identify external actions, as usual.
  - Synchronize time-passage steps globally.
  - Pasting and projection theorems.
- Hiding operation
- Levels of abstraction, simulation relations

Timed I/O Automata (TIOAs)

- Extension of GTAs in which time-passage steps are replaced by trajectories, which describe state evolution over time intervals.
  - Formally, mappings from time intervals to states.
  - Allows description of interesting state evolution, such as:
    - Clocks that evolve at approximately-known rates.
    - Motion of vehicles, aircraft, robots, in controlled systems.
- Composition, hiding, abstraction.

Proof methods for GTAs and TIOAs.

- Like those for untimed automata.
- Compositional methods.
- Invariants, simulation relations.
  - They work for timed systems too.
  - Now they generally involve time-valued state components as well as “ordinary” state components.
  - Still provable using induction, on number of discrete steps + trajectories.

Example: Two-task race

- Invariant 1: count $\leq \lfloor$ now / $l_1 \rfloor$.
  - count can't increase too much in limited time.
  - Largest count results if each increment takes smallest time, l_1.
- Prove by induction on number of discrete + time-passage steps? Not quite:
  - Property is not preserved by increment steps, which increase count but leave now unchanged.
- So we need another (stronger) invariant.
- Q: What else changes in an increment step?
  - Before the step, first(Main) $\leq$ now; afterwards, first(Main) = now + l_1.
  - So first(Main) should appear in the stronger invariant.

- Invariant 2: If not reported then count $\leq \lfloor$ first(Main) / l_1 - 1$\rfloor$.
- Use Invariant 2 to prove Invariant 1.

Two-task race

- Invariant 2: If not reported then

  $$count \leq \lfloor first(Main) / l_1 - 1 \rfloor$$

- Proof:
  - By induction.
  - Base: Initially, LHS = RHS = 0.
  - Inductive step: Dangerous steps either increase LHS (increment) or decrease RHS (report).
    - Time-passage steps: Don't change anything.
    - report: Can't cause a problem because then reported = true.
    - increment:
      - count increases by 1
      - first(Main) increases by at least l_1: Before the step, first(Main) $\leq$ now, and after the step, first(Main) = now + l_1.
      - So the inequality is preserved.

Modeling timed systems (summary)

- MMT automata [Merritt, Modugno, Tuttle]
  - Simple, special-cased timed model
  - Immediate extension of I/O automata
  - Add upper and lower bounds for tasks.
- GTA, more general timed automata
  - Explicit time-passage steps
- Timed I/O Automata
  - Still more general
  - Instead of time-passage steps, use trajectories, which describe evolution of state over time.
  - [Kaynar, Lynch, Segala, Vaandrager] monograph
  - Tempo support

Simulation relations

- These work for GTAs/TIOAs too.
- Imply inclusion of sets of timed traces of admissible executions.
- Simulation relation definition (from A to B):
  - Every start state of A has a related start state of B. (As before.)
  - If s is a reachable state of A, u a related reachable state of B, and (s, π, s') is a discrete step of A, then there is a timed execution fragment α of B starting with u, ending with some u' of B that is related to s', having the same timed trace as the given step, and containing no time-passage steps.
  - If s is a reachable state of A, u a related reachable state of B, and $(s, \upsilon(t), s')$ is a time-passage step of A, then there is a timed execution fragment of B starting with u, ending with some u' of B that is related to s', having the same timed trace as the given step, and whose total time-passage is t.

Example: Two-task race

- Prove upper bound of $l + l_2 + (l_2 / l_1)\, l$ on time until report.
- Intuition:
  - Within time l, set flag true.
  - During time l, can increment count to at most approximately l / l_1.
  - Then it takes time at most $(l / l_1)\, l_2$ to decrement count to 0.
  - And at most another l_2 to report.
- Could prove a simulation relation, to a trivial GTA that just outputs report, at any time $\leq l + l_2 + (l_2 / l_1)\, l$.
- Express this using time variables:
  - now
  - last(report), initially $l + l_2 + (l_2 / l_1)\, l$.
- The simulation relation has an interesting form: inequalities involving the time variables:

Simulation relation

- s = state of race automaton, u = state of time bound spec automaton
- u.now = s.now, u.reported = s.reported
- u.last(report) ≥

  s.last(Int) + (s.count + 2) l_2 + (l_2 / l_1) (s.last(Int) – s.first(Main)),

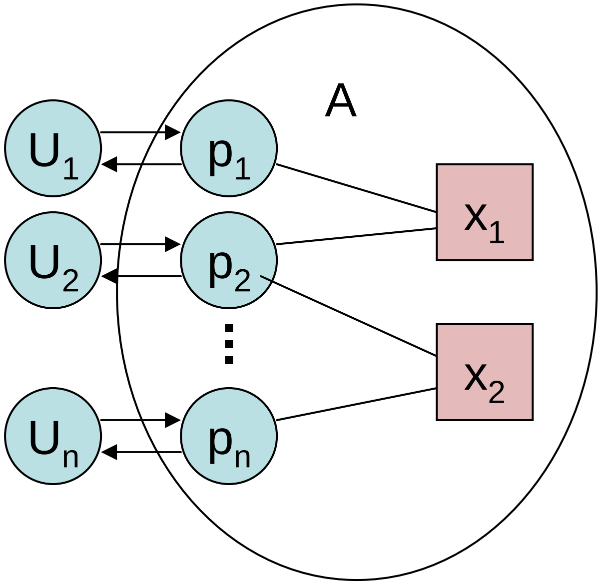
  if s.flag = false and s.first(Main) ≤ s.last(Int),

  s.last(Main) + (s.count) l_2 , otherwise.

- Explanation:
  - If flag = true, then time until report is the time until the next decrement, plus the time for the remaining decrements and the report.
  - Same if flag = false but must become true before another increment.
  - Otherwise, at least one more increment can occur before flag is set.
  - After set, it might take time (s.count + 1) l_2 to count down and report.
  - But current count could be increased some more:
    - At most 1 + (last(Int) – first(Main)) / l_1 times.
  - Multiply by l_2 to get extra time to decrement the additional count.

Timed Mutual Exclusion Algorithms

Timed mutual exclusion



- Model as before, but now the Us and the algorithm are MMT automata.
- Assume one task per process, with bounds $[l_1,l_2]$, $0 < l_1 \leq l_2 < \infty$.
- Users: Arbitrary tasks, boundmaps.

- Mutual exclusion problem: guarantee well-formedness, mutual exclusion, and progress, in all admissible timed executions.
- No high-level fairness guarantees, for now.
- Now, algorithm’s correctness is allowed to depend on timing assumptions.

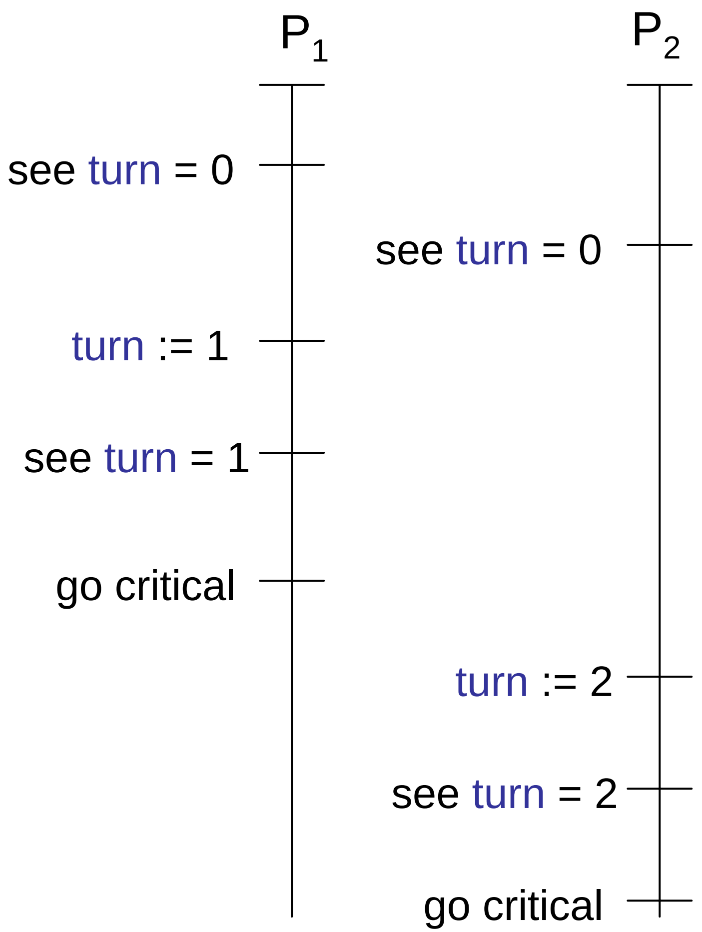
Fischer mutual exclusion algorithm

- Famous, “published” only in email from Fischer to Lamport.
- A toy algorithm, widely used as a benchmark for modeling and verification methods for timing-based systems.
- Uses a single read/write register, turn.
- Compare: In asynchronous model, need n variables.

- Incorrect, asynchronous version (process i):
  - Trying protocol:
    - wait for turn = 0
    - turn := i
    - if turn = i, go critical; else go back to beginning
  - Exit protocol:
    - turn := 0

Incorrect execution

- To avoid this problem, add a timing constraint:
  - Process i waits long enough between set_i and $check_i$ so that no other process j that sees turn = 0 before set_i can set turn := j after $check_i$.
  - That is, interval from set_i to $check_i$ is strictly longer than interval from $test_j$ to set_j.
- Can ensure by counting steps:
  - Before checking, process i waits k steps, where $k > l_2 / l_1$.
  - Shortest time from set_i to $check_i$ is $k\, l_1$, which is greater than the longest time l_2 from $test_j$ to set_j.



Fischer mutex algorithm

- Pre/effect code, p. 777.
- Not quite in the assumed model:
  - That has just one task/process, with bounds $[l_1, l_2]$.
  - Here we use another task for the check, with bounds $[a_1, a_2]$, where $a_1 = k\, l_1$, $a_2 = k\, l_2$,
  - This version is more like the ones used in most verification work.
- Proof?
  - Easy to see the algorithm avoids the bad example, but how do we know it's always correct?

Proof of mutex property

- Use invariants.
- One of the earliest examples of an assertional proof for timed models.
- Key intermediate assertion:
  - If pc_i = check, turn = i, and pc_j = set, then $first(check_i) > last(main_j)$.
  - That is, if i is about to check turn and get a positive answer, and j is about to set turn, then the earliest time when i might check it is strictly after the latest time when j might set it.
  - Rules out the bad interleaving.
- Can prove this by an easy induction.
- Use it to prove main assertion:
  - If $pc_i \in \{$ leave-try, crit, reset $\}$, then turn = i, and for every j, $pc_j \neq$ set.
- Which immediately implies mutual exclusion.

Proof of progress

- Easy event-based argument:
  - By contradiction: Assume someone is in T, and no one is thereafter ever in C.
  - Then eventually region changes stop, everyone is in either T or R, at least one process is in T.
  - Eventually turn acquires a contender's index, then stabilizes to some contender's index, say i.
  - Then i proceeds to C.
- Refine this argument to a time bound, for the time from when someone is in T until someone is in C:
  - $2 a_2 + 5 l_2 = 2 k l_2 + 5 l_2$
  - Since k is approximately $L = l_2 / l_1$ (timing uncertainty ratio), this is $2 L l_2 + O(l_2)$
  - Thus, timing uncertainty stretches the time complexity.

Stretching the time complexity

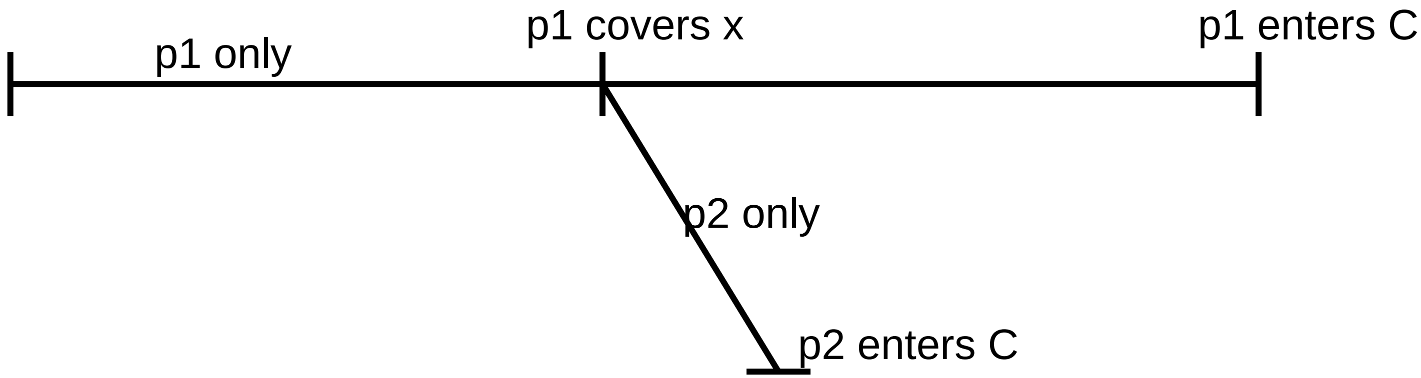
- Q: Why is the time complexity "stretched" by the timing uncertainty $L = (l_2 / l_1)$, yielding an $L\ l_2$ term?
- Process i must ensure that time $t = l_2$ has elapsed, to know that another process has had enough time to perform a step.
- Process i determines this by counting its own steps.
- Must count at least t / l_1 steps to be sure that time t has elapsed, even if i's steps are fast (l_1).
- But the steps could be slow (l_2), so the total time could be as big as $(t / l_1)\ l_2 = (l_2 / l_1)\ t = L\ t$.
- Requires real time Lt for process in a system with timing uncertainty L to be sure that time t has elapsed.
- Similar stretching phenomenon arose in timeout example.

Lower bound on time

- Theorem: There is no timed mutex algorithm for 2 processes with 1 shared variable, having an upper bound of $L\, l_2$ on the time for someone to reach C.
- Proof:
  - Like the proof that 1 register is insufficient for 2-process asynchronous mutual exclusion.
  - By contradiction; suppose such an algorithm exists.
  - Consider admissible execution α in which process 1 runs alone, slowly (all steps take l_2).
  - By assumption, process 1 must enter C within time $L\, l_2$.
  - Must write to the register x before $\rightarrow$C.
  - Pause process 1 just before writing x for the first time.

Lower bound on time

- Proof, cont'd:
  - Now run process 2, from where process 1 covers x.
  - p2 sees initial state, so eventually $\rightarrow$C.
  - If p2 takes steps as slowly as possible (l_2), must $\rightarrow$C within time L l_2.
  - If we speed p2 up (shrink), p2 $\rightarrow$C within time L l_2 (l_1 / l_2) = L l_1.
  - So we can run process 2 all the way to C during the time p1 is paused, since l_2 = L l_1.
  - Then as in asynchronous case, can resume p1, overwrites x, enters C, contradiction.



The Fischer algorithm is fragile

- Depends on timing assumptions, even for the main safety property, mutual exclusion.
- It would be nice if safety were independent of timing (e.g., like Paxos).
- Can modify Fischer so mutual exclusion holds in all asynchronous runs, for n processes, using 3 registers [Section 24.3].
- But this fails to guarantee progress, even assuming timing eventually stabilizes (like Paxos).
- In fact, progress depends crucially on timing:
  - If time bounds are violated, then algorithm can deadlock, making future progress impossible.
- In fact, we have:

Another impossibility result!

- It's impossible to guarantee n-process mutual exclusion in all asynchronous runs, progress if timing stabilizes, with < n registers:

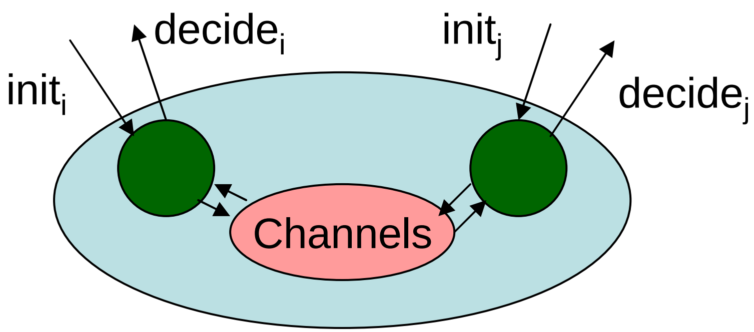
- Theorem: There is no asynchronous read/write shared-memory algorithm for $n \geq 2$ processes that:
  - Guarantees well-formedness and mutual exclusion when run asynchronously,
  - Guarantees progress when run so that each process' step bounds eventually are in the range $[l_1, l_2]$, and
  - Uses < n shared registers.
- !!!

- Proof: Similar to that of impossibility of asynchronous mutex for < n registers (tricky).

Timed Consensus Algorithms

Consensus in timed systems

- Network model:
- Process:
  - MMT automaton, finitely many tasks.
  - Task bounds $[l_1,l_2]$, $0 < l_1 \leq l_2 < \infty$, $L = l_2 / l_1$
  - Stopping failures only.
- Channels:
  - GTA or TIOA
  - Reliable FIFO channels, upper bound of d for every message.
- Properties:
  - Agreement,
  - Validity (weak or strong),
  - Failure-free termination
  - f-failure termination, wait-free termination



- In general, we're allowed to rely on time bounds for both safety + liveness.
- Q: Can we solve fault-tolerant agreement? How many failures? How much time does it take?

Consensus in timed systems

- Assumptions:
  - V = { 0, 1 },
  - Completely connected graph,
  - $l_1, l_2 << d$ (in fact, $n\, l_2 << d$, $L\, l_2 << d$).
  - Every task always enabled.
- Results:
  - Simple algorithm, for any number f of failures, strong validity, time bound $\approx f\, L\, d$
  - Simple lower bound: (f+1) d.
  - More sophisticated algorithm: $\approx Ld + (2f+2)\, d$
  - More sophisticated lower bound: $\approx Ld + (f-1)\, d$
- [Attiya, Dwork, Lynch, Stockmeyer]

Simple algorithm

- Implement a perfect failure detector, which times out failed processes.
  - Process i sends periodic "alive" messages.
  - Process i determines process j has failed if i doesn't receive any messages from j for a large number of i's steps ($\approx (d + l_2) / l_1$ steps).
  - Time until detection at most $\approx L\ d + O(L\ l_2)$.
  - Ld is the time needed for a timeout.
- Use the failure detector to simulate a round-based synchronous consensus algorithm for the required f+1 rounds.
- Time for consensus at most $\approx f\ L\ d + O(f\ L\ l_2)$.

Simple lower bound

- Upper bound (so far): $\approx$ f L d + O(f L l_2).
- Lower bound (f+1)d
  – Follows from (f+1)-round lower bound for synchronous model, via a model transformation.
- Note the role of the timing uncertainty L:
  – Appears in the upper bound: f Ld, time for f successive timeouts.
  – But doesn't appear in the lower bound.
- Q: How does the real cost depend on L?

Better algorithm

- Time bound: $Ld + (2f+2)d + O(f\, l_2 + L\, l_2)$
  - Time for just one timeout!
  - Tricky algorithm, LTTR.
    - Uses a series of rounds, each involving an attempt to decide.
    - At even-numbered rounds, try to decide 0; at odd-numbered rounds, try to decide 1.
    - Each failure can cause an attempt to fail, move on to another round.
    - Successful round takes time at most ≈ Ld.
    - Unsuccessful round k takes time at most ≈ $(f_k + 1)$ d, where f_k is the number of processes that fail at round k.

Better lower bound

- Upper bound: $\approx$ Ld + (2f+2)d
- Lower bound: Ld + (f-1) d
- Interesting proof---uses practically every lower bound technique we've seen:
  - Chain argument, as in Chapter 6.
  - Bivalence argument, as in Chapter 12.
  - Stretching and shrinking argument for timed executions, as in Chapter 24.
- LTTR

[Dwork, Lynch, Stockmeyer 88] consensus results

- 2007 Dijkstra prize
- Weaken the time bound assumptions so that they hold eventually, from some point on, not necessarily always.
- Assume $n > 2f$ (unsolvable otherwise).
- Guarantees agreement, validity, f-failure termination.
  - Thus, safety properties (agreement and validity) don't depend on timing.
  - Termination does---but in a nice way: guaranteed to terminate if time bound assumptions hold from any point on.
  - Similar to problem solved by Paxos.
- Algorithm:
  - Similar to Paxos (earlier), but allows less concurrency.

[DLS] algorithm

- Rotating coordinator as in 3-phase commit, pre-allocated “stages”.
- In each stage, one pre-determined coordinator takes charge, tries to coordinate agreement using a four-round protocol:
  1. Everyone sends “acceptable” values to coordinator; if coordinator receives “enough”, it chooses one to propose.
  2. Coordinator sends proposed value to everyone; anyone who receives it “locks” the value.
  3. Everyone who received a proposal in round 2 sends an ack to the coordinator; if coordinator receives “enough” acks, decides on the proposed value.
  4. Everyone exchanges lock info.
- “Acceptable” means opposite value isn’t locked.

- Implementing synchronous rounds:
  – Use the time assumptions.
  – Emulation may be unreliable until timing stabilizes.
  – That translates into possible lost messages, in earlier rounds.
  – Algorithm can tolerate lost messages before stabilization.

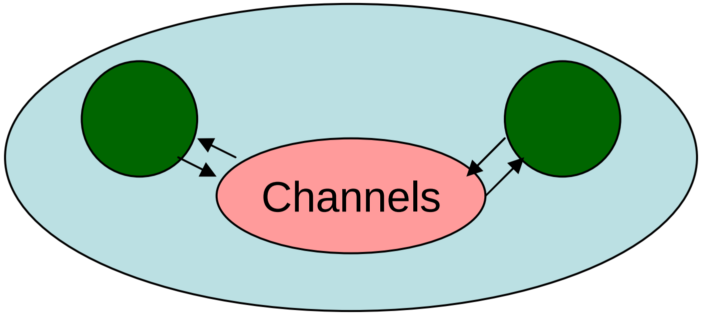
Mutual exclusion vs. consensus

- Mutual exclusion with < n shared registers:
  - Asynchronous systems:
    - Impossible
  - Timed systems:
    - Solvable, time upper bound O(L l_2), matching lower bound
  - Systems where timing assumptions hold from some point on:
    - Impossible to guarantee both safety (mutual exclusion) and liveness (progress).
- Consensus with f failures, $f \geq 1$:
  - Asynchronous systems:
    - Impossible
  - Timed systems:
    - Solvable, time upper bound L d + O(d), matching lower bound.
  - Systems where timing assumptions hold from some point on:
    - Can guarantee both safety (agreement and validity) and liveness (f-failure termination), for n > 2f.

Clock Synchronization Algorithms

Clock synchronization

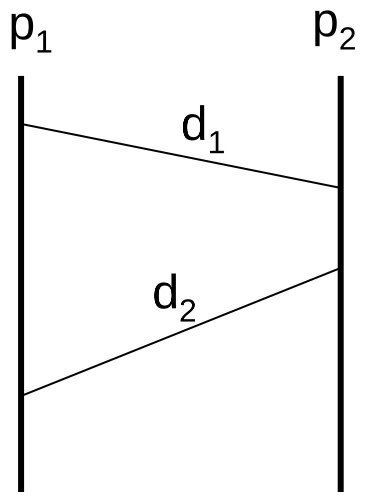
- Network model:
- Process:
  - TIOA
  - Includes a physical clock component that progresses at some (possibly varying) rate in the range [1 - ρ, 1+ ρ].
  - Not under the process' control.
- Channels:
  - GTA or TIOA
  - Reliable FIFO channels, message delay bounds in interval [d_1, d_2].
- Properties:
  - Each node, at each time, computes the value of a logical clock
  - Agreement: Logical clocks should become, and remain, within a small constant ε of each other.
  - Validity: Logical clock values should be approximately within the range of the physical clock values.

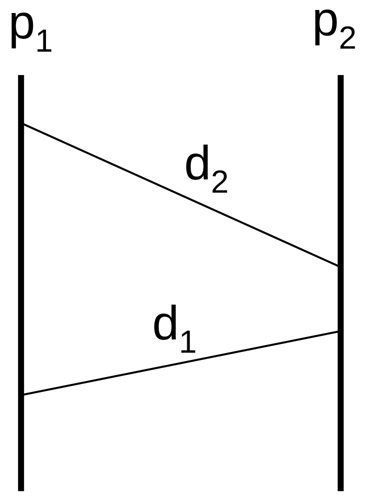


- Issues:
  - Timing uncertainty
  - Tolerating failures
  - Scalability
  - Accommodating external clock inputs

Timing uncertainty

- E.g., 2 processes:
  - Messages from p_1 to p_2 might always take the minimum time d_1.
  - Messages from p_2 to p_1 might always take the maximum time d_2.
  - Or vice versa.
  - Either way, the logical clocks are supposed to be within ε of each other.
  - Implies that $\varepsilon \geq (d_2 - d_1) / 2$
- Can achieve $\varepsilon \approx (d_2 - d_1) / 2$, if clock drift rate is very small and there are no failures.
- For n processes in fully connected graph, can achieve $\varepsilon \approx (d_2 - d_1)(1 - 1/n)$, and that's provably optimal.



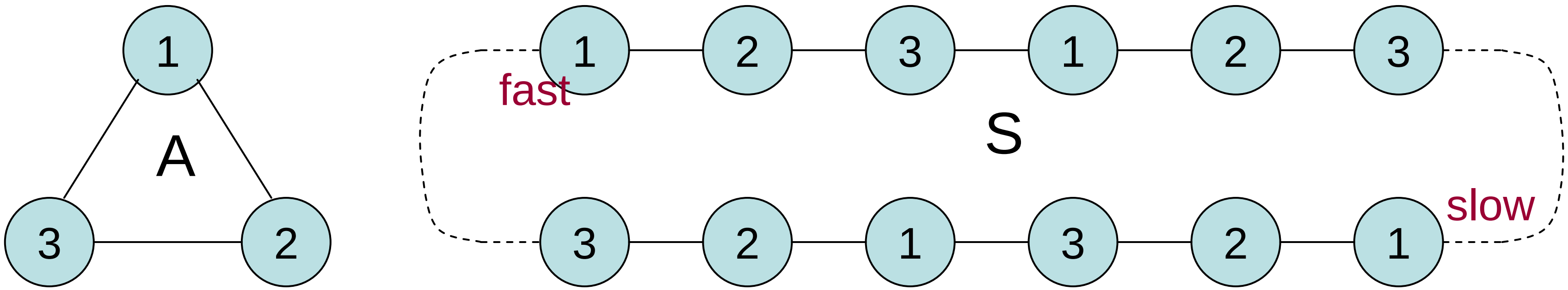


Accommodating failures

- Several published algorithms for n > 3f processes to establish and maintain clock synchronization, in the presence of up to f Byzantine faulty processes.
  - [Lamport], [Dolev, Strong], [Lundelius, Lynch],…
  - Some algorithms perform fault-tolerant averaging.
  - Some wait until f+1 processes claim a time has been reached before jumping to that time.
  - Etc.
- Lower bound: n > 3f is necessary.
  - Original proof: [Dolev, Strong]
  - Cuter proof: [Fischer, Lynch, Merritt]
    - By contradiction: Assume (e.g.) a 3-process clock synch algorithm that tolerates 1 Byzantine faulty process.
    - Form a large ring, from many copies of the algorithm:

Accommodating failures

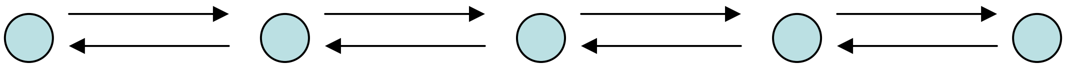
- Lower bound proof: n > 3f necessary
  - By contradiction: Assume a 3-process clock synch algorithm that tolerates 1 Byzantine faulty process.
  - Form a large ring, from many copies of the algorithm:



  - Let the physical clocks drift progressively, as we move around the ring, fastest and slowest at opposite sides of the ring.
  - Any consecutive pair's logical clocks must remain within ε of each other, by agreement, and must remain approximately within the range of their physical clocks, by validity.
  - Can't satisfy this everywhere in the ring.

Scalability

- Large, not-fully-connected network.
- E.g., a line:



- Can't hope to synchronize distant nodes too closely.
- Instead, try to achieve a gradient property, saying that neighbors' clocks are always closely synchronized.
- Impossibility result for gradient clock synch [Fan 04]: Any clock synch algorithm in a line of length D has some reachable state in which the logical clocks of two neighbors are $\Omega(\log D / \log \log D)$ apart.
- Algorithms exist that achieve a constant gradient "most of the time".
- And newer algorithms that achieve O(log D) all of the time.

External clock inputs

- Practical clock synch algorithms use reliable external clock sources:
  - NTP time service in Internet
  - GPS in mobile networks
- Nodes with reliable time info send it to other nodes.
- Recipients may correct for communication delays
- Typically ignore failures.

Mobile Wireless Network Algorithms

Mobile networks

- Nodes moving in physical space, communicating using local broadcast.
- Mobile phones, hand-held computers; robots, vehicles, airplanes
- Physical space:
  - Generally 2-dimensional, sometimes 3
- Nodes:
  - Have uids.
  - May know the approximate real time, and their own approximate locations.
  - May fail or be turned off, may restart.
  - Don't know a priori who else is participating, or who is nearby.
- Communication:
  - Broadcast, received by nearby listening nodes.
  - May be unreliable, subject to collisions/losses, or
  - May be assumed reliable (relying on backoff mechanisms to mask losses).
- Motion:
  - Usually unpredictable, subject to physical limitations, e.g. velocity bounds.
  - May be controllable (robots).
- Q: What problems can/cannot be solved in such networks?

Some preliminary results

- Dynamic graph model
  - Welch, Walter, Vaidya,…
  - Algorithms for mutual exclusion, k-exclusion, message routing,…
- Wireless networks with collisions
  - Algorithms / lower bounds for broadcast in the presence of collisions [Bar-Yehuda, Goldreich, Itai], [Kowalski, Pelc],…
  - Algorithms / lower bounds for consensus [Newport, Gilbert, et al.]
- Rambo atomic memory algorithm
  - [Gilbert, Lynch, Shvartsman]
  - Reconfigurable Atomic Memory for Basic (read/write) Objects
  - Implemented using a changing quorum system configuration.
  - Paxos consensus used to change the configuration, runs in the background without interfering with ongoing reads/writes.
- Virtual Node abstraction layers for mobile networks
  - Gilbert, Nolte, Brown, Newport,…

Some preliminary results

- Neighbor discovery, counting number of nodes, maintaining network structures,…

- Leave all this for another course.

VN Layers for mobile networks

- Add Virtual Nodes: Simple state machines (TIOAs) located at fixed, known geographical locations (e.g., grid points).
- Mobile nodes in the vicinity emulate the VSNs, using a Replicated State Machine approach, with an elected leader managing communication.
- Virtual Nodes may fail, later recover in initial state.
- Program applications over the VSN layer.
  - Geocast, location services, point-to-point communication, bcast.
  - Data collection and dissemination.
  - Motion coordination (robots, virtual traffic lights, virtual air-traffic controllers).

- Other work: Neighbor discovery, counting number of nodes, maintaining network structures,…
- Leave all this for another course.

Next time…

- There is no next time!
- Have a very nice break!

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