6.852: Distributed Algorithms
Fall, 2009

Class 23

Today's plan

- Shared memory vs. networks
- Consensus in asynchronous networks
- Reading:
  - Chapter 17
  - [Lamport] The Part-Time Parliament (Paxos)
- Next time:
  - Self-stabilization
  - [Dolev book], Chapter 2

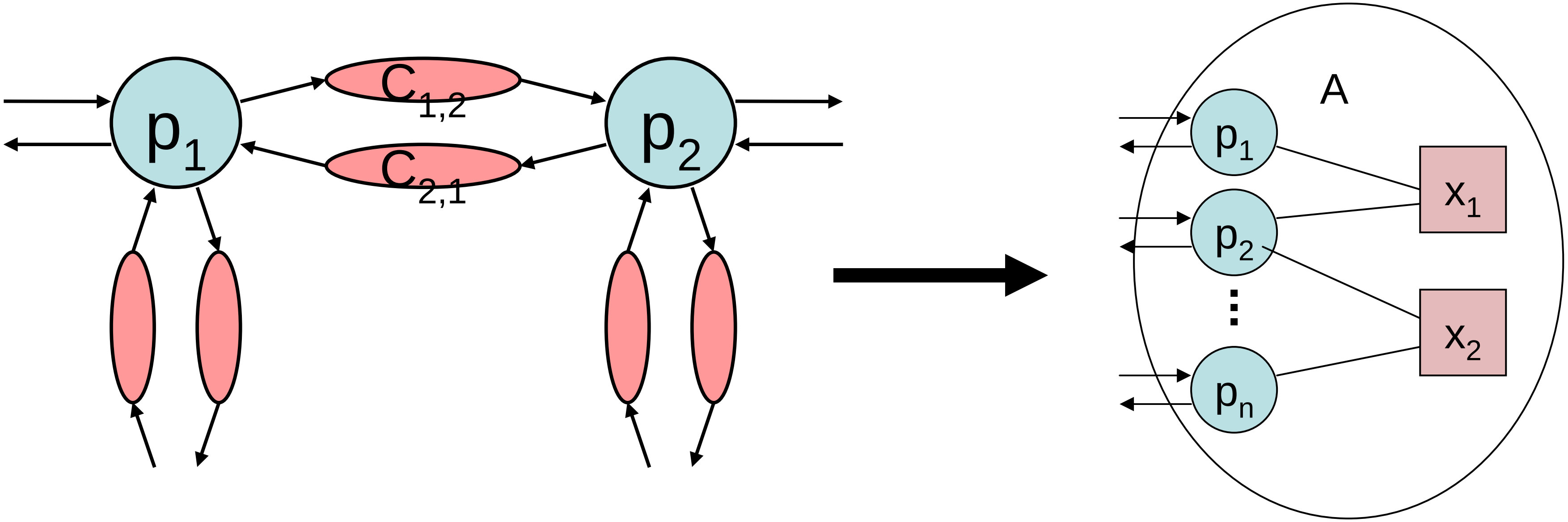
Shared memory vs. Networks

- Simulating shared memory in distributed networks:
  - Popular method for simplifying distributed programming.
  - Distributed shared memory (DSM).
  - Easy if there are no failures.
  - Possible if $n > 2f$; impossible if $n \leq 2f$.
  - [Attiya, Bar-Noy, Dolev] fault-tolerant algorithm
- Simulating networks using shared memory:
  - Easier, because shared memory is “more powerful”.
  - Works for any number of failures.
  - Useful mainly for lower bounds, impossibility results.
    - Carry over impossibility results for shared memory model to network model
    - E.g., for fault-tolerant consensus.

Paxos

- A fault-tolerant consensus algorithm for distributed networks.
- Can use it to implement a fault-tolerant replicated state machine (RSM) in a distributed network.
- Generalizes Lamport’s timestamp-based non-fault-tolerant RSM algorithm.

Simulating networks using shared-memory systems



Simulating networks using shared-memory systems

- Easy transformation from networks to shared-memory, because shared-memory model is more powerful:
  - Has reliable, instantaneously-accessible shared memory.
  - No arbitrary delays as in channels.
- Transformation preserves fault-tolerance, even for $f \geq n/2$.
- Assume:
  - Asynchronous network system A, running on undirected graph network G.
  - Failures: $stop_i$ event disables P_i and has no effect on channels.
- Produce:
  - Asynchronous read/write shared-memory system B simulating A, in the same sense as for atomic objects:
  - For any execution α of the shared-memory system $B \times U$, there is an execution α' of the network system $A \times U$ such that:
    - $\alpha \mid U = \alpha' \mid U$ and
    - $stop_i$ events occur for the same i in α and α'.
    - If α is fair then α' is also fair.

Algorithm

- Replace channel $C_{i,j}$ with a 1-writer, 1-reader shared variable x(i,j), writable by i, readable by j.
- x(i,j) contains a queue of messages, initially empty.
- Process i adds messages, never removes any.

- Process i simulates automaton P_i, step by step.
  - To simulate $\text{send}(m)_{i,j}$, process i adds m to end of x(i,j).
    - Does this using a write operation, by remembering what it wrote there earlier.
  - Meanwhile, process i keeps checking its incoming variables x(j,i), looking for new messages.
    - Does this by remembering what it saw there before.
    - When it finds a new message, process i handles it the same way P_i would handle it.

Some pseudocode

- State variables for process i
  - pstate : states(P_i)
  - sent(j) for each out-neighbor j: sequence of M, initially empty
  - rcvd(j), processed(j) for each in-neighbor j: seq of M, initially empty

- Transitions for i
  - Internal send(m,j)$_i$:
    - pre: send(m)$_{i,j}$ enabled in pstate$_i$
    - eff: append m to sent(j); x(i,j) := sent(j);
      update pstate as for send(m)$_{i,j}$
  - Internal receive(m,j)$_l$
    - pre: true
    - eff: rcvd(j) := x(j,i);
      update pstate using messages in rcvd(j) - processed(j);
      processed(j) := rcvd(j)
  - All others: As for P_i, using pstate.

An important corollary

- Theorem: This simulation produces an asynchronous shared-memory system B simulating A, in the sense that, for any execution α of the shared-memory system B × U, there is an execution α' of the network system A × U such that:
    - $\alpha \mid U = \alpha' \mid U$.
    - $stop_i$ events occur for the same i in α and α'.
    - If α is fair then α' is also fair.
- Corollary: Consensus is impossible in asynchronous networks, with 1 stopping failure [Fischer, Lynch, Paterson].
- Proof:
    - If such an algorithm existed, we could simulate it in an asynchronous shared-memory system using the simulation just given.
    - This would yield a 1-fault-tolerant consensus algorithm for (1-writer 1-reader) read/write shared memory.
    - We already know this is impossible [Loui, Abu-Amara].

Another corollary

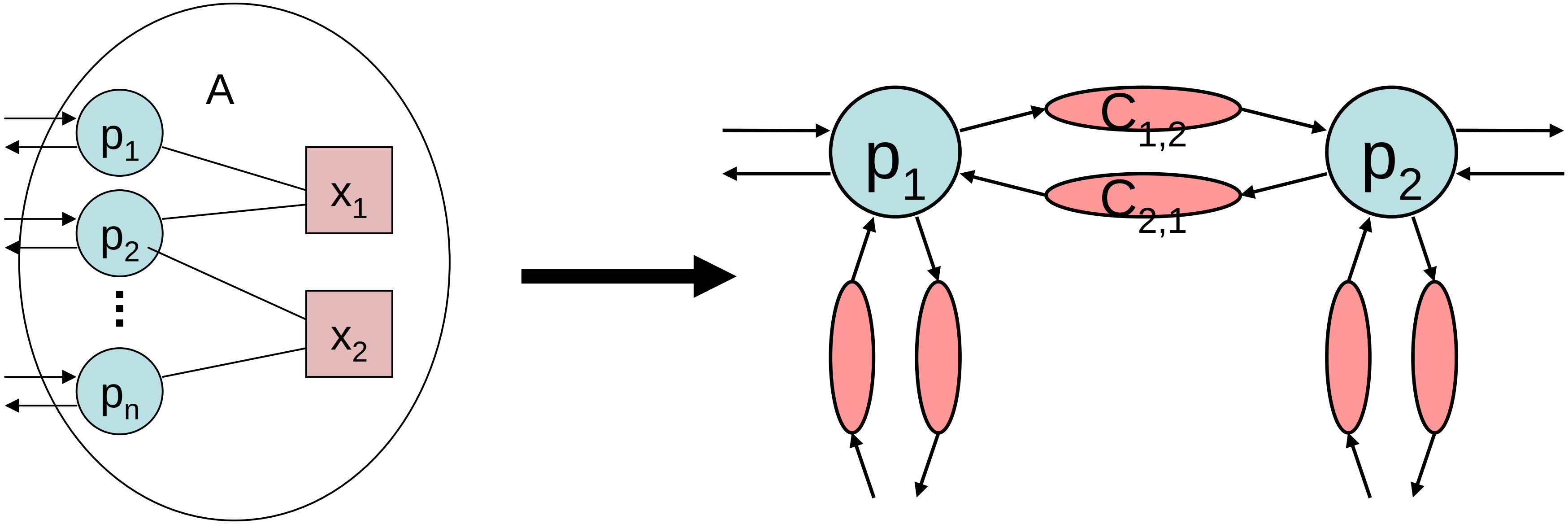
- Corollary: Consensus is impossible in asynchronous broadcast systems, with 1 stopping failure [Fischer, Lynch, Paterson].
- Asynchronous broadcast system: Process can put a message in all its outgoing channels in one step, and all are guaranteed to eventually be delivered.
  - Process cannot fail in the middle of a broadcast.
- Proof:
  - If such an algorithm existed, we could simulate it in an asynchronous shared-memory system using a simple extension of the simulation above.
  - Extension uses 1-writer multi-reader shared variables to represent the broadcast channels.
  - This would yield a 1-fault-tolerant consensus algorithm for 1-writer multi-reader read/write shared memory.
  - We already know this is impossible [Loui, Abu-Amara].
- Q: Is this counterintuitive?

Is this counterintuitive?

- Corollary: Consensus is impossible in asynchronous broadcast systems, with 1 stopping failure [Fischer, Lynch, Paterson].
- Asynchronous broadcast system: Process can put a message in all its outgoing channels in one step, and all are guaranteed to eventually be delivered.
  - Process cannot fail in the middle of a broadcast.

- Recall in synchronous model, impossibility results for consensus depended heavily on processes failing in the middle of a broadcast.
- Now every broadcast is completed, and guaranteed to be delivered everywhere.
- But we still get impossibility.

Simulating shared-memory systems using networks



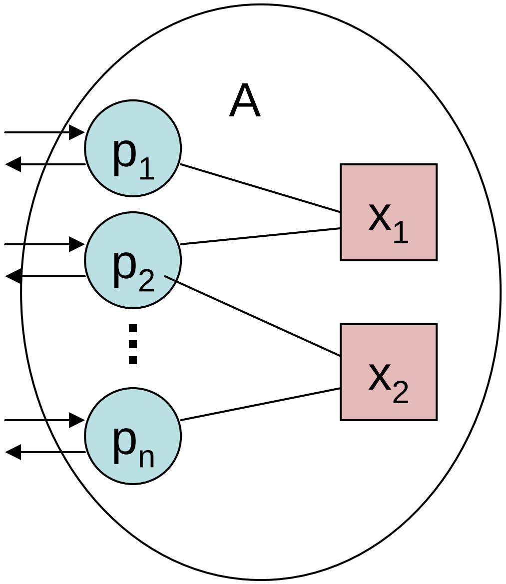
Simulating shared-memory in distributed networks

- Popular method for simplifying distributed programming.
- Non-fault-tolerant algorithms:
  - Single-copy
  - Multi-copy
  - Majority voting
- Fault-tolerant algorithms:
  - [Attiya, Bar-Noy, Dolev] algorithm for $n > 2f$.
  - Impossibility result for $n \leq 2f$.

Non-fault-tolerant simulation of shared memory in distributed networks

Shared memory in networks

- Assume shared memory system A:
  - Ports 1,…,n
  - User U_i interacts with process i on port i
  - Technical restriction: For each i, it's always either the user's turn, or process's turn to take steps (not both).
    - So we can replace shared variables with atomic object implementations without introducing new behavior.



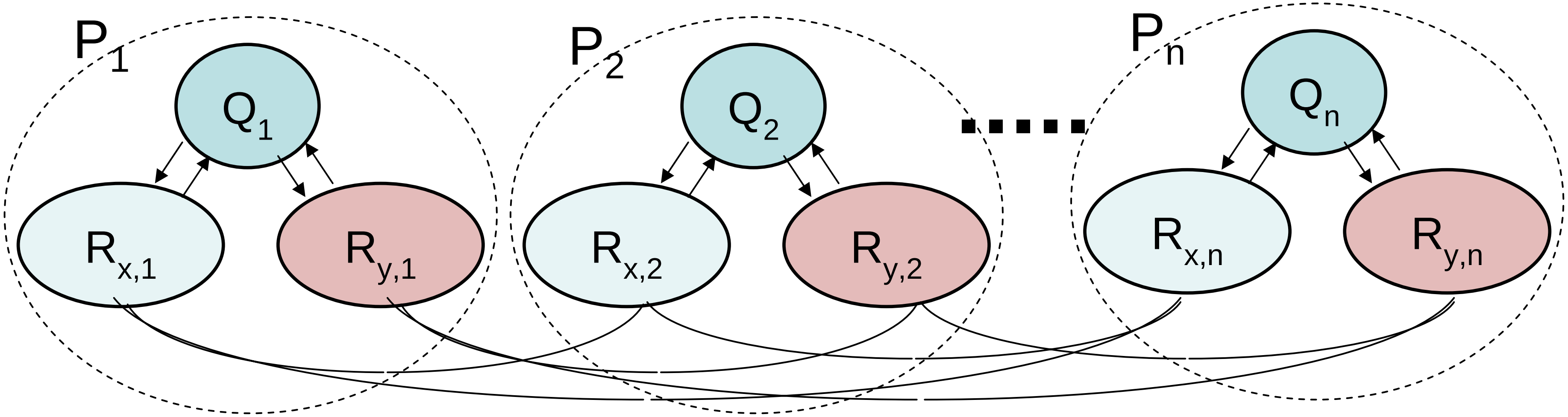
- Design asynchronous network system B:
  - Same ports/user interface.
  - Processes and FIFO reliable channels.
  - For any execution α of the network system $B \times U$, there is an execution α' of the shared memory system $A \times U$ such that:
    - $\alpha \mid U = \alpha' \mid U$ and
    - $stop_i$ events occur for the same i in α and α'.
    - If α is fair then α' is also fair (will change for FT case).

Single-copy simulation

- Non-fault-tolerant.
- Works for any object type.
- Locate each shared variable x at some known process, owner(x).
- Handle each shared variable independently.
- Automaton P_i simulates process i of A, step by step.
  - All actions other than shared-memory accesses as before.
  - To access variable x, P_i sends a message to owner(x) and waits for a response; when response arrives, uses it and resumes the simulation.
  - Meanwhile, P_i handles requests to perform accesses to all variables x for which i = owner(x).
    - Performs on local copy, in one indivisible step.
    - Sends response.

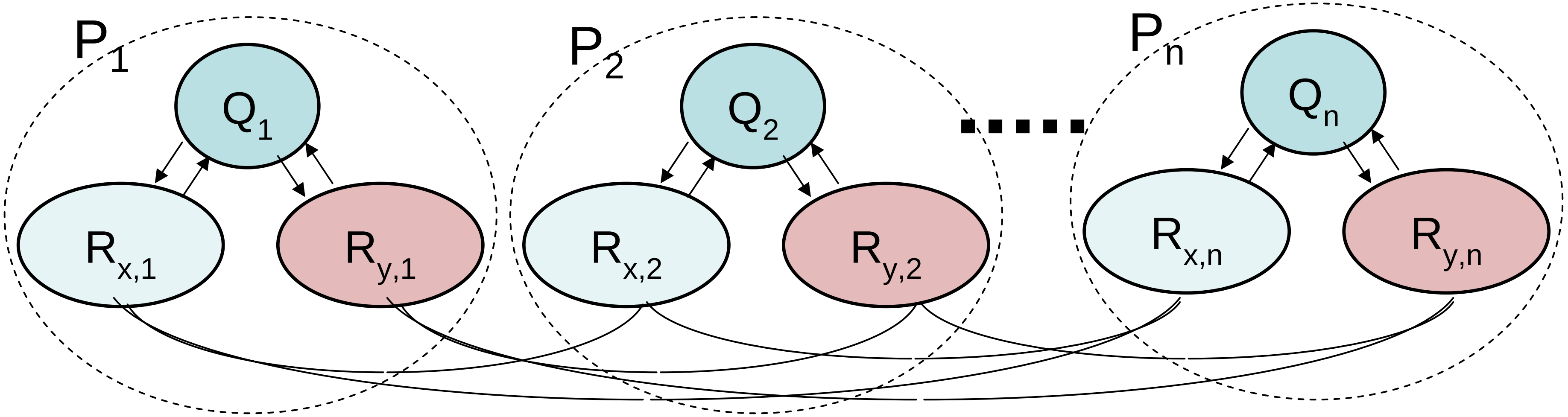
More formally…

- Each automaton P_i is the composition of:
  - Q_i, an automaton that simulates process i of the shared-memory system A,
    - Use same automata as when replacing shared variables by atomic objects.
  - $R_{x,i}$, for every shared variable x, an automaton that manages variable x and its requests.



More formally…

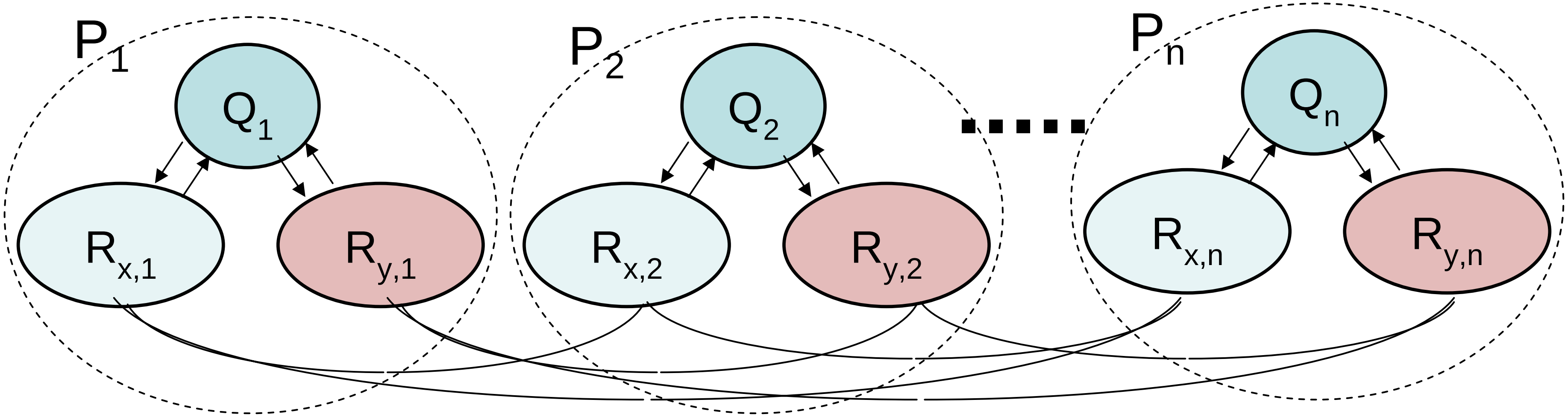
- Q_i and $R_{x,i}$ interact using invocations and responses on object x.
- For each x, the $R_{x,i}$ automata communicate over FIFO send/receive channels, and cooperate to implement an atomic object for x.
- Owner(x): Collects requests via local invocations and messages from others, processes on local copy.
- Non-owners: Send invocation to owner(x), await response.



More formally…

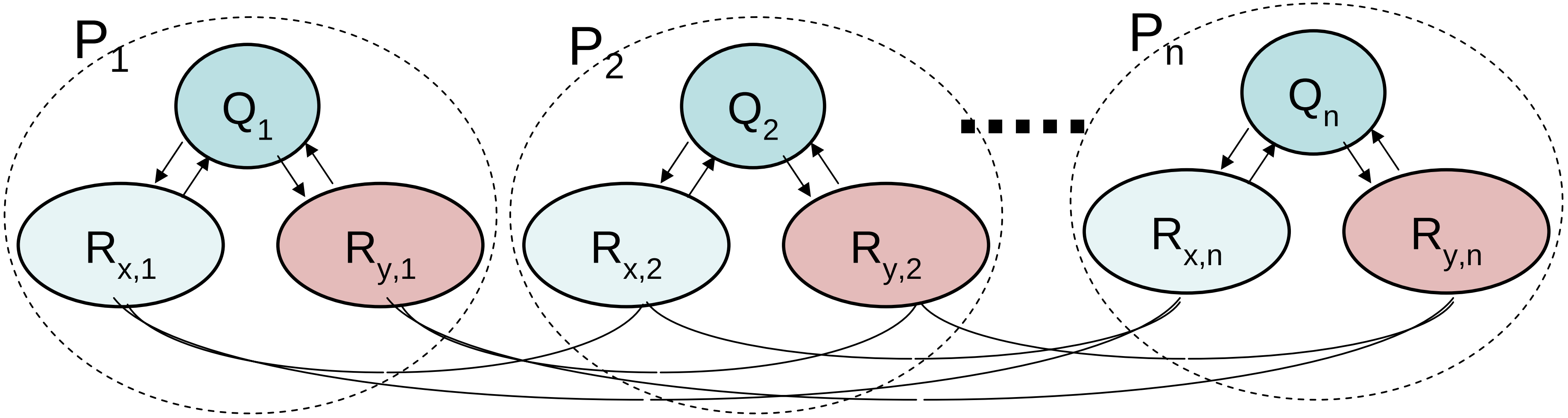
- Correctness: Pretty obvious, since clearly the $R_{x,i}$ automata (and the channels between them) implement an atomic object for x.
- Serialization point for each operation: When the owner performs the operation on the local copy.

- Fault-tolerance: None. Any process failure kills its variables, which can block everyone.



Some issues

- Optimization: Avoid busy-waiting on a remote shared variable: Send one request, let owner notify sender when the value of the variable changes, or when some condition on this value becomes true.
- Q: Where to put the copies?



Multi-copy simulation

- Still not fault-tolerant.
- Just for read/write objects.
- Locate each shared variable x at some known collection of processes, owners(x).
- Handle each shared variable independently.
- How P_i accesses variable x:
  - READ: Read any copy.
  - WRITE: Write all copies, asynchronously, in any order.
  - “Read-one, write-all.”
- Can be faster than single-copy, on average, if reading is much more common than writing.
  - E.g., in peer-to-peer systems, sharing files.
- But, without some constraints, we get coherence issues…

Multi-copy simulation: Bad examples

- Example 1: Multi-writer, inconsistent order of WRITEs
  - P_1 and P_2 want to WRITE the same shared variable x.
  - owners(x) = $\{P_3, P_4\}$.
  - P_1 and P_2 send write request messages to both P_3 and P_4.
  - P_3 and P_4 receive the write requests in different orders, so end up with different values.
  - Later READs may get either value, inconsistent.
- Example 2: Single-writer, inconsistent READs
  - owners(x) = $\{P_2, P_3\}$.
  - Writer P_1 sends write request messages to P_2 and P_3.
  - Message arrives at P_2, P_2 writes its local copy.
  - Then a READ happens at P_2, getting the new value.
  - Later, a READ happens at P_3, getting the old value.
  - Then P_1's write message arrives at P_3, P_3 writes its local copy.
  - The READs do not overlap, but are concurrent with the WRITE.
  - Out-of order READ behavior is not allowed by atomic R/W object.

Multi-copy simulation

- So we need some more clever protocols…
- Idea: Use atomic transactions:
- E.g., to do a WRITE(x), perform all the writes to all copies as a single atomic transaction, so that they appear to occur instantaneously, as far as READ operations can tell.
- Can implement such a transaction using 2-phase locking:
  - Phase 1: Lock all copies of x and write them.
  - Phase 2: Release all the locks.
- Must solve problems of deadlock for lock acquisition.
- Works because serialization point for WRITE can be placed at the “lock point”, where all the locks have been acquired.

Majority-voting algorithms

- Still not fault-tolerant.
- Just for read/write objects.
- Locate each shared variable x at some known collection of processes, owners(x).
- Handle each shared variable independently.
- How P_i accesses variable x:
  - READ: Read from a majority of copies.
  - WRITE: Write to a majority of copies.
- Concurrency anomalies suggest that we run each READ or WRITE as an atomic transaction, using an underlying concurrency-control strategy like 2-phase locking.
- More precisely:…

Majority-voting algorithms

- Each copy of x includes an integer tag, initially 0, as well as a value for x.
- How P_i accesses variable x:
  - Performs an atomic transaction, implemented by 2-phase locking.
  - READ:
    - Read from a majority of copies.
    - Return the value associated with the largest tag.
  - WRITE(v):
    - First do an embedded-read of a majority of copies.
    - Determine the largest tag t.
    - Write (v,t+1) to a majority of copies.
  - Each READ or WRITE appears to be instantaneous, because they are implemented as transactions.

Majority-voting algorithms

- To see that this implements an atomic R/W object for x:
  - Choose serialization points for the READ and WRITE operations to be the serialization points for their transactions.
  - These are guaranteed by the transaction implementation, e.g., lock points for 2-phase locking.
- Show that the R/W operations behave as if they occurred at their transactions' serialization points:
  - WRITE operations are assigned tags 1,2,…in order of their transactions' serialization points.
  - READ or embedded-read obtains the largest tag that has been written by a WRITE operation serialized before it (0 if there are none), together with the associated value for x.
  - These two facts depend, in turn, on the fact that each READ or embedded-read reads a majority of the copies, the largest tag gets written to a majority of the copies, and all majorities intersect.

Some issues

- Still no fault-tolerance:
  - Standard transaction impls like 2-phase locking aren't fault-tolerant.
  - A process that fails while holding locks "kills" the locked objects.

- Can generalize majorities to quorum configurations.
- Quorum configuration:
  - A set of read-quorums, finite subsets of process indices,
  - A set of write-quorums, finite subsets of process indices, such that
  - $R \cap W \neq \varnothing$ for every read-quorum R and write-quorum W.
- READ operation accesses any read-quorum.
- WRITE operation accesses both a read-quorum and a write-quorum (in its two phases).
- Allows tuning for smaller read-quorums, which can speed up READs.
  - E.g., read-one, write-all is a special case.

Fault-tolerant simulation of shared memory in distributed networks

Fault-tolerant simulation of shared memory in distributed networks

- [Attiya, Bar-Noy, Dolev] algorithm.
- Tolerates f stopping failures, requires n > 2f.
- Assume reliable channels.
- Just for read/write objects, in fact, 1-writer multi-reader objects (exercise: extend to MWMR).
- Modeling failures:
  - Use a $stop_i$ input at each external port (of the shared-memory system A, or of the network system B).
  - $stop_i$ disables all locally-controlled actions of process i, in either system.
  - Does not affect messages in transit (in system B).

- Q: What is guaranteed by the [ABD] simulation?

[ABD] Guarantees

- Tolerates f stopping failures, requires n > 2f.
- For any execution α of network system B × U, there is an execution α' of shared-memory system A × U such that:
  - $\alpha \mid U = \alpha' \mid U$ and
  - stop_i events occur for the same i in α and α'.
- Moreover, if α is fair and contains stop_i events for at most f different ports, then α' is also fair.
- This means that in the simulated shared-memory execution, all non-failed processes continue taking steps---the failed processes in the network system don't introduce any new blocking.

- Assume shared-memory system A has only 1-writer multi-reader read/write shared variables.

[ABD] algorithm

- Tolerates f stopping failures, requires n > 2f.
- Implement atomic object for each shared variable x, then combine.
- No transactions, no synchronization.
- Each process keeps:
  - val, a value for x, initially v_0
  - tag, initially 0
- P_1 does WRITE(v):
  - Let t be the first unused tag (P_i knows this because it's the only writer, hence the only process generating tags).
  - Set local variables to (v,t).
  - Send message ("write", v,t) to all other processes.
  - When anyone receives such a message:
    - Updates local variables to (v,t) if t > current tag.
    - In any case, sends ack to P_1.
  - When P_1 knows a majority have received (v,t), returns ack.

[ABD] atomic object algorithm

- Any process P_i does a READ:
  - Read own copy; send ("read") messages to all other processes.
  - When anyone receives this message, responds with its current (v,t).
  - When P_i has heard from a majority, prepares to return the v from the (v,t) pair with the largest t.
  - However, before returning v, P_i propagates this (v,t).
    - As in the [Vitanyi, Awerbuch] algorithm.
    - And for a similar reason (prevent out-of-order reads).
  - When anyone receives this propagated (v,t):
    - Updates local variables to (v,t) if t > current tag.
    - Sends ack to P_i.
  - When P_i knows a majority have received (v,t), returns ack.

ABD algorithm

STATE VARIABLES per process

```
val: V, initially $v_0$
tag: **N**, initially 0
readtag: **N**, initially 0
lots of “bookkeeping” variables
```

WRITER

```
on write(v)
  (val,tag) := (v,tag+1)
  send “write(val,tag)” to all readers
    - wait for ack from majority
  return ack
```

READERS

```
on receiving “write(v,t)” from writer
 if t > tag then
    (val,tag) := (v,t)
  send “write-ack(t)” to writer
```

READERS

```
on read
  readtag := readtag+1
  send “read(readtag)” to all other processes
    - wait for ack from majority
  let t be largest tag received
  if t > tag then (val,tag) := (v,t)
    where v is value received with t
  send “propagate(val,tag,readtag)” to all readers
    - wait for ack from majority
  return val
```

ALL PROCESSES

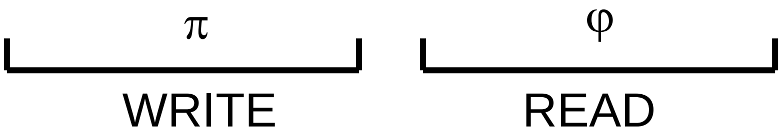
```
on receiving “read(rt)” from $_j$
  send “read-ack(val,tag,rt)” to j
```

READERS

```
on receiving “propagate(v,t)” from j
  if t > tag then (val,tag) := (v,t)
  send “prop-ack(t)” to j
```

Correctness of [ABD] atomic object algorithm

- Well-formedness √
- f-failure termination, for $n > 2f$ √
- Atomicity:
  - Algorithm is similar to [Vitanyi, Awerbuch], so use similar proof, based on partial order lemma.
  - Here, define the partial order by:
    - Order WRITEs by tags.
    - Order READ right after WRITE whose value it gets.
  - Key: Condition 2: If operation π finishes before operation φ starts, then φ is not ordered before π.
  - Consider cases, based on operation types.
  - Case 1:



  - Because majorities intersect, φ gets a tag $\geq$ the tag written by π.
  - So φ is ordered after π.

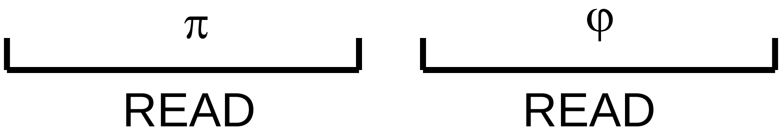
Correctness of [ABD] atomic object algorithm

- linearization point of write with tag t
  - when majority of processes have tag ≥ t
  - may linearize multiple writes at same point
- linearization point of read returning value associated with tag t
  - immediately after linearization point of write with tag t, or
  - immediately after invocation of read, (why do we need this?)
  - whichever is later

Atomicity, cont’d

- Partial order:
  - Order WRITEs by tags.
  - Order READ right after WRITE whose value it gets.
- Condition 2: If operation π finishes before operation φ starts, then φ is not ordered before π.

- Case 2:



- Then φ gets a tag $\geq$ the tag obtained by π, because of propagation and majority intersection.
- So φ is not ordered before π.

- Other cases: Simpler, LTTR.

[ABD] Simulation

- Now use [ABD] atomic object algorithm to construct a distributed simulation of any fault-tolerant shared-memory algorithm A that uses 1-writer multi-reader shared vars:
- Simply replace shared variables by [ABD] atomic object implementations.

- Guarantees:
  - For any execution α of network system B × U, there is an execution α' of shared-memory system A × U such that:
    - $\alpha \mid U = \alpha' \mid U$ and
    - stop_i events occur for the same i in α and α'.
  - Moreover, if α is fair and contains stop_i events for at most f (< n/2) different ports, then α' is also fair.
- That is, we have a correct simulation, provided that there are at most f failures in the network system B.

[ABD] Simulation Corollaries

- Guarantees:
  - For any execution α of network system $B \times U$, there is an execution α' of shared-memory system $A \times U$ such that:
    - $\alpha \mid U = \alpha' \mid U$ and
    - $stop_I$ events occur for the same i in α and α'.
    - If α is fair and contains $stop_i$ events for at most f different ports, then α' is also fair.
- Corollary: Wait-free atomic snapshot algorithm using 1WmR registers (Chapter 13) can be transformed, using [ABD], to a distributed network memory-snapshot algorithm.
- Corollary: [Vitanyi, Awerbuch] wait-free mWmR register implementation using 1WmR registers can be transformed, using [ABD], to a distributed network register implementation.
- But note:
  - The transformed versions are not wait-free, but guarantee only f-failure termination, where $n > 2f$.
  - Since the [ABD] implementation of atomic 1WmR registers tolerates only $f < n/2$ failures, so do the algorithms that use it.

Some issues

- Can generalize majorities to quorum configuration:
  - Set of read-quorums, set of write-quorums.
  - $R \cap W \neq \emptyset$ for every read-quorum R, write-quorum W.
- Then
  - READ operation accesses both a read-quorum and a write-quorum.
  - WRITE operation accesses just a write-quorum.
- So, we cannot improve READ performance by using smaller read-quorums!
- Q: So how can we get faster READ performance?
- A: Optimize to eliminate "most" propagation phases.
  - When a WRITE with tag t completes, or a READ completes propagation of tag t, then tag t doesn't require further propagation.
  - So, an operation that completes t can send messages to everyone saying that t is complete; everyone who receives such a message marks t as complete.
  - A READ that gets tag t and sees it marked (anywhere) as complete doesn't need to propagate t.

Impossibility of n/2-fault-tolerance

- General "fact" about the distributed network model: hardly anything interesting can be computed with ≥ n/2 failures.
- Contrast with shared-memory model: There are many interesting wait-free shared-memory algorithms.
- Theorem: In the asynchronous network model with n = m+p processes, no implementation of m-writer p-reader atomic registers guarantees f-failure termination for f ≥ n/2.
- Proof: (Same structure as for other proofs showing impossibility of n/2-fault-tolerance.)
  - By contradiction. Suppose f ≥ n/2 and we have an algorithm…
  - Assume WLOG that:
    - Initial value of implemented register = 0.
    - P_1 is a writer and P_n is a reader.
  - Partition the n processes into two subsets, each with size ≤ f:
    - $G_1 = \{1,\ldots,f\}$, $G_2 = \{f+1,\ldots,n\}$.
  - By f-fault-tolerance, even if one entire group fails, the other group must still give correct atomic register responses.

Impossibility of n/2-fault-tolerance

- Theorem: In the asynchronous network model with n = m+p processes, no implementation of m-writer p-reader atomic registers guarantees f-failure termination for $f \geq n/2$.
- Proof, cont'd:
  - Partition the processes into $G_1 = \{1,\ldots,f\}$, $G_2 = \{f+1,\ldots,n\}$.
  - If one group fails, the other group must still give correct atomic register responses.
  - Execution α_1:
    - G_2 processes fail initially.
    - P_1 invokes WRITE(1).
    - WRITE must eventally terminate with ack.
    - Let α_1' be the portion of α_1 up to the ack.
  - Execution α_2:
    - G_1 processes fail initially.
    - P_n invokes READ.
    - READ must eventally terminate with response 0.
    - Let α_2' be the portion of α_2 up to the response.

Proof, cont'd

- Execution α_1:
  - G_2 processes fail initially.
  - P_1 invokes WRITE(1).
  - WRITE must eventally terminate with ack.
  - Let α_1' be the portion of α_1 up to the ack.
- Execution α_2:
  - G_1 processes fail initially.
  - P_n invokes READ.
  - READ must eventally terminate with response 0.
  - Let α_2' be the portion of α_2 up to the response.
- Execution α_3: Paste…
  - Don't fail anyone.
  - Do all the steps of α_1' first, including the ack.
  - Then do all the steps of α_2', including the response of 0.
  - Meanwhile, delay all messages between G_1 and G_2.
- Activity in α_1' and α_2' is independent, so α_3 is an execution.
- But not correct for an atomic register, since the WRITE(1) completes before the start of the READ that returns 0.
- Contradiction.

An implication

- This theorem implies that there is no general simulation of shared-memory systems by networks, preserving f-fault-tolerance, for f $\geq$ n/2.
  - See book, p. 567, for a definition of f-simulation, which formalizes "preserving f-fault-tolerance".
  - It's essentially the overall guarantee we gave earlier for [ABD].
- Because if there were, then we could use it to convert a (trivial) wait-free shared-memory implementation of a multi-writer, multi-reader atomic register into an f-fault-tolerant distributed network implementation, f $\geq$ n/2.
- Since the example shows that no such algorithm exists, neither does such a general simulation.

Fault-Tolerant Agreement in Asynchronous Networks: The Paxos Algorithm

Agreement in asynchronous networks

- It’s impossible to reach agreement in asynchronous networks, even if we know that at most one failure will occur.
- But what if we really need to?
  - For transaction commit.
  - For agreeing on the order in which to perform operations.
  - …

- Some possibilities:
  - Randomized algorithm (Ben-Or), terminates with high probability.
  - Approximate agreement.
  - Use a failure detector service, implemented by timeouts.

Best approach

- Guarantee agreement, validity in all cases.
- Guarantee termination if the system eventually “stabilizes”:
  - No more failures, recoveries, message losses.
  - Timing of messages, process steps within “normal” bounds.
- Termination should be fast when system is stable.
- Actually, stable behavior need not continue forever, just long enough for computation to terminate.

Eventually stable approach: Some history

- [Dwork, Lynch, Stockmeyer] first presented a consensus algorithm with these properties (2007 Dijkstra Prize)
- [Cristian] used similar approach for group membership algorithms.
- [Lamport, Part-Time Parliament]
  - Introduced the Paxos algorithm.
  - Relationship with [DLS]:
    - Achieves similar guarantees.
    - Paxos allows more concurrency, tolerates more kinds of failures.
    - Basic strategy for assuring safety similar to [DLS].
  - Background:
    - Paper unpublished for 10 years because of nonstandard style.
    - Eventually published "as is", because others began recognizing its importance and building on its ideas.

Paxos consensus protocol

- Called Single-Decree Synod protocol.
- Assumptions:
  - Asynchronous processes, stopping failures, also recovery.
  - Messages may be lost.
- Lamport’s paper also describes how to cope with crashes, where volatile memory is lost in a crash (we’ll skip this).
- We’ll present the algorithm in two stages:
  - Describe a very nondeterministic algorithm that guarantees the safety properties (agreement, validity).
  - Constrain this to get termination soon after stabilization.

The nondeterministic “safe” algorithm: Ballots

- Uses ballots, each of which represents an attempt to reach consensus.
- Ballot = (identifier, value) pair.
  - Identifier is an element of Bid, some totally-ordered set of ballot identifiers.
  - Value in $V \cup \{ \perp \}$, where V is the consensus domain.
- Somehow, ballots get started, and get values assigned to them.
- Processes can vote for, or abstain from, particular ballots.
  - Abstention from a ballot is a promise never to vote for it.

The safe algorithm: Quorums

- The fate of a ballot depends on the actions of quorums of processes on that ballot.
- Quorum configuration:
  - A set of read-quorums, finite subsets of process index set I, and
  - A set of write-quorums, finite subsets of I, such that
  - $R \cap W \neq \emptyset$ for every read-quorum R and write-quorum W.
- Generalization of majorities.
- Ballot becomes dead if every node in some read-quorum abstains from it.
- A ballot can succeed only if every node in some write-quorum votes for it.

Safe algorithm, centralized version

- Anyone can create a new ballot with Bid b:
  - make-ballot(b)
  - Provided no ballot with Bid b has yet been created.
  - val(b) is set to $\perp$.
- A process i can abstain, in one step, from an entire set of ballots:
  - abstain(B,i), $B \subseteq$ Bid
  - Provided i has not previously voted for any ballot in B.
  - We allow B to be any set of Bids, not necessarily associated with already-created ballots.
    - For example, B = all Bids in some range $[b_{min}, b_{max}]$.
    - This is important…

Safe algorithm, centralized version

- Anyone can assign a value v to a ballot id b, assign-val(b,v), provided:
  - A ballot with id = b has been created.
  - val(b) is undefined.
  - v is someone’s consensus input.
  - (**) For every b′ ∈ Bid, b′ < b, either val(b′) = v or b′ is dead.
- Notes on (**):
  - Recall: b′ dead means some read-quorum has abstained from b′.
  - (**) Refers to every b′ ∈ Bid, not just created ones.
    - Relies on “set abstentions”.
- Thus, we can assign a value to a ballot b only if we know it won’t make b conflict with lower-numbered ballots b′.
- Motivation:
  - Several ballots can be created, can collect votes.
  - More than one might succeed in collecting write-quorum of votes.
  - But we don’t want successful ballots to conflict.

Safe algorithm, centralized version

- A process i can vote for a ballot b, vote(b,i), if b is a created ballot from which i hasn't abstained.
- A ballot may succeed, succeed(b), if a write-quorum W has voted for it.
- A process can decide on the value that is associated with any successful ballot, decide(v).

Safety properties

- Validity:
  - Immediate. Only initial values ever get assigned to ballots.
- Agreement:
  - Because of the careful way we avoid assigning different values to ballots that might succeed.
  - Key Invariant: If val(b) $\neq \perp$, b′ $\in$ Bid, and b′ < b, then either val(b′) = val(b) or b′ is dead.
  - Implies that all successful ballots have the same value.

Modifying the ** condition for assigning ballot values

- Instead of checking:

  (**) For every $b' \in$ Bid, $b' < b$, either $val(b') = v$ or b' is dead.
- Check the apparently-weaker condition:

  (***) Either:

  Every $b' \in$ Bid, $b' < b$, is dead, or

  there exists $b' < b$ with $val(b') = v$, and such that every b'' with $b' < b'' < b$ is dead.

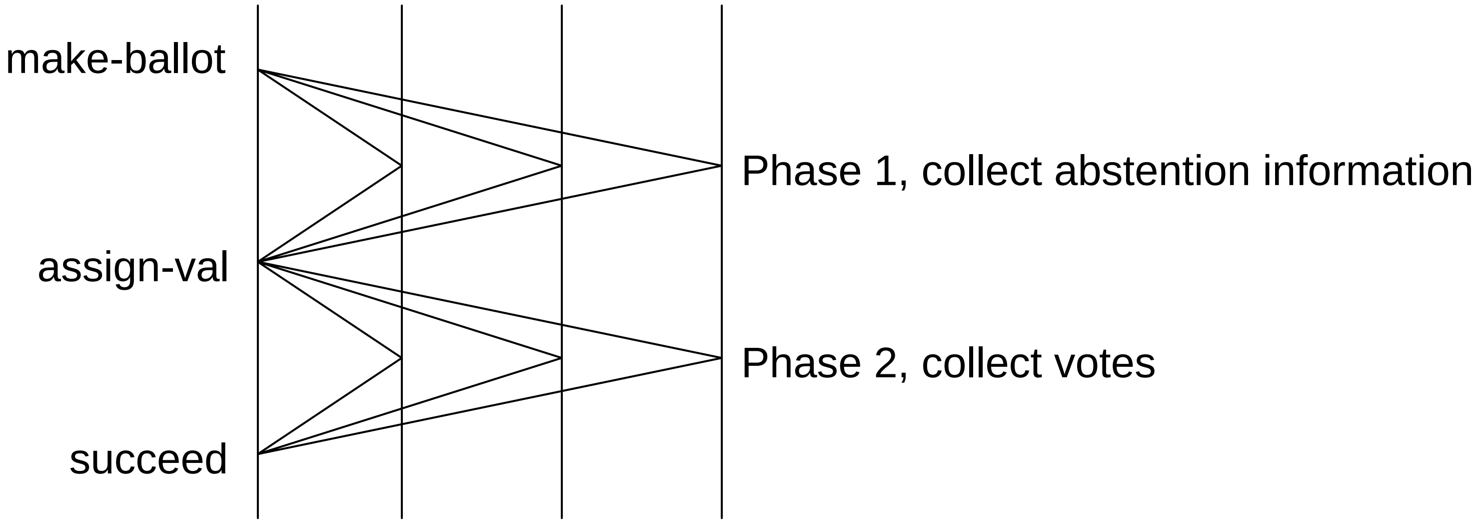
- (***) is easier to check in a distributed algorithm (will show how).
- And (***) implies (**), by easy induction on the number of steps in an execution.

Safe algorithm, distributed version

- Any process i can create a ballot, at any time.
  - Use locally-reserved ballot id b.
  - Ballot start is triggered by signal from a separate BallotTrigger service that decides who should start ballots and when, based on monitoring system behavior.
  - Precise choices don’t affect the safety properties, so for now, leave them nondeterministic.
- Phase 1:
  - Process i starts a ballot when told to do so by BallotTrigger, but doesn’t assign a value to it yet.
  - Rather, first tries to collect enough abstention information for smaller ballots to guarantee (***).
  - If/when it collects that, assigns val(b).

Safe algorithm, distributed version

- Phase 2:
  - Tries to get enough other processes to vote for its new ballot.
- Communication pattern:



Ensuring (***)

(***) Either every b′ < b is dead, or there exists b′ < b with val(b′) = v, such that every b′′ with b′ < b′′ < b is dead.

- Phase 1:
  - Originator process i tells other processes the new ballot number b.
  - Each recipient j abstains from all smaller-numbered ballots it hasn't yet voted for.
  - Each j sends back to i:
    - The largest ballot number < b that it has ever voted for, if any, together with that ballot's value.
    - Else (if no such ballot), sends a message saying there is none.
  - When process i collects this information from a read-quorum R, it assigns a value v to ballot b:
    - If anyone in R says it voted for a ballot < b, then v = the value associated with the largest-numbered of these ballots.
    - If not, then v = any initial value.
- Claim this choice satisfies (***):

Ensuring (***)

- (***) Either every b′ < b is dead, or there exists b′ < b with val(b′) = v, such that every b′′ with b′ < b′′ < b is dead.

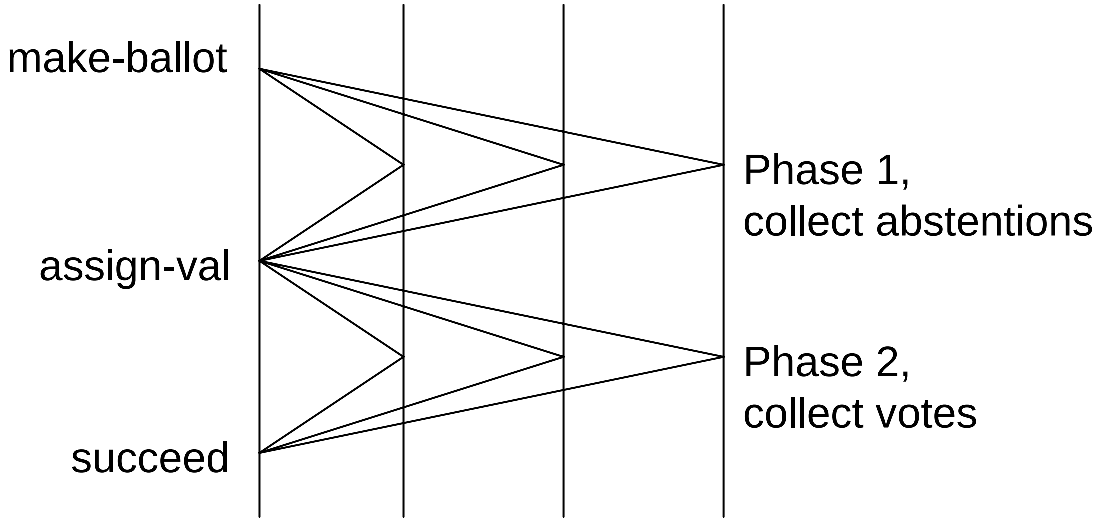
- Why does this choice satisfy (***)?

- Case 1: Someone in R says it voted for a ballot < b.
  - Say b′ is the largest such ballot number.
  - Then everyone in R has abstained from all ballots between b′ and b.
  - So all ballots between b′ and b are dead.
  - So, choosing v = val(b′) ensures the second clause of (***).

- Case 2: Everyone in R says it did not vote for a ballot < b.
  - Then everyone in R has abstained from all ballots < b.
  - So all ballots < b are dead.
  - Satisfies the first clause of (***).

Safe algorithm, distributed version, cont'd

- After assigning val(b) = v, originator i sends Phase 2 messages asking processes to vote for b.
- If i collects such votes from a write-quorum W, it can successfully complete ballot b and decide v.
- Note:
  - Originator i, or others, could start up new ballots at any time.
  - (***) guarantees that all successful ballots will have the same value v.
  - Arbitrary concurrent attempts to conduct ballots are OK, at least with respect to safety.



Liveness

- To guarantee termination when the algorithm stabilizes, we must restrict its nondeterminism.
- Most importantly, must restrict BallotTrigger so that, after stabilization:
  - It asks only one process to start ballots (leader).
  - It doesn't tell the leader to start new ballots too often---allows enough time for ballots to complete.
- E.g., BallotTrigger might:
  - Use knowledge of "normal case" time bounds to try to detect who has failed.
  - Choose smallest-index non-failed process as leader (refresh periodically).
  - Tell the leader to try a new ballot every so often---allowing enough "normal case" message delays to finish the protocol.
- Notice that BallotTrigger uses time information---not purely asynchronous.
- We know we can't solve the problem otherwise.
- Algorithm tolerates inaccuracies in BallotTrigger: If it "guesses wrong" about failures or delays, termination may be delayed, but safety properties are still guaranteed.

Replicated state machines (RSMs)

- Paper also deals with repeated consensus, in particular, on a sequence of operations for an RSM.
- Yields an RSM that tolerates stopping failures/recoveries, message loss/duplication.
- Strategy:
  - Use infinitely many instances of Paxos to agree on first operation, second, third,…
  - Similar to Herlihy's universal construction, which uses repeated consensus to decide on successive operations for an atomic object.
- Lamport's paper also includes various optimizations, LTTR.
- Considerable follow-on work, engineering Paxos to work for maintaining real data.
  - Disk Paxos
  - HP, Microsoft, Google,…

Next time

- Self-stabilization
- [Dolev book], Chapter 2

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