PROFESSOR: All right. So today we resume efficient origami design. And we had our guest lecture from Jason Ku which was definitely a different style of lecture. More survey, lots of different artwork. And it had some practical hands-on experience with TreeMaker, which you're welcome to do more of on your problem set.

And so there weren't a lot of questions because this is not a very technical lecture, so I thought I'd show you some more examples of artistic origami, things not covered by Jason, and some other different types of origami. So we start with a bunch of models by Jason because he didn't show his own models, so I thought it'd be fun. We've seen a bunch already in this class, but this is a really nice F16 that he designed.

And these are all done with Tree method of origami design. Another lobster. We saw a Robert Lang's lobster before. This one's different. This is a version of the crab that he showed. So the one you saw was like the very preliminary, very rough folding. But with some refinement, especially in the shaping stage, it looks pretty nice. Even on the back side you get some nice features.

We have a little rabbit. This is kind of in the traditional style that he showed where you've got sharp crease lines that really define the form. I assume that's what he was going for here.

This is a non-tree method design. This is using what's called box pleating. We've heard about box pleating. And it means you have horizontal, vertical, and 45 degree diagonal folds. But you can use it just to shape box-like shapes. It originally was used by Moser to make a train out of one rectangle of paper.

But here we've got a pretty nice sports car convertible, even with a color reversal. So it's pretty cool. This is one of my favorite designs of Jason's. Bicycle, one square paper, color reversal. Really thin features. Probably lots of layers up there, but pretty awesome.

This is using tree method. Obviously, the paper is not connected with a hole there, so there's some part here that's attached just by folding to another part. Yeah, questions?

AUDIENCE: How big is that?

PROFESSOR: I'm trying to remember. I think the bicycle's about that big. Anyone remember? It's been a while. So presuming he started from a piece of paper maybe twice the size or so. Looks big here.

And this is a really complicated butterfly, very exact features, very cool. These are all from his website if you want to check out that. I'm just giving a selection. Some of them have crease patterns and you can very clearly see the different parts of the model, and the rivers, and so on. Others do not.

This is one of-- we're going back in time, so this is when Jason was just starting at MIT as an undergrad, I believe. This is the dog of someone who works at the admissions office. It's very cool. And this is one of his earliest models, 2004. I think it's pretty elegant on the ice skate with color reversal.

So that was Jason, for fun. One question we had is what about origami from other materials, not just paper? And we've seen a few examples of that, but I thought it'd be a fun theme. And we'll come back to this a couple times today.

This is-- I don't if you call dollar bills paper-- but there is this whole style of dollar bill origami, as my t-shirt last class indicated. And this is one of the more famous dollar bill folders. And he has hundreds and hundreds of designs. One of his latest is the alien face hugger for Prometheus and so on. So there's a ton of stuff done.

There's the particular proportion of the rectangle of a dollar bill. And it's also just plentily available. The US is one of the cheapest currencies to do bill folding because it has one of the lowest value bills. So there's that.

These are all folded from toilet paper rolls, so moving up to cardboard. This definitely is pretty different in the way it acts relative to standard paper. And there's this guy who makes these incredible masks. Very impressive. And I'm guessing crayon or some kind of rubbed color. So that's pretty awesome.

Here's something called Hydro-Fold. This just came out this year by this guy Christophe Guberan, where he's got an inkjet printer. He's filled it with a particular kind of ink that he custom makes. And as it comes out of the printer, it folds itself. It's been printed on both sides. So one side you get mountains, the other side you get also mountains, but relative to that, it's valleys.

So there's some fun thing happening as the liquid dries out that causes the paper to curve. You can't get 180 degree folds, but you can get some pretty nice creases. I don't know exactly how much accelerated that is, but he's hopefully visiting MIT later on and we'll find out more.

So it's using regular paper, but a different folding style, a different material for folding. You can also take casts of existing paper models. So Robert Lang has done a bunch of these with a guy named Kevin Box where they take a paper model and cast or partially cast it. In this case, in bronze. In these cases, stainless steel.

So these are two. This is like traditional origami crane and Robert Lang complex crane. And for fun, the crease pattern for those two looks like this. And this is I think mostly on a 22.5 degree grid system. May actually be-- you can see here there's a river that's not orthogonal. So it's not intended to be box pleated.

So that gives you these 22.5 degrees. There's some other features out here, but the most of it is this 22.5 degree system. So as you might guess from now, there's some questions about this. You don't necessarily entirely use the tree method. You use a mix of different things.

In particular, there's a technique called grafting where you can combine two models. If you're interested in that, check out Origami Design Secrets. And for things like the dragon where you have this textured pattern-- which we'll get to, it's called a tessellation-- and you want to combine that with doing tree method stuff, you can do that. But it's not necessarily mathematical formal how to do that. It's just people figure it out by trial and error. There's probably interesting open problems there, haven't been formalized.

Here's another cardboard design. This is by our friend Tomohiro Tachi. That's him. So this was initially a bed. And you fold it up. And you need a pillow, of course. It turns into a chair. So that's pretty awesome. So that's one of the great things about using non-paper is you get a lot more structural integrity and support.

And that leads us into steel, which also makes for stronger models. And this is another design by Tomohiro. We made it here at MIT using a waterjet cutter in CSAIL. And it makes a pretty nice table. This is based on a curve crease design which initially drafted on paper, and then in plastic.

And then when it seemed to be working pretty well, we waterjet cut this steel and these perforation lines. And then many hours of painful bending or difficult bending later, some hamming and so on, we got it to fold into a pretty nice shape. So that's one example.

I have another example. This is out of much thinner steel. And this happens to be laser cut using a newer laser cutter in the Center for Bits and Atoms in the Media Lab building. So a little bit of a cheat. This is not from a square paper. It's been cut a little bit smaller. I need chalk. Jason.

So take a square of paper. You can cut out-- these are 22.5 degree angles. You can cut out material like this from your square and still make a good crane. But it substantially reduces the number of layers you get, especially at the corners. And so we exploited that because this is pretty thick material.

And this is just the Center for Bits and Atoms logo. But pretty cool. You can make a crane. And we added these crease lines to get the nice bow of the crane. So pretty nice. This is made by Kenny Cheung who just graduated, PhD. So that was some metal.

Next topic is tessellations. So this is a particular style of origami. It goes back-- probably the earliest tessellation folder is Ron Resch. The early history's a little hard to know for sure.

Ron Resch was an artist starting in the '60s. He died just a few years ago. We've met him. Pretty crazy guy. Did a lot of cool origami foldings early in the day. There's a patent that describes this particular folding.

And what makes a tessellation is essentially a repeated pattern of some sort. It could be periodic. It could be aperiodic. You've probably heard of tessellations like the square grid or some kind of mesh of two dimensions.

Origami tessellations are in some sense trying to represent such a tessellation. Here you've got the triangular grid, if you look closely, after folding. But also if you look at the crease pattern itself, it is a tessellation. It's going to be a repeated pattern of polygons.

So you've got sort of two levels of tessellation going on. It's like a double rainbow or something. And so there are lots of examples of this. Here's some kind of traditional flat origami tessellations. Some of these are more traditional than others.

You've some very simple-- well not simple, but beautiful repeating patterns. Octagons and squares here. You can count them. And this is still periodic. Then we get to some less periodic stuff.

And so there are techniques for designing these kinds of tessellation. If you start with a regular 2D tessellation, there's a transformation from that tessellation into a crease pattern, which then makes things like this. You can see here there's sort of clear edges here. And that represents the tessellation it's based on. It's just been kind of shrunk a little bit.

Each of these is a pleat. There's a mountain and a valley crease. And so on all of these, I believe, that style. You've got essentially a twist fold at each of the vertices. And you've got a pleat along each of the edges.

And if you want to play with these, there's software called Tess freely available online. And I'll show it to you. And it lets you design things like this, following a particular algorithm.

So you start with some geometry. And I don't really know these by heart. So it has a fixed set of geometries that you can play with. We'll try this one.

And you get a regular 2D tessellation of polygons. And then you increase the-- then I hit, Show Creases. And it's applying a particular algorithm which is essentially-- it's maybe more dramatic if I increase this value or change it dynamically. It's rotating each of the polygons, so a twisting. Sorry, that's negative.

As it rotates them, you get-- let me know you. It'd be nice if this is color-coded, but it's not. So these two squares are two original squares of the tessellation. They've been twisted.

And then these edges which used to be-- so they're shrunk and twisted. And then these edges used to be attached. We're now going to put in a little parallelogram there. And you just do that everywhere. And this is a crease pattern. It will fold flat.

Doesn't work for all tessellation. And there's a paper characterizing which tessellations it works for. They're called spider webs. But it's a very simple algorithm and it's led to tons of tessellations over the years.

And you can export this to PDF, print it out, and fold it. It obviously takes a little while. One of the fun surprises of this algorithm, which this is made by Alex Bateman and this was just sort of a surprise by accident. I think there's a slider at the top, the pleat angle slider. And by accident, he didn't require it to be positive.

And he realized that if you made it negative-- whoa, that's a little too negative-- you actually get the folded state. This is what that crease pattern will look like after you fold it flat, because it's essentially reflecting across each crease. So this is with all the layers stacked up. So you get sort of an x-ray view.

But it gives you a sense of-- it's hard to see the thickness here so we actually wrote a little thing here which is a little bit slow-- we'll see if it works-- called Light Pattern. And it's just measuring how many layers are stacked up at each point and it will hopefully give you a shaded pattern so that if you held it up to light where the dark spot's going to be, where the bright spot's going to be. So the idea is this will help you figure out whether something's going to be interesting or not interesting ahead of have time.

Then you can go fold once you've set the parameters exactly like you like. I've just shown one of the parameters there. There's another one, pleat ratio. So this is cool. I think an interesting project would be to extend this tool. It's open source. Lots of interesting things to do with it. Add more tessellations. Improve the interface.

Maybe try to show 3D visualization as it folds. There are existing 3D origami tools which we'll see in the very next lecture, Rigid Origami Simulator, that might make that not too hard actually. It'd be cool to try. Put it on the web I think would be interesting. Point it to JavaScript or something.

Because I think there's really cool tessellation here. Not many people have actually used the software because it's a little awkward and as you can see, Light pattern doesn't always work. But I think that's just because this tessellation's a little too big.

All right. So that was Tess. And that style of tessellation. You can see that you could some really cool thing. This is what a light pattern looks like. So you get the different shades of gray. 50 shades of gray?

Then there are more three dimensional tessellations. So this is in a different style. And this is folding a very simple origami base called water bomb. And the resulting thing is not flat, but it's very simple crease pattern and pretty cool three dimensional result.

This is not captured by Tess. And that would be a different style project to generalize to 3D tessellations. That'd be very cool. Here's that same tessellation, I think, or a very similar one, but made out of stainless steel. So you can see there's big cuts here. So this is probably made on a waterjet cutter.

And then you leave little tabs. So you wear gloves so you can fold this by hand. Probably not easy, but possible.

Here's some more back to paper, some more 3D tessellations. And if you're interested in playing with tessellations, you could try Tess. Or there's this really good book came out recently by Eric Gjerde, *Origami Tessellations*. And this is actually one of the models that's described in here.

Unlike traditional origami, there's no sequence of steps. All of these are based on here's a crease pattern, fold along all the lines, and then collapse all the lines simultaneously. Like a lot of mathematical origami design, but there's great stuff in here. Really cool tessellations and some of the best photographs of tessellations. So definitely check out that book if you want to do tessellations.

This is the crease pattern. Give you an idea for this guy. It's also periodic. This is triangular twists. You can kind of recognize that, but it's very cool.

More alternate materials. This is polypropylene. And there's this great Flickr site, polyscene by Polly Verity. And tons of examples of foldings by polypropylene. So it's a kind of plastic. It gets scored by a machine and then folded by hand.

And so really striking results. You get this nice semi-transparency. It works really well with tessellations. Here's some recent ones we just found making things out of mirror and plywood and copper as like the surface material, and then polyester and fabric, or polyester and Tyvek. Tyvek is like those envelopes, plasticy envelopes that you can't really stretch or tear. Really great stuff.

And you can buy it in sheets. So that's sort of the base layer that's holding everything together. At the creases here, you can see through to the fabric material. And then this is plywood on the surface. So these are all different tessellations, kind of tessellations.

These have been wrapped around to make vessels or to make-- they call it a shoulder cape. Looks like a set of armor. But really cool stuff when you work with other materials. It'd be a great project in this class, I think, to try some of these techniques. Combining some basic foldable sheet material with some richer material, you can make some really cool stuff.

Once you have a computer model of it, you can-- and we'll see in the next lecture different computer tools for doing that-- then actually building them I think is really striking. Back to paper, although this barely looks like paper. These are some really cool kind of traditional style tessellations, but folded in a very unusual and beautiful way by Joel Cooper who's one of the leading tessellation folders in a certain sense.

He's best known for tessellations like this, however. So these are all based on a regular triangular grid, but not quite identical. It's definitely not periodic here. Going for human forms. He has whole busts and heads.

And these are really striking. They're not designed particularly algorithmically. My understanding is he comes up with little gadgets for certain features like cheeks and so on, and he starts composing them in ways that seem to work. And he has a collection of different pieces that work together well. And he can get really intricate, really beautiful 3D surfaces out of that. So this is kind of begging to be studied mathematically in some way, but pretty challenging.

This is an interesting tessellation style by Goran Konjevod. He was a co-author on the "Folding a Better Checkerboard" paper that I talked about. And the crease pattern here is extremely boring. It's a square grid. But the mountain valley assignment is not quite trivial.

And because of the thickness of the material, it actually gets this curving behavior. So this thing is technically, mathematically it's flat. It's like this really boring pleated square. But the way it goes is you sort of take a square and you pleat the edge and then you pleat the edge and you pleat the edge. So you do mountain valley, mountain valley.

And here you're alternating between this side and this side and this side and this side. And that gives you this kind of corner. But because the material is nonzero thickness, you get these really cool curves. And when you change which order you fold the pleats in, you can really control a lot of this surface.

It's kind of magical. He has a bunch of designs like this. You can check out his images on the web if you want to see more and diagrams. And I think this is our last tessellation example. So here, goal is to make US flag. And there's a video of this being made, but it's just fold along the lines and then collapse.

You're using a tessellation element to get the stars in the flag. And this is what the crease pattern looks like. So you've got a nice tessellation here and then sort of a simpler tessellation out here, which is just some pleats. And getting those pleats to resolve to the outside. This is by Robert Lang. Very cool.

So next, I want to transition to kind of modular origami where you use multiple parts. But before we get there, this is I guess the oldest recorded example of a picture of origami. So this is from 1734. This is a reference. This is the actual object-- I believe, a newspaper article. And it's a little rough to see here, but there's an origami crane and a bunch of other classic origami things like water bomb. So the assumption is by 1734, origami was well-known. All the classic models were out there.

We don't know how far back it goes. It could be as early as when paper was invented which was like 50 AD. Somewhere between 50 and 1734, origami really hit it big. That's the big range.

But I wanted to show this because of the cranes. And one way to combine multiple parts together is to combine multiple cranes together. And there's this whole world, hiden senbazuru, which is connected cranes. And orikata means you're cutting in addition to folding.

So this is a rectangle of paper. It's been split along two lines and then folded into three cranes. So that's pretty cool. And there's much more intricate ones where you take a square of paper or a rectangle paper, do lots of cuts, subdivide your thing into a bunch of squares. Each square gets folded into a crane. The tips of the cranes stay connected at these tabs. And the challenge when you're folding these is to not tear at the tabs.

But then you'll get these really cool folds. This is an old book from 1797, not much later than that last reference. We have a copy of this book if you're interested in checking it out. Lots of different designs.

There have been some recent works in making really nice. These are spheres out of connected cranes by Linda Tomoko. And here's one out of silver foil. So really cool connected cranes.

So that's a traditional origami style. I want to transition to modular origami where you combine lots of identical parts, but now they're actually disconnected. And this is a very simple unit. I think it's just water bomb based. And then they nest into each other.

You've probably seen these kind of swans, modular swans. I think they're a very old tradition. Possibly China? I'm not sure exactly. So a kind of traditional model.

But you get a lot of geometric models like this. So these are examples of different units. You take typically a square of paper. You do maybe 10 or 20 folds and you get a unit. And then you combine a bunch of these units together.

So one of the classic units is called a Sonobe unit. Sonobe units use sort of backwards, but you can get these kinds of cool polyhedra. Robert Neale, he's a magician and an origami designer. Has some units. This one's called the penultimate unit.

And so you can see each of these green strips is one unit-- blue strip, pink strip. There's a lot of units in here. 90 in total. Typically, one per edge of the polyhedron, sometimes two per edge. And they lock together in certain ways to really hold these nice shapes here.

Tom Hull folds a lot of modular origami. And one of his units is called a PHiZZ unit. I think it can make anything as long as you have three units coming together at each vertex. So as long as every vertex has degree three, you can kind of make your polyhedron. I guess the lengths also have to be the same or else you have to adjust the units to be different. So each of the units here is identical, except different color patterns.

Here's a big example of a PHiZZ unit construction. So this is 270 units. Take a long time to fold probably and even more time to weave them together. Usually putting the last piece in is the hardest.

Here's some more examples by Tom Hull. He has another unit called the hybrid unit. And this is what three of them look like woven together. So this paper is probably red on one side, black on the other. And there's one unit that comes here, wraps around the tetrahedron, and two more.

And you combine them and you can make all these different regular solids. And you get these spiky tetrahedra on each of the faces which is pretty cool. So this like icosahedra, a regular 20-sided die, on the inside here, but then each of them has a spike from there.

And here's a big one he made. This is actually one of my favorite polyhedra, the rhombicosidodecahedron. It's got all the polygons-- squares, triangles, hexagons, if I recall correctly. It's obvious, right?

And one of the challenges here is getting the color patterns to be nice and symmetric and even. And Tom Hull is one of the experts in that. He's a mathematician, but also an origamist. And then he started combining the two because of problems like this.

Next we get to polypolyhedra. This is the idea of taking multiple polyhedra and weaving them together and then making that out of origami. And this is one of the most famous designs in this family called FIT, or Five Intersecting Tetrahedra, designed by Tom Hull.

This is a photograph of one that I am the proud owner. It was folded by Vanessa Gould, who directed *Between the Folds*, which is the documentary you all heard about when Jason mentioned it. And it's available free streaming on Netflix, so you should all watch it. Or we could have a showing here. Actually, how many people are interested? Haven't seen the movie or would like to see it again related to this class some evening?

OK. That's maybe enough to do a showing. Anyway, she folded this. Cool. And then Robert Lang enumerated all possible polypolyhedra that are symmetric in a certain sense. And these are two examples that he thought were so cool. He made them out of paper. Most of them just exist as virtual designs.

People have been folding them, but there's hundreds if not thousands in his list. So if you're interested, check out his website on polypolyhedra. These are, again, modular.

And finally, we come to modules of cubes. And this is why you have business cards. And I thought we could play with this. This is a life-size chair made from a particular unit, which is out of business cards, folding these individual cubes and then sticking them together in a particular way.

Unfortunately, the material's not strong enough to actually support much weight. So you can't sit on this chair, but it looks just like a real chair. It's very cool.

You can make any set of cues you like and interlock them together. One of the craziest experimenters with this cube module is Jeannine Mosely, who's a MIT alum and lives in the area. And she became really famous for making this Menger Sponge out of 66,000 business cards. It took something like five years to make this. She made a lot of the units herself.

And so this is trying to represent a particular fractal, which is pretty cool. You start by taking a cube and then drilling holes through each of the sides in the center third. So this is one iteration. You just drill through that hole, that hole, same on each side. Remove that material. That leaves you with-- how many cubes?

Eight cubes on top. Eight cubes on the bottom. Four cubes in the middle, which is 20. For each of the 20 cubes, you recurse. So for each of those 20 cubes, you drill holes, drill holes from all the sides. And after two iterations, you have this structure.

After three iterations, you have this structure. After infinitely many iterations-- well, no, this is not infinite. But this is actually the same number of iterations as that.

So in principle, you keep going. But at any fixed point, you can treat the smallest little unit that hasn't been recursed as one of these cubes, build that, and then assemble them together. It's challenging.

You could not take this-- with the business cards, you could not go to the next level-- not because it would take forever, but also because it would collapse under its own weight. So trade-off there. That was 66,000 business cards, five years. I thought, man, that was a big project.

But then Jeannine says, what else can we make? And she got more volunteers for these future projects so they were made a lot faster. This is a cool fractal. Not quite as many, 50,000 business cards. And this is a fractal that she designed.

Kind of complimentary. You take a cube and subdivide it into three by three by three, and then remove all the corner cubes, and then recurse. And she calls it the Moseley Snowflake because if you look at it from the corner, you get this nice Koch snowflake outline. And this is the real one from the same view.

It's a little big, so it's hard to see it all in one shot. And so that's pretty awesome. And then her most recent project was 100,000 business cards. This is I guess the world record for origami made from business cards. And this is a model of Union Station in Worcester, Massachusetts.

Hundreds of volunteers here to make this. This was done for first night celebration a year or so ago. Pretty amazing. And you can see, you can really sculpt with these cube units, do lots of cool stuff. And there's a few extra details on the surface there.

So I thought we would make something. So these are diagrams you can start working or I can tell you about how they work. Each cube is made from six identical business cards. I have here my own business cards from when I first arrived, old classic. So you start by taking two of your business cards.

You have to decide whether you want the white face up on your cube and make it nice and clean or you want the pattern side up. Whichever one you want to expose, you keep that on the outside and you bring the two cards together. So in this case, I'm going to make the pattern side out. And you want to align these approximately evenly.

You want them as perpendicular as possible and then roughly evenly spaced. And then you just mountain fold both sides. So you want mountain folds on the side that you care about. And that gives you a nice square. Now I've got two nice squares folded like this. Repeat three times, you get six units.

Four and six. OK, once you've got the six units, you want to combine them together. This is where it gets fun. And it's helpful to look at this diagram down here. These are some diagrams by Ned Batchelder.

And so this idea of making cubes has been around. I think it was Jeannine's idea to combine them together. So this is what one cube looks like. Why don't I fold, make one of them. Basically, you want the tabs going on the outside. And you need to alternate so they lock together. And you need to alternate between oriented horizontal and oriented vertical.

So they recommend starting by making a corner, three of them like that, and then fill around the outside. And then as usual, putting in the last piece is the hardest. So I've got to get-- I want all the tabs on the outside like that.

And I probably should've mentioned-- fold the creases really hard. You can do that to a certain extent afterwards, make it nice and cubey. But in this case, I got my six-sided cube out of those six units. It's got my name right in the center. So you can design business card specifically for this purpose. I accidentally did.

And that's how you make one cube. Once you've got two cubes, you can lock them together by just twisting them 90 degrees relative to each other and just sliding the tabs in, just sliding the tabs in. This is also like doing that last move.

So this tab's got to go in here between these two tabs. It wouldn't hold together very well if it wasn't hard to put together. So once you've got them together, you've got two cubes. Now if you want, for a finishing touch, you can also make another unit and cover the surfaces.

So all of Jeannine Moseley's examples are done this way where at the end-- I haven't tried this lately-- you stick on a business card just on the surface so it interlocks here and then interlocks over here. Ho boy, this is challenging. And then you get a full square business card on the outside.

And you can use this to, if you have different colored business cards or you want a nice, clean white surface, no seams. So you have these tabs right now, but once you add something like this, you have a nice seamless square on the outside. So you use up more business cards, but it can make for a nicer surface.

So any questions about making these? I thought we would make some and then build something. But for that, I need suggestions on what to build.

Oh, an MIT. I like that. Let's make an MIT. So let's design. So by MIT, do you mean MIT logo or like an M?

1, 2, 3, 4, 5, 6, 7, 8, 9, 10 cubes. Easy. Exploding cubes. I wonder if you can use these to make pinatas? MIT. We could also just make a row at the bottom. One cube higher. Four more minutes.

AUDIENCE: We could do Minecraft origami.

AUDIENCE: Ohh.

AUDIENCE: Oh, yes!

AUDIENCE: That's a good idea.

PROFESSOR: Minecraft is a good source.