6.832 Midterm

Name: _____

April 22, 2009

Please do not open the test packet until you are asked to do so.

- You will be given 90 minutes to complete the exam.

- Please write your name on this page, and on any additional pages that are in danger of getting separated.

- We have left workspace in this booklet. Scrap paper is available from the staff. Any scrap paper should be handed in with your exam.

- YOU MUST WRITE ALL OF YOUR ANSWERS IN THIS BOOKLET (not the scrap paper).

- The test is open notes.

- The test is out of 30 points.

Good luck!

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Problem 1 (Optimal control of the simple pendulum) *(8 points total) Consider the following four phase portraits, θ-histories and u-histories which are the result of various closed-loop policies being executed on the simple pendulum:*

1)

2)

3)

4)
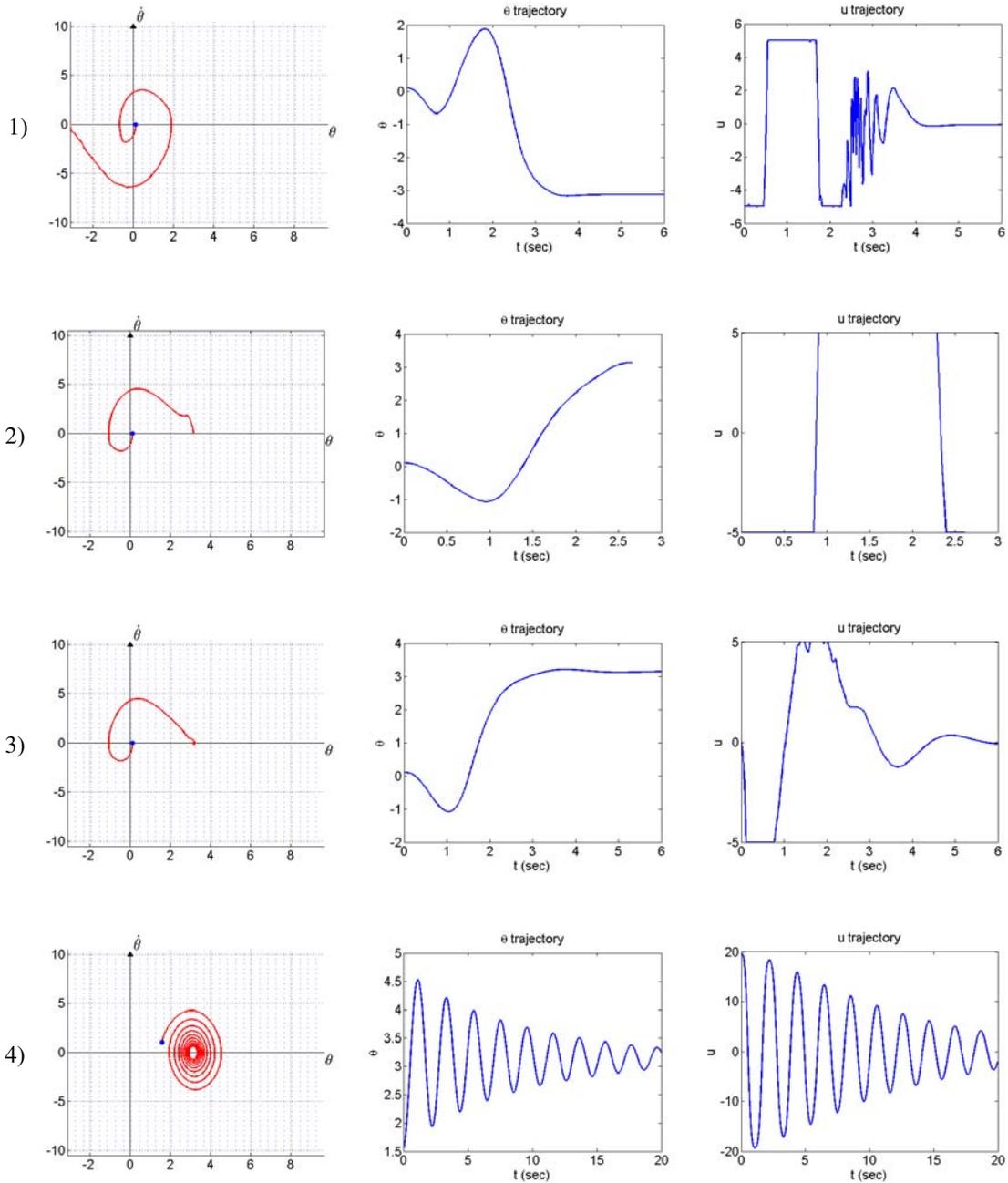
Match the closed-loop dynamics above with the following candidate control methods. In each case, explain what features of the plot allowed you to determine the algorithm and cost function which generated it (8 points):

a) *Feedback linearizing controller which inverts gravity.*

   Matching Trajectory (1-4): _____

   Justification:

b) *Miniminum-time cost function, solved with finely discretized value iteration*

   Matching Trajectory (1-4): _____

   Justification:

c) *Quadratic regulator cost, with torque-limits imposed but not used in the controller design, solved with value iteration.*

   Matching Trajectory (1-4): _____

   Justification:

d) *Minimum-time cost function, solved with direct collocation.*

   Matching Trajectory (1-4): _____

   Justification:

Problem 2 (Pendubot) *(7 points total) The Pendubot is the companion system to the Acrobot - a two-link arm with an actuator at the shoulder but no actuator at the elbow.*
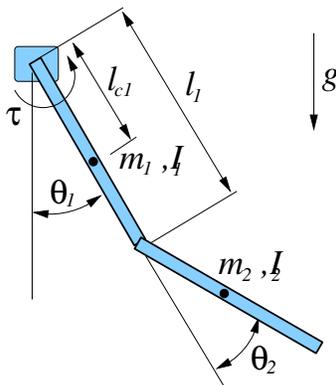
The equations of motion are:

$$(I_1 + I_2 + m_2l_1^2 + 2m_2l_1l_{c2}c_2)\ddot{q}_1 + (I_2 + m_2l_1l_{c2}c_2)\ddot{q}_2 - 2m_2l_1l_{c2}s_2\dot{q}_1\dot{q}_2$$
$$-m_2l_1l_{c2}s_2\dot{q}_2^2 + (m_1l_{c1} + m_2l_1)gs_1 + m_2gl_2s_{1+2} = \tau$$
$$(I_2 + m_2l_1l_{c2}c_2)\ddot{q}_1 + I_2\ddot{q}_2 + m_2l_1l_{c2}s_2\dot{q}_1^2 + m_2gl_{c2}s_{1+2} = 0,$$

where I_1, I_2 are the moments of inertia of the first and second links, respectively, about the pivots, m_1, m_2 are the masses, l_1, l_2 are the lengths, l_{c1}, l_{c2} are the distances from the pivots ot the center of mass, g is gravity, and $s_1, ...$ are the usuals short-hands for $\sin(q_1), ...$

a) *(4 points) Is there any configuration of the pendubot in which non-collocated partial feedback linearization will fail? If so, which configurations, and why?*

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b) *(3 points) Consider a change of coordinates to an "absolute frame". The new coordinates are given as $\psi_1 = q_1$ and $\psi_2 = q_1 + q_2$. In this new frame, the equations of motion are:*

$$(I_1 + m_2 l_1^2 + m_2 l_1 l_{c2} c_{2-1})\ddot{\psi}_1 + (I_2 + m_2 l_1 l_{c2} c_{2-1})\ddot{\psi}_2$$
$$-m_2 l_1 l_{c2} s_{2-1}(\dot{\psi}_1^2 + \dot{\psi}_2^2) + (m_1 l_{c1} + m_2 l_1)g s_1 + m_2 g l_2 s_2 = \tau$$
$$m_2 l_1 l_{c2} c_{2-1}\ddot{\psi}_1 + I_2 \ddot{\psi}_2 + m_2 l_1 l_{c2} s_{2-1}\dot{\psi}_1^2 + m_2 g l_{c2} s_2 = 0.$$

Our goal is now to control ψ_2. Does the system represented by this new coordinate frame have any configurations in which non-collocated partial feedback linearization will fail? If there are any differences between the two coordinate frames, explain why. (Also note that c_{2-1} now means $\cos(\psi_2 - \psi_1)$).

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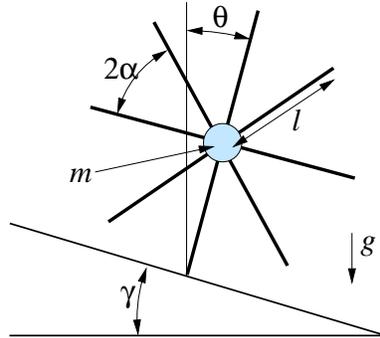
Problem 3 (Control of a Rimless Wheel) *(15 points total) Consider a modified version of the rimless wheel, where the length of the spokes can be changed (all simultaneously) by a single motor, which applies a radial force, f. The kinetic, T, and potential, U, energies of the system are given by*

$$T = \frac{1}{2}m(\dot{l}^2 + l^2\dot{\theta}^2), \quad U = mgl\cos(\theta).$$

The resulting equations of motion during the stance phase are,

$$ml^2\ddot{\theta} + 2ml\dot{l}\dot{\theta} - mgl\sin\theta = 0$$
$$m\ddot{l} - ml\dot{\theta}^2 + mg\cos\theta = f,$$

where f is the force applied to change the length of the leg (positive force extends the leg). The impact conditions do not change; to keep the impact dynamics simple we will assume that the legs are instantaneously clamped ($\dot{l} = 0$) at the impact. This results in the impact dynamics:

$$\dot{\theta}(t_c^+) = \dot{\theta}(t_c^-)\cos(2\alpha), \quad \dot{l}(t_c^+) = 0.$$

At the impact, θ is also reset to the coordinate system of the new stance leg. For reference, you may wish to observe that the distance (chord length) between two feet is $2l\sin(\alpha)$.

  a) *(5 points) Design an energy-shaping controller which drives the rimless wheel into it's vertical (inverted-pendulum balancing) configuration.*

   (i) *Give a feedback law for the stance phase, [f =?]. Make sure you consider the case when $\dot{l} = 0$.*

   (ii) *Give the resulting energy dynamics in the stance phase, [$\dot{E}$ =?].*

   (iii) *The impacts will add or remove energy from the system. Describe, in words, any additional constraints that this puts on your feedback design.*

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b) *(3 points) Can we stabilize the upright balancing configuration with feedback?*

    (i) *Linearize the dynamics around the upright fixed point and write the equations in the form $\dot{\bar{x}} = A\bar{x} + B\bar{u}$. For simplicity, you may set all parameters to one: $m = l = g = 1$.*

   (ii) *Compute the controllability matrix for this system. Is it controllable at the fixed point? Can you give an intuitive reason for your answer?*

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c) *(4 points) Design a controller which drives the system to a desired rolling fixed point defined by $\dot{\theta}(t_c^+) = \omega_d^* > 0$.*

   (i) If we design a controller such that the leg length is constant on the nominal limit cycle, $l(t) = l^*$, like in the passive rimless wheel, what must this leg length be? Your answer should be in the form $[l^* =?]$ and be in terms of the system parameters and w_d^*.

   (ii) Give a feedback controller, $[f =?]$, which stabilizes this rolling fixed point. You may assume that at the initial conditions that $\dot{\theta}(0) > 0$ and $E(0) > mgl^*$.

   (iii) Is your controller a dead-beat controller? If so, for what choice of parameters? If not, is it possible to produce a dead-beat controller for this problem (answer this part with a sentence or two explanation - there is no need to design the controller)?

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d) *(3 points) Optimal control algorithms*

   (i) *If we wished to find feedback control law, $f = \pi(\mathbf{x})$ for this problem with value iteration, using N bins per state variable, then how many total bins would be required? Are their any major potential concerns of this approach?*

   (ii) *If we wished to use value iteration to find a find a feedback control law on the Poincare map (decisions made once per step), using N bins per dimension, then how many total bins would be required? (This could be implemented with $[l_d, \dot{l}_d] = \pi(\mathbf{x}_p)$, where l_d and $\dot{l}_d$ are tracked by a PD controller inside the step). How are the concerns for this approach different that in part i?*

   (iii) *Could we use a shooting method to optimize the nominal trajectory of this system?*

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