6.866

Quizz #1 Oct. 19 Quizz #2 Nov. 30

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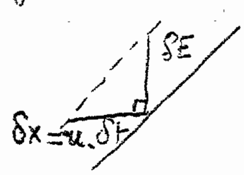
(1) Estimating motion

1st example, 1 vector (x, y), 1 variable $E =$ brightness

$x =$ pos. on the image

shift + meas. noise

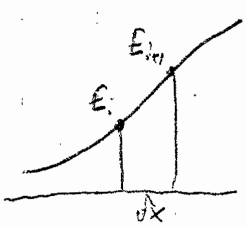


slope $= \frac{dE}{dx}$

$u =$ velocity $= \frac{\delta x}{\delta t}$

$\frac{dE}{dx} = -\frac{\delta E}{u \cdot \delta t}$

$u \cdot E_x + E_t = 0.$ or: $u \frac{dE}{dx} + \frac{\delta E}{\delta t} = 0$



$E_x \approx \frac{E_{i+1} - E_i}{\delta x}$ and $E_t \simeq \frac{E_{i+1} - E_i}{\delta t}$ thus $\boxed{u = -\frac{E_t}{E_x}}$

but image noisy so poor method

and what if $E_t = 0$? (blank area)

and what if $E_t \approx 0$? not good either

Better method:

$$u \simeq \frac{-1}{x_2 - x_1} \int_{x_1}^{x_2} \frac{E_x}{E_t} dx \quad + \text{ filtering } E_t \simeq 0$$

$$u \simeq \frac{-1}{x_2 - x_1} \int_{x_1}^{x_2} w(x) \frac{E_x}{E_t} dx \Big/ W \quad \text{where } W = \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} w(x) dx$$

<u>so</u> $$u = -\int_{x_1}^{x_2} w(x) \frac{E_x}{E_t} dx \Big/ \int_{x_1}^{x_2} w(x) dx$$

<u>let's take</u> $w(x) = E_t$

$$u = \underbrace{-\int_{x_1}^{x_2} E_x dx}_{-E_2 - E_1} \Big/ \int_{x_1}^{x_2} E_t \, dx$$

<u>not good</u>! %?

<u>but what about</u> $w(x) = |E_t|$

$$u = -\int_{x_1}^{x_2} E_x \, \mathrm{sign}(E_t) dx \Big/ \int_{x_1}^{x_2} |E_t| dx$$

not optimal... discontinuities

<u>so</u> $w(x) = E_t^2$

$$\boxed{u = -\int_{x_1}^{x_2} E_x E_t \, dx \Big/ \int_{x_1}^{x_2} E_t^2 dx}$$

⊕ takes info from all the image

Another way:

$$u E_x + E_t = 0$$ constraint equation

$$\int_{x_1}^{x_2} (u E_x + E_t)^2 dx$$
$\nearrow 0$ perfect case
$\searrow \varepsilon$ error real case

u = min value for which ε

$$\frac{d}{du}\left(\ldots\right) = 0 \implies \int_{x_1}^{x_2} 2E_x(uE_x + E_t)E_x\,dx = 0$$

$$u\int_{x_1}^{x_2} E_x^2\,dx + \int_{x_1}^{x_2} E_x E_t\,dx = 0$$

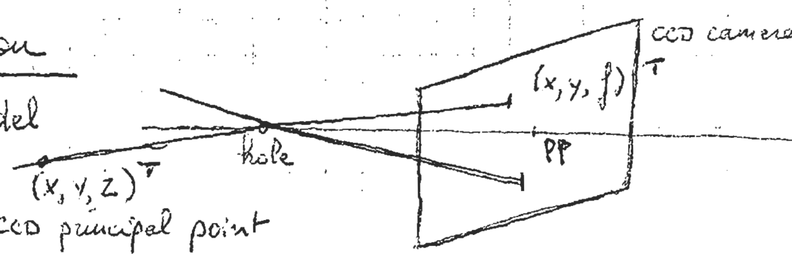
$$u = -\int_{x_1}^{x_2} E_x E_t\,dx \Big/ \int_{x_1}^{x_2} E_x^2\,dx$$

Least Square Solution
"optical flow"

if motion = constant then average on time also
→ more accurate

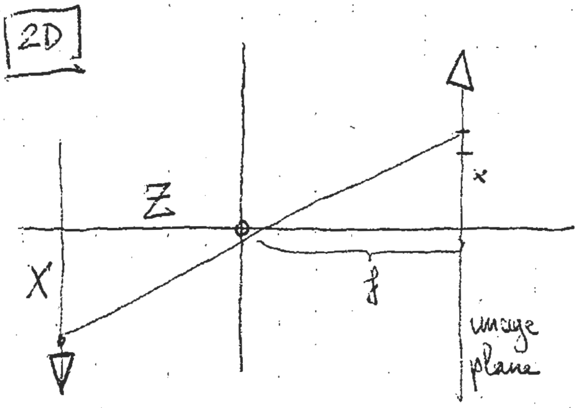
② Image projection

pinhole model



f = dis. bet. hole & CCD principal point

2D



3D

$$\frac{x}{f} = \frac{X}{Z} \qquad \frac{y}{f} = \frac{Y}{Z}$$

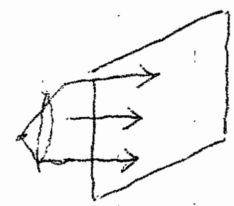
perspective projection equation

f = zoom factor
+ scale ambiguity

H] Z constant (Z = optical axis)

$$\frac{x}{f} = \frac{X}{Z_0} \Rightarrow x = \frac{f}{Z_0} \cdot X \qquad y = \frac{f}{Z_0} Y.$$

case $f = Z_0$: $\underline{x = X,\ y = Y}$ orthographic projection



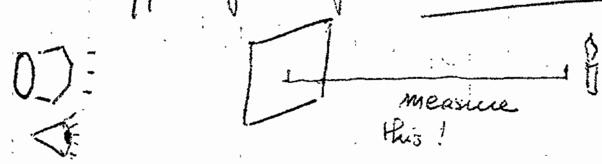
good approx. for telephoto lenses.

③ Brightness

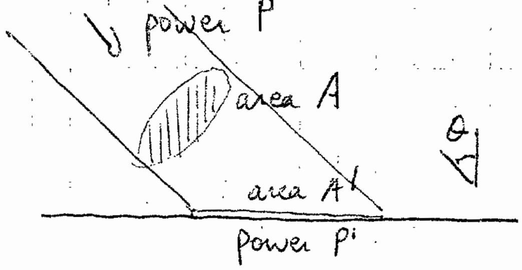
Parameters : (i) material of surface
(ii) illumination : power & distribution
(iii) viewer direction

Model of a particular type of surface : "Lambertian" (ideal)

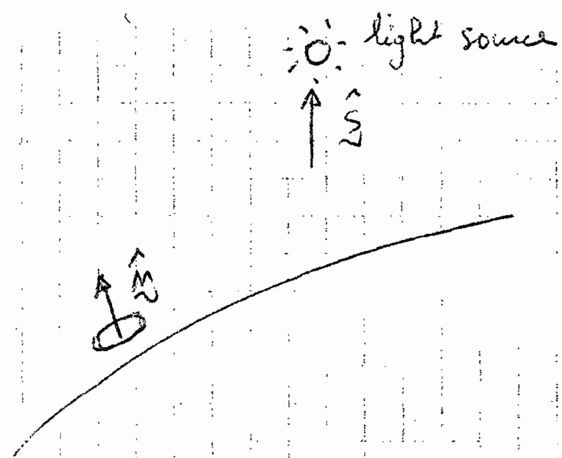
Lambert,



(i) reflects all incident light
(ii) appears equally bright from all viewpoints (mat)



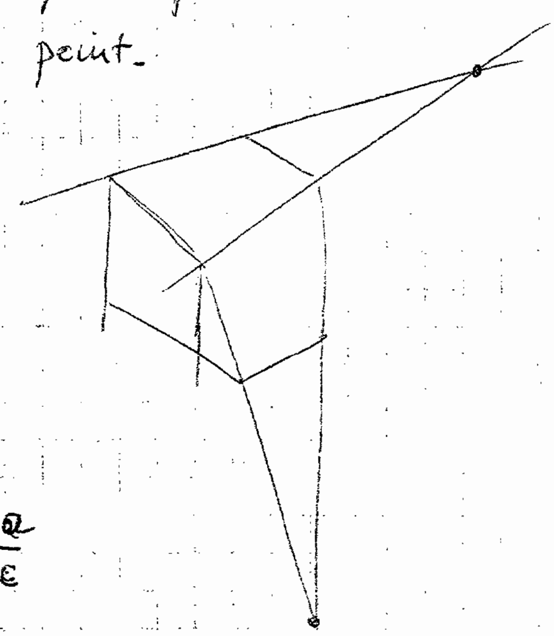
$$\frac{P'}{A'} = \frac{P}{A} \cos\theta$$



Power received $= P \times \hat{n} . \hat{s}$

<u>where</u> $\hat{n} . \hat{s} = \|\vec{n}\| . \|\vec{s}\| . \cos\theta$

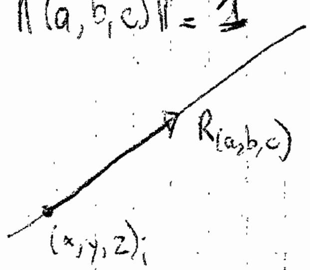
1. A line in 3D becomes a line in 2D camera
2. Two parallele lines in 3D are generally not parallel in 2D.
3. The common point is the vanishing point.



Equation of a line:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} + \begin{pmatrix} Ra \\ Rb \\ Rc \end{pmatrix}$$

$\|(a,b,c)\| = 1$



$$\begin{cases} \dfrac{x}{f} = \dfrac{X}{Z} = \dfrac{X_i + Ra}{Z_i + Rc} \\[2ex] \dfrac{y}{f} = \dfrac{Y}{Z} = \dfrac{Y_i + Rb}{Z_i + Rc} \end{cases}$$

if $R \to \infty$: $\dfrac{x}{f} \to \dfrac{a}{c}$ $\quad \dfrac{y}{f} \to \dfrac{b}{c}$ <u>vanishing point</u>