6.780
Semiconductor Manufacturing
Spring 2003

QUIZ #2

Wednesday, April 23, 2003: 9:30 am - 10:55 am

Name: SOLUTIONS

For each of the following questions, please be sure to **show your work**. If you need to make substantial assumptions, be sure to state them.

Question 1 (10 POINTS)

A semiconductor chip is 4 mm x 5 mm in size. At a particular process step, the defect density is 2/cm$^{\cancel{3}\,2}$, and the critical area is 25% of the total chip area.

(a) Assuming a Poisson yield model, what is the chip yield at this step?

$A = 0.4 \times 0.5 \text{ cm}^2 = 0.20 \text{ cm}^2$ total chip area

$A_c = 0.25 \cdot A = 0.05 \text{ cm}^2$ critical area

$D_0 = 2 \text{ cm}^{-2}$ defect density

Using poisson model, yield Y is

$$Y = e^{-A_c \cdot D_0} = e^{-0.1} = \boxed{0.905}$$

(b) Assuming a negative binomial yield model with clustering coefficent $\alpha = 2$, what is the chip yield at this step?

$$Y = \left(1 + \frac{A_c \cdot D_0}{\alpha}\right)^{-\alpha}$$

$$= \left(1 + \frac{0.1}{2}\right)^{-2}$$

$$= 1.05^{-2} = \boxed{0.907}$$

Question 2 (20 POINTS)

In a thermal oxidation process, two random sites are measured on each of two wafers, with the measured oxide thickness as shown in Table 1.

Table 1: Oxide thickness measurements (nm)

Wafer 1	Site 1	18	> 19	> 17
	Site 2	20		
Wafer 2	Site 1	14	> 15	
	Site 2	16		

(a) Estimate $\sigma^2_{\overline{W}}$, the variance in the wafer averages: $W = \#$ wafers $= 2$

$$s^2_{\overline{W}} = \frac{1}{W-1}\sum_{i=1}^{W}(\overline{w}_i - \overline{\overline{w}})^2 = \frac{1}{1}\left\{(19-17)^2 + (17-15)^2\right\} = \boxed{8}$$

(b) Estimate σ^2_S, the site to site (or within wafer) variances:

$$s^2_S = \frac{1^2+1^2+1^2+1^2}{4-2} = \frac{4}{2} = \boxed{2}$$

4 terms, but used two d.o.f. in wafer means

$$= \frac{1}{W}\sum_{i=1}^{W}\sum_{j=1}^{M}\frac{(w_{ij}-\overline{w}_i)^2}{M-1}$$

(c) Estimate σ^2_W, the wafer to wafer variance: $M = \#$ sites per wafer

$$s^2_W = s^2_{\overline{W}} - \frac{s^2_S}{M} = 8 - 1 = \boxed{7}$$

(d) An $\overline{X}$ control chartwith $\pm 3\sigma$ control limits is to be used to monitor for mean shifts from wafer to wafer. Each wafer average will be based on five site measurements within that wafer. What should the centerline (CL), upper control limit (UCL) and lower control limit (LCL) be?

$$s^2_{\overline{W}} = s^2_W + \frac{s^2_S}{M} = 7 + \frac{2}{5} = 7.4$$

NOTE use of $\sigma_{\overline{W}}$, since we're charting $\overline{w}$ based on 5 point measurements...

So our 3σ limits are $\pm 3\sqrt{7.4} = \pm 3 \cdot 2.72$

$= \pm 8.16$

$$\boxed{\begin{array}{l} UCL = 25.16 \\ CL = 17 \\ LCL = 8.84 \end{array}}$$

Question 3 (40 POINTS)

A CVD process is under investigation, where the goal is to model the thickness of a barrier metal layer as a function of two process parameters, the pressure (P) and temperature (T). The following experiment has been performed. The process conditions are shown in normalized units (-1 to +1) in Table 2.

Table 2: CVD experimental design and results

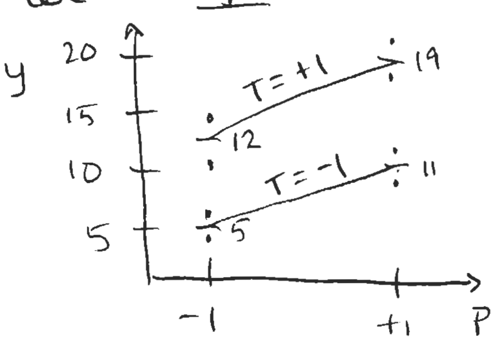
P Pressure	T Temperature	y_i thickness (nm)	$\bar{y}_{P,T}$	# replicate	$\hat{y}$ estimate	$\varepsilon_i = y_i - \hat{y}$ residual	
-1	-1	4	5	1	4.75	-0.75	
-1	-1	6		1		1.25	
-1	+1	10	12	2	12.25	-2.25	
-1	+1	14		2		1.75	
+1	-1	13	11	2	11.25	1.75	
+1	-1	9		2		-2.25	
+1	+1	21	19	2	18.75	2.25	
+1	+1	17		2		-1.75	

Your task is to produce, and justify, a model for this process. Thorough explanation and justification must include analysis of variance, effects or interaction plots, lack of fit analysis, estimates of any significant effects and interactions, confidence intervals (e.g. standard errors) on model parameter estimates, and R^2 and adjusted R^2 values.

10 pts

5 pts   5 pts   5 pts   5 pts   10 pts for effects/model

- First, we note that this is a two-factor, two-level DOE, with two replicates at each experimental point. The average at each condition is shown as $\bar{y}_{P,T}$ above.
- To help decide if we should include interaction terms, we'll begin with a plot of our data in an interaction plot:



So our lines are not quite parallel, but nearly so given the size of our replication error. So we will proceed assuming no interaction is significant, and check this assumption later with lack of fit analysis.

Question 3, continued

- To estimate the size of the effects, we look at

$\bar{y}$ = grand average = 11.75

$\bar{y}_{P+}$ = 15.0 = average of all pressure @ +1 levels = $\frac{11+19}{2}$ = 15

$\bar{y}_{P-}$ = 8.5 = $\frac{12+5}{2}$ = $\frac{17}{2}$

$\bar{y}_{T+}$ = 15.5 = ave of all temp @ +1 levels. = $\frac{12+19}{2}$ = $\frac{31}{2}$

$\bar{y}_{T-}$ = 8.0 = $\frac{11+5}{2}$ = $\frac{16}{2}$

- Our model is therefore

$$\hat{y} = \bar{y} + \frac{(\bar{y}_{P+} - \bar{y}_{P-})}{2} \cdot P + \frac{(\bar{y}_{T+} - \bar{y}_{T-})}{2} \cdot T$$

$$\boxed{\hat{y} = 11.75 + 3.25P + 3.75T}$$

- Analysis of variance helps us understand the significance of these effects, as well as lack of fit and replication errors (calculations shown on next page).

	dof	Sum of Squares	Mean Square	F ratio
Pressure	1	2 84.5	84.5	15.94
Temperature	1	3 112.5	112.5	21.23
Error	5	6 26.5	5.3	
LOF	1	0.5	0.5	= 0.077
Pure	4	1 26.0	6.5	
Average	1	5 1104.5	1104.5	
Total	8	4 1328		

Question 3, continued

① Pure replication error – $\varepsilon_{replicates}$ are shown on Table 2.
The $SS_{pure} = 1^2 + 1^2 + \cancel{2} 6 \cdot 2^2 = 26$

② $SS_P = 4(\bar{y}_{P+} - \bar{y})^2 + 4(\bar{y}_{P-} - \bar{y})^2$
$= 4(15.0 - 11.75)^2 + 4(8.5 - 11.75)^2$
$= 42.25 + 42.25 = 84.5$

③ $SS_T = 4(\bar{y}_{T+} - \bar{y})^2 + 4(\bar{y}_{T-} - \bar{y})^2$
$= 4(15.5 - 11.75)^2 + 4(8.0 - 11.75)^2$
$= 56.25 + 56.25 = 112.5$

④ $SS_{TOTAL} = \sum_{i=1}^{8} y_i^2 = 1328$

⑤ $SS_{ave} = 8(11.75)^2 = 1104.5$

⑥ Errors – ε in table 2, based on deviations from $\hat{y}$
$SS_{ER} = \sum_{i=1}^{8} \varepsilon_i^2 = 0.75^2 + 1.25^2 + 3(2.25)^2 + 3(1.75)^2$
$= 0.5625 + 1.5625 + 15.1875 + 9.1875 = 26.5$

Alternatively (easier, but can serve as a check with above).
$SS_{Error} = SS_{TOTAL} - SS_{AVE} - SS_P - SS_T$
$= 1328 - 1104.5 - 84.5 - 112.5 = 26.5$

∴ LOF Sum of squares is $26.5 - 26 = 0.5$

- Before evaluating significance of factor effects, we'll look at lack of fit. In this case

$$F = \frac{MS_{LOF}}{MS_{Pure}} = \frac{0.5}{6.5} = 0.077$$

Since this is << 1, there is no evidence of lack of fit ... compared to replication error, model error is small.

Question 3, continued

- So, we'll use full error (both replication and model fit error) as or error term:

$$F_{pressure} = \frac{84.5}{5.3} = 15.94$$

$$F_{temp} = \frac{112.5}{5.3} = 21.23$$

For significance, we'll use 95% as confidence cutoff, or

$$F_{0.05,1,5} = 6.61 \text{ from stat charts.}$$

$\therefore$ Both pressure & temp are significant, to better than 95% confidence.

- To establish the standard error, we also use the error term, and recognize that the error mean square gives us estimate of process variance as $\hat{\sigma}^2 \approx 5.3$. So

$$Var(\bar{y}) = \frac{5.3}{8} \Rightarrow \text{std. err}(\bar{y}) = \sqrt{\frac{5.3}{8}} = 0.814$$

$$Var(E_p) = Var\left\{\frac{\bar{y}_{P+} - \bar{y}_{P-}}{2}\right\}$$

$$= \frac{1}{4}\left(\frac{5.3}{4} + \frac{5.3}{4}\right) \Rightarrow \text{std. err}(E_P) = \sqrt{\frac{5.3}{8}} = 0.814$$

So our model can be stated as

$$\hat{y} = (11.75 \pm 0.814) + (3.25 \pm 0.814) P + (3.75 \pm 0.814) T$$

and confidence intervals to any desired degree of confidence could be derived from this.

Question 3, continued

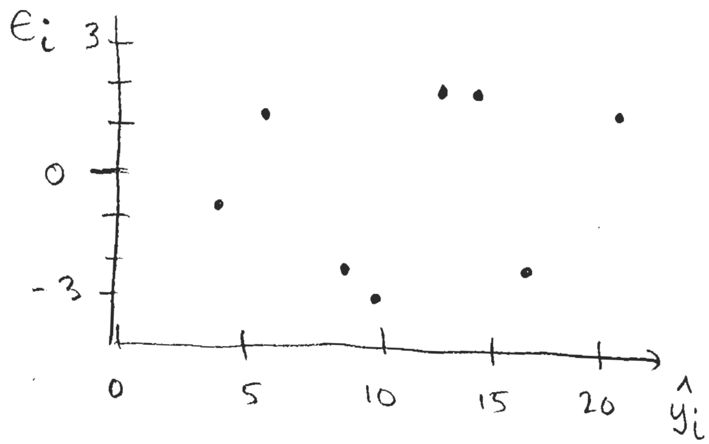
- Finally, we can calculate R^2 and adjusted-R^2

$$R^2 = \frac{SS_{MODEL}}{SS_{TOTAL} - SS_{AVE}} = \frac{84.5 + 112.5}{1328 - 1104.5} = \frac{197}{223.5} = \boxed{0.881}$$

$$R^2_{adj} = 1 - \frac{MS_{error}}{MS_{DEV}}$$

$$= 1 - \frac{MS_{error}}{(1328 - 1104.5)/7} = \boxed{0.834}$$

- Although not explicitly requested, residual analysis is also good practice:



No dependence on size of estimate is apparent.

We don't have run or sequence number; if we did, we should use this too, to check for trends.