6.776
High Speed Communication Circuits
Lecture 4
S-Parameters and Impedance Transformers

Massachusetts Institute of Technology

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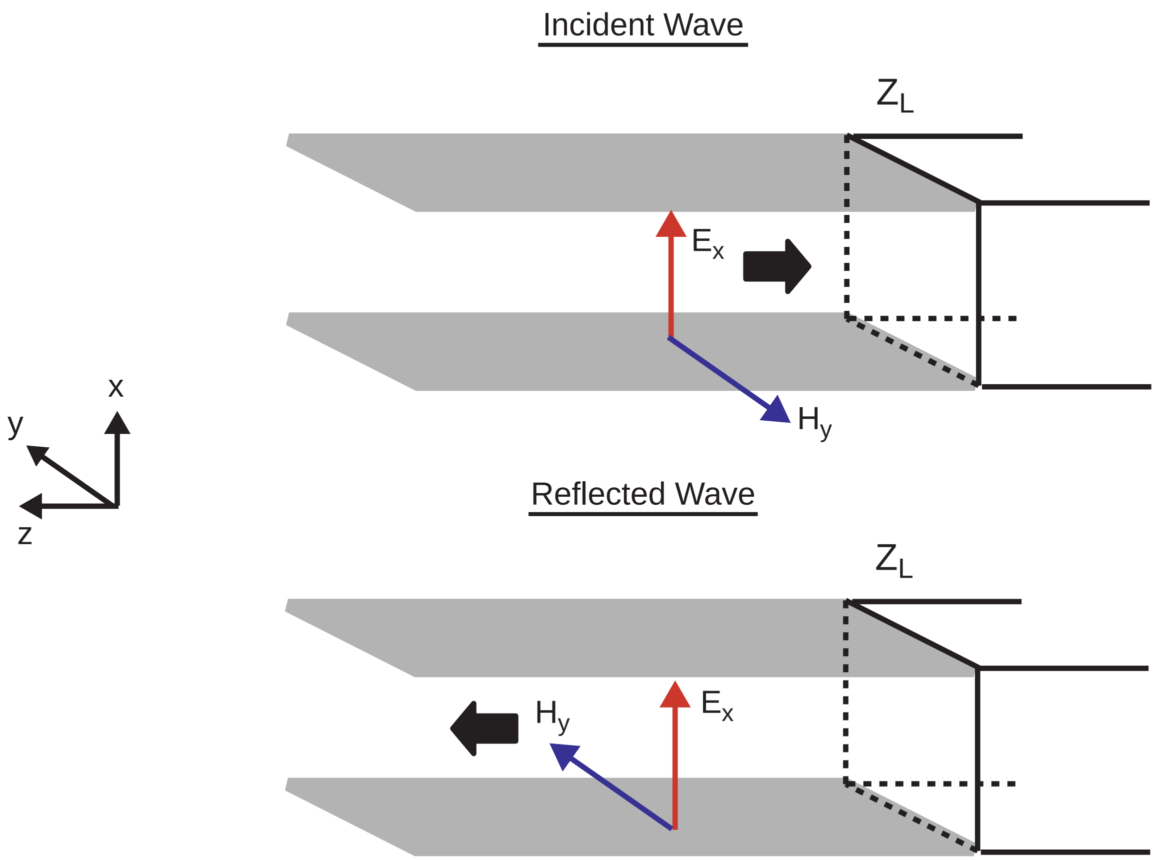
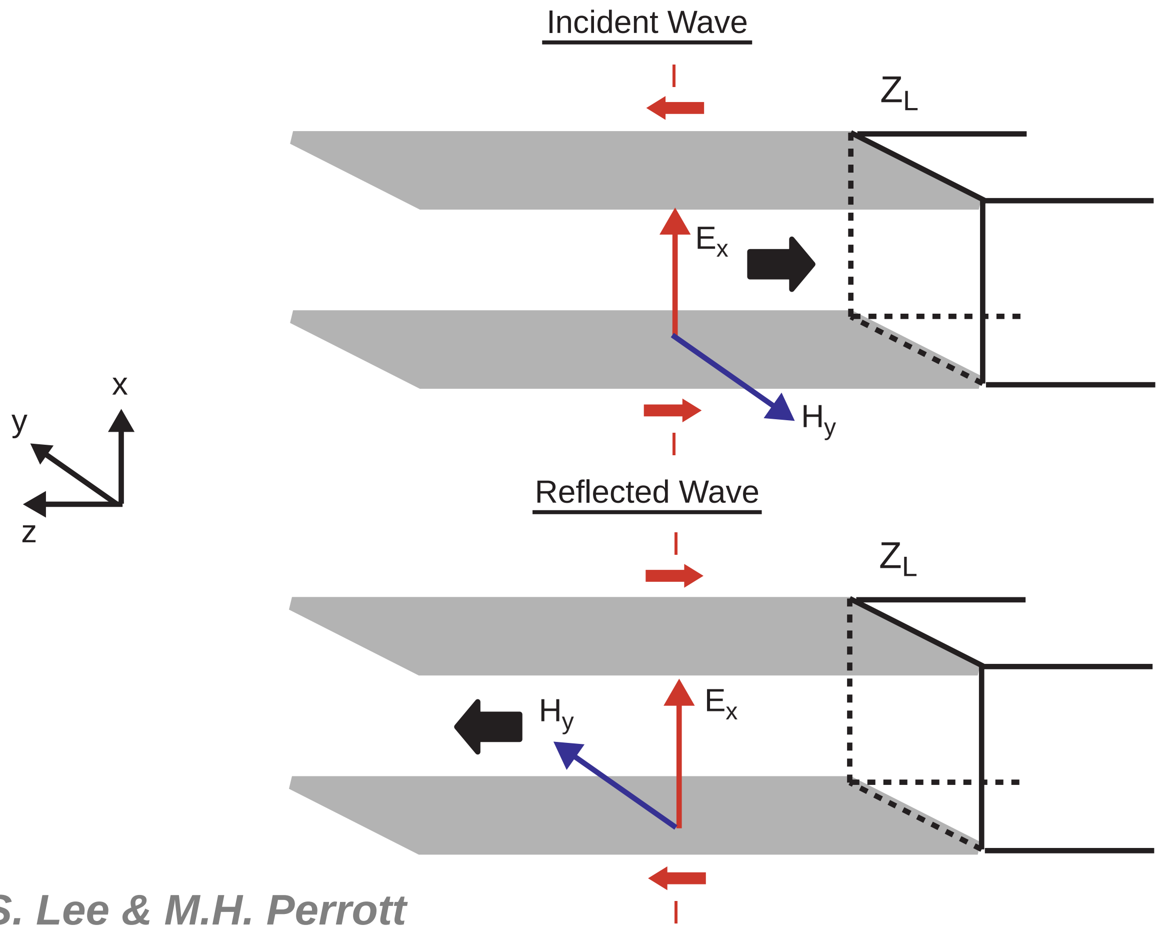
What Happens When the Wave Hits a Boundary?

- Reflections can occur



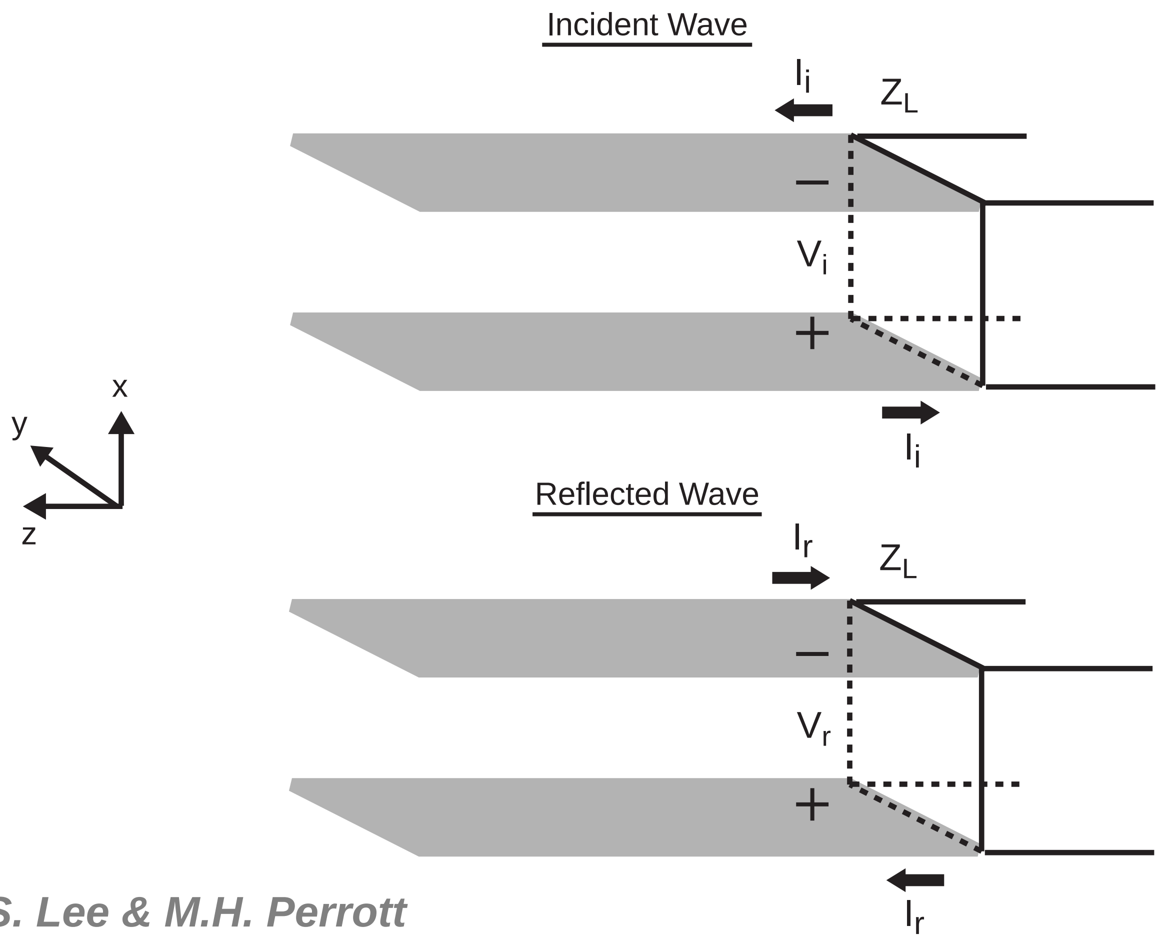
What Happens When the Wave Hits a Boundary?

- At boundary
  - Orientation of H-field flips with respect to E-field
    - Current reverses direction with respect to voltage



What Happens At The Load Location?

- Voltage and currents at load are ratioed according to the load impedance



Voltage at Load

$$V_i + V_r$$

Current at Load

$$I_i - I_r$$

Ratio at Load

$$\frac{V_i + V_r}{I_i - I_r} = Z_L$$

Relate to Characteristic Impedance

- **From previous slide**

$$\frac{V_i + V_r}{I_i - I_r} = \frac{V_i}{I_i}\left(\frac{1 + V_r/V_i}{1 - I_r/I_i}\right) = Z_L$$

- **Voltage and current ratio in transmission line set by it characteristic impedance**

$$\frac{V_i}{I_i} = \frac{V_r}{I_r} = Z_o \quad \Rightarrow \quad \frac{I_r}{I_i} = \frac{V_r}{V_i}$$

- **Substituting:**

$$Z_o\left(\frac{1 + V_r/V_i}{1 - V_r/V_i}\right) = Z_L$$

Define Reflection Coefficient

- **Definition:** $$\Gamma_L = \frac{V_r}{V_i}$$
  - **No reflection if $\Gamma_L = 0$**
- **Relation to load and characteristic impedances**

$$Z_o\left(\frac{1+\Gamma_L}{1-\Gamma_L}\right) = Z_L$$

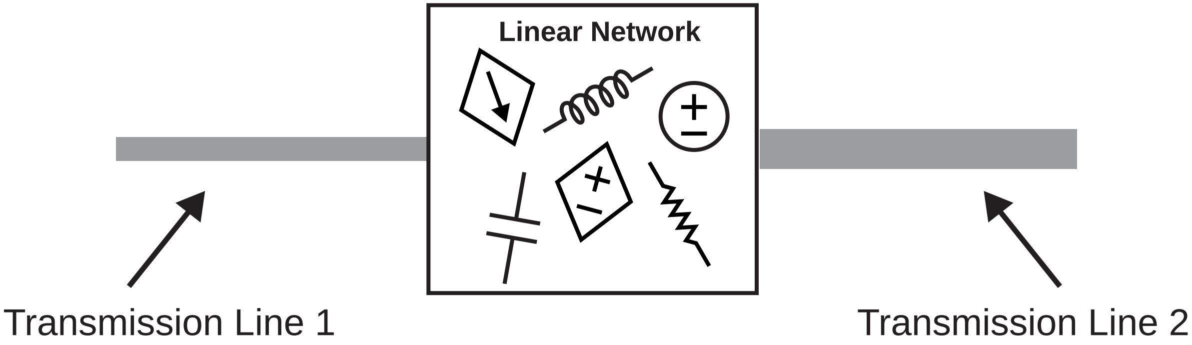
- **Alternate expression**

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

  - **No reflection if $Z_L = Z_o$**

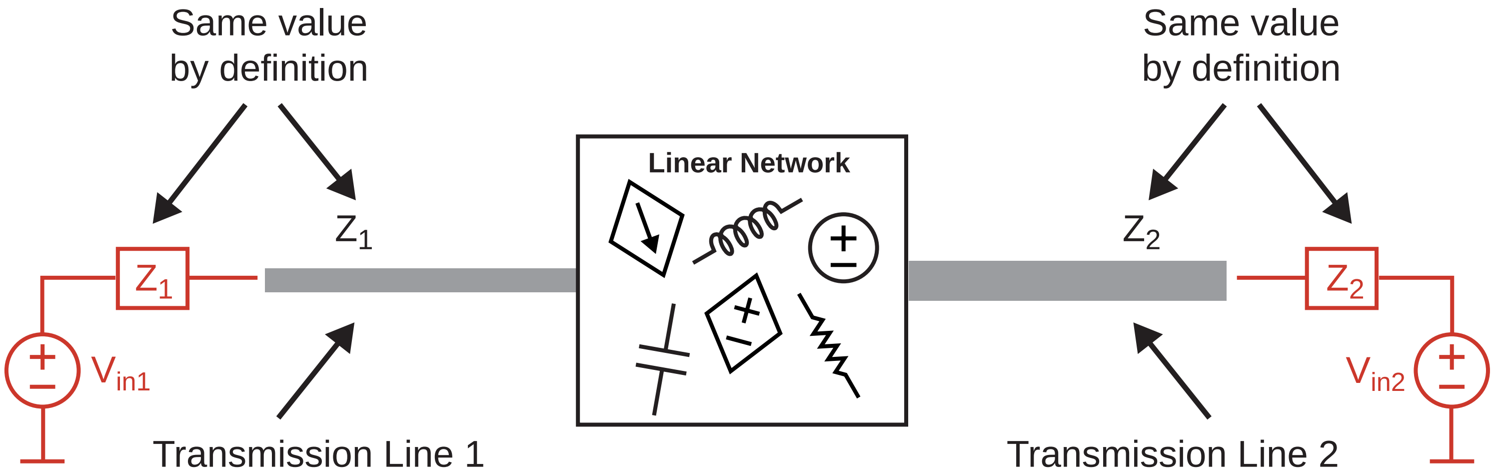
Parameterization of High Speed Circuits/Passives

- **Circuits or passive structures are often connected to transmission lines at high frequencies**
  - **How do you describe their behavior?**



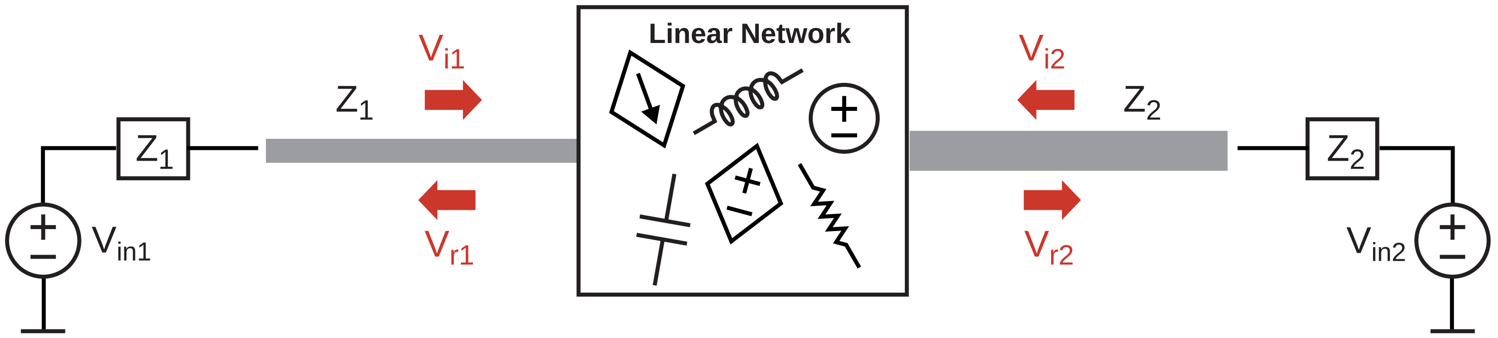
Calculate Response to Input Voltage Sources

- **Assume source impedances match their respective transmission lines**



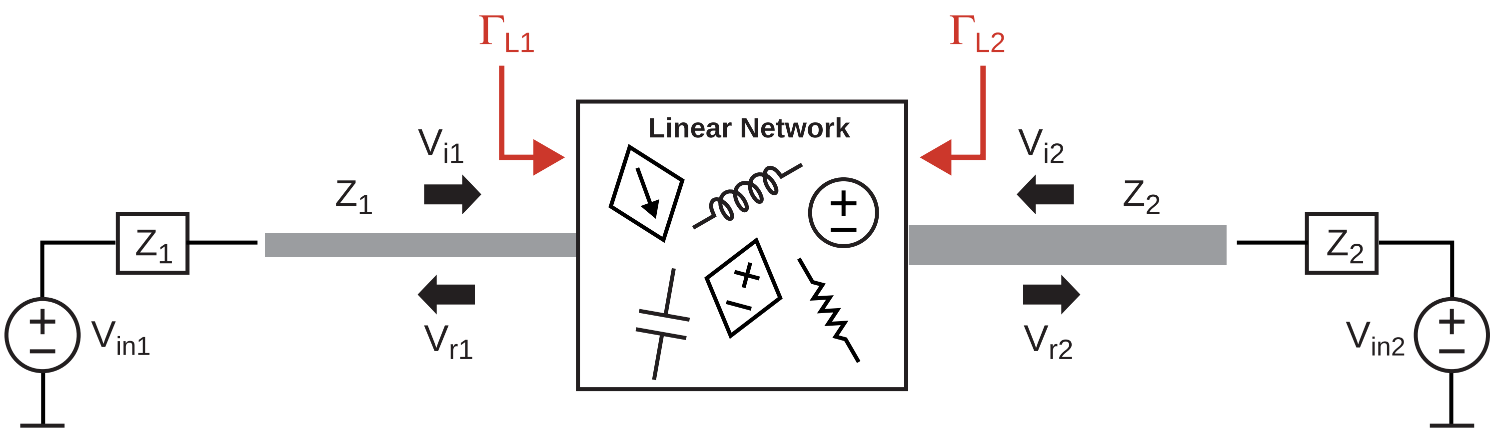
Calculate Response to Input Voltage Sources

- **Sources create incident waves on their respective transmission line**
- **Circuit/passive network causes**
  - **Reflections on same transmission line**
  - **Feedthrough to other transmission line**



Calculate Response to Input Voltage Sources

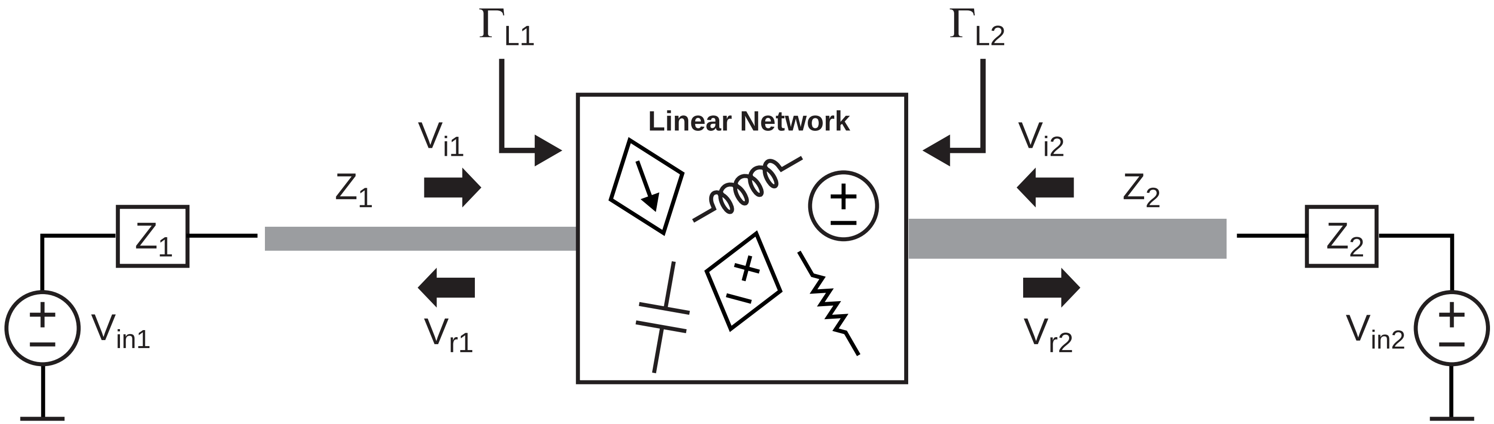
- **Reflections on same transmission line are parameterized by Γ_L**
  - **Note that Γ_L is generally different on each side of the circuit/passive network**



How do we parameterize feedthrough to the other transmission line?

S-Parameters – Definition

- **Model circuit/passive network using 2-port techniques**
  - **Similar idea to Thevenin/Norton modeling**

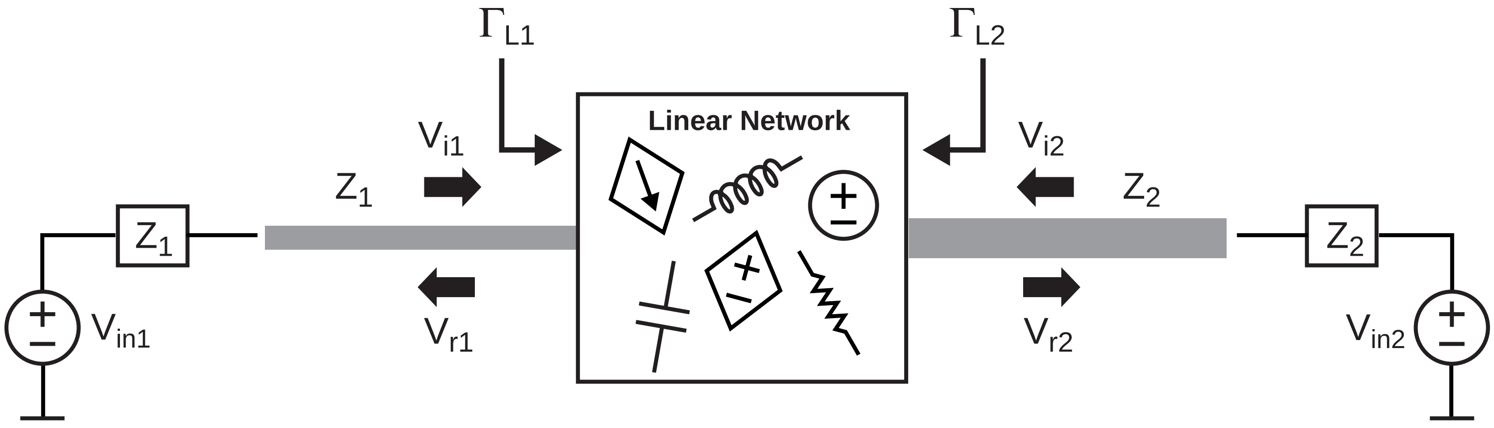


- **Defining equations:**

$$\frac{V_{r1}}{\sqrt{Z_1}} = S_{11}\frac{V_{i1}}{\sqrt{Z_1}} + S_{12}\frac{V_{i2}}{\sqrt{Z_2}}$$

$$\frac{V_{r2}}{\sqrt{Z_2}} = S_{21}\frac{V_{i1}}{\sqrt{Z_1}} + S_{22}\frac{V_{i2}}{\sqrt{Z_2}}$$

S-Parameters – Calculation/Measurement



$$\frac{V_{r1}}{\sqrt{Z_1}} = S_{11}\frac{V_{i1}}{\sqrt{Z_1}} + S_{12}\frac{V_{i2}}{\sqrt{Z_2}} \qquad \frac{V_{r2}}{\sqrt{Z_2}} = S_{21}\frac{V_{i1}}{\sqrt{Z_1}} + S_{22}\frac{V_{i2}}{\sqrt{Z_2}}$$

set $V_{in2} = 0$

$$\Rightarrow \quad S_{11} = \frac{V_{r1}}{V_{i1}} = \Gamma_{L1}$$

$$\Rightarrow \quad S_{21} = \sqrt{\frac{Z_1}{Z_2}}\left(\frac{V_{r2}}{V_{i1}}\right)$$

set $V_{in1} = 0$

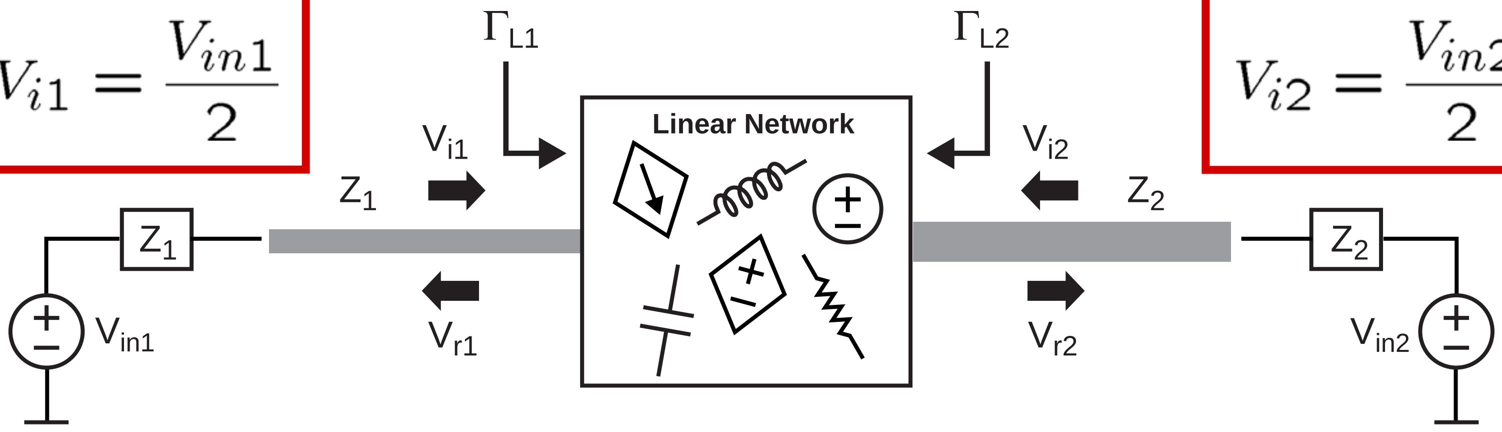
$$\Rightarrow \quad S_{22} = \frac{V_{r2}}{V_{i2}} = \Gamma_{L2}$$

$$\Rightarrow \quad S_{12} = \sqrt{\frac{Z_2}{Z_1}}\left(\frac{V_{r1}}{V_{i2}}\right)$$

Note: Alternate Form for S_{21} and S_{12}

$$V_{i1} = \frac{V_{in1}}{2}$$

$$V_{i2} = \frac{V_{in2}}{2}$$



set $V_{in2} = 0$

$$\Rightarrow \quad S_{11} = \frac{V_{r1}}{V_{i1}} = \Gamma_{L1}$$

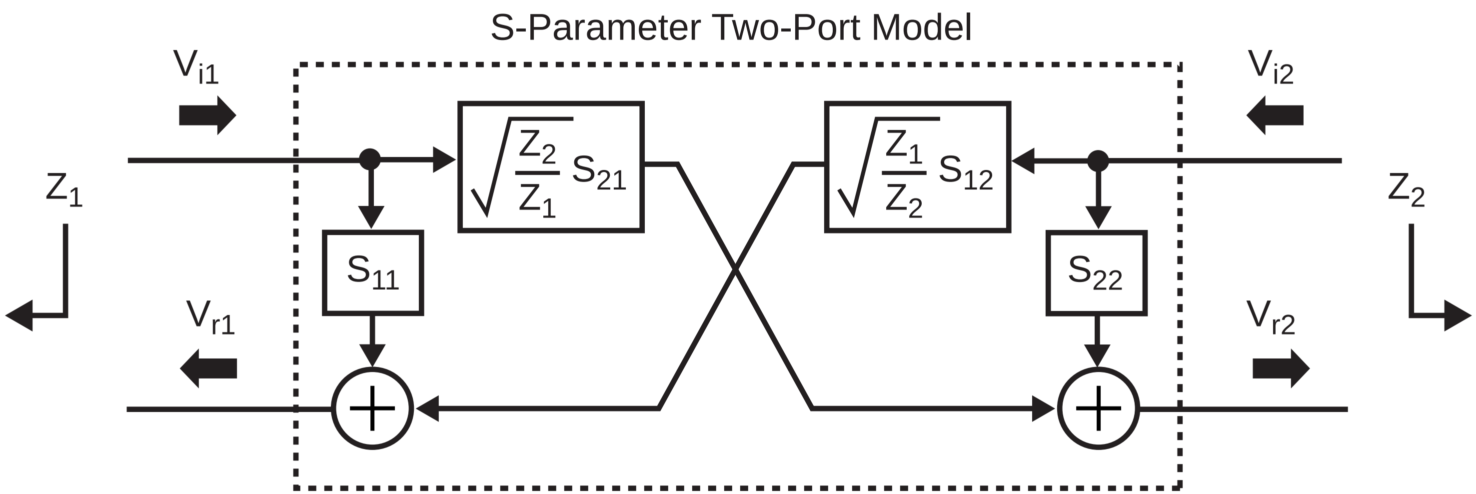
$$\Rightarrow \quad S_{21} = 2\sqrt{\frac{Z_1}{Z_2}}\left(\frac{V_{r2}}{V_{in1}}\right)$$

set $V_{in1} = 0$

$$\Rightarrow \quad S_{22} = \frac{V_{r2}}{V_{i2}} = \Gamma_{L2}$$

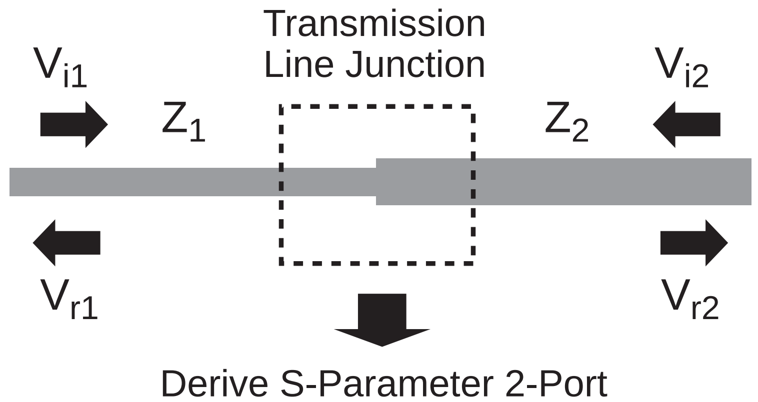
$$\Rightarrow \quad S_{12} = 2\sqrt{\frac{Z_2}{Z_1}}\left(\frac{V_{r1}}{V_{in2}}\right)$$

Block Diagram of S-Parameter 2-Port Model



- **Key issue – two-port is parameterized with respect to the left and right side load impedances (Z_1 and Z_2)**
  - **Need to recalculate S_{11}, S_{21}, etc. if Z_1 or Z_2 changes**
  - **Typical assumption is that $Z_1 = Z_2 = 50$ Ohms**

S-Parameter Calculations – Example 1



- **Set $V_{i2} = 0$**

$$V_{r1} = \Gamma_1 V_{i1} = \frac{Z_2 - Z_1}{Z_2 + Z_1} V_{i1}$$

$$V_{r2} = V_{i1} + V_{r1} = (1 + \Gamma_1) V_{i1}$$

$$\Rightarrow \quad S_{11} = \Gamma_1$$

$$\Rightarrow \quad S_{21} = \sqrt{\frac{Z_1}{Z_2}} (1 + \Gamma_1)$$

- **Set $V_{i1} = 0$**

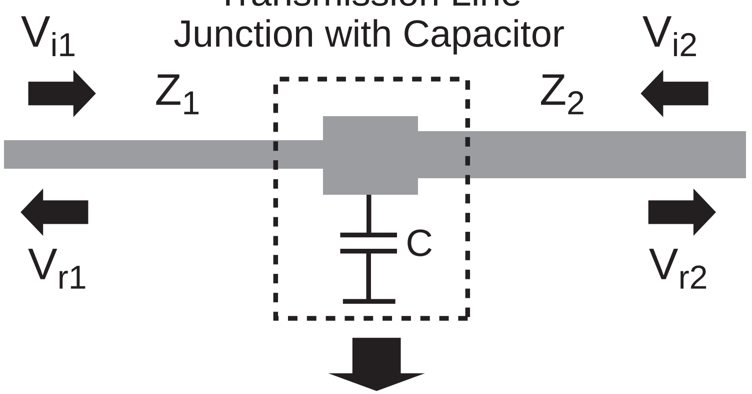
$$V_{r2} = \Gamma_2 V_{i2} = \frac{Z_1 - Z_2}{Z_1 + Z_2} V_{i1}$$

$$V_{r1} = V_{i2} + V_{r2} = (1 + \Gamma_2) V_{i2}$$

$$\Rightarrow \quad S_{22} = \Gamma_2$$

$$\Rightarrow \quad S_{12} = \sqrt{\frac{Z_2}{Z_1}} (1 + \Gamma_2)$$

S-Parameter Calculations – Example 2



- **Same as before:**

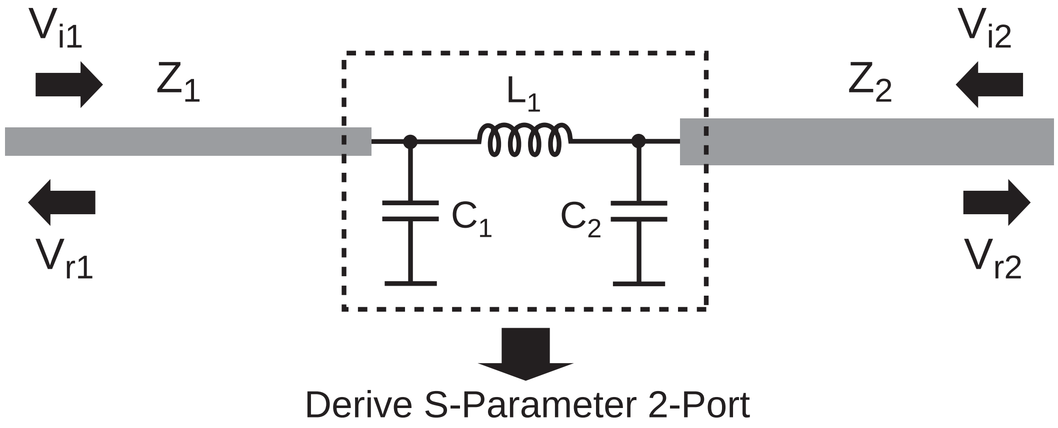
$$\Rightarrow \; S_{11} = \Gamma_1 \qquad\qquad \Rightarrow \; S_{22} = \Gamma_2$$

$$\Rightarrow \; S_{21} = \sqrt{\frac{Z_1}{Z_2}}(1+\Gamma_1) \qquad\qquad \Rightarrow \; S_{12} = \sqrt{\frac{Z_2}{Z_1}}(1+\Gamma_2)$$

- **But now:**

$$\Gamma_1 = \frac{Z_2||(1/sC) - Z_1}{Z_2||(1/sC) + Z_1} \qquad\qquad \Gamma_2 = \frac{Z_1||(1/sC) - Z_2}{Z_1||(1/sC) + Z_2}$$

S-Parameter Calculations – Example 3



- **The S-parameter calculations are now more involved**
  - **Network now has more than one node**
- **This is a homework problem**

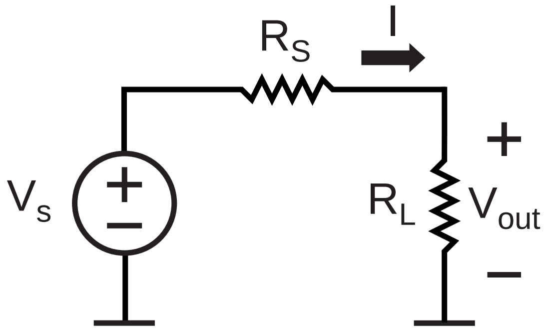
Impedance Transformers

Matching for Voltage versus Power Transfer

- **Consider the voltage divider network**

  Given the Thevenin equivalent source with V_s and R_s, how do we deliver maximum voltage or power to the load?

- **For maximum voltage transfer**



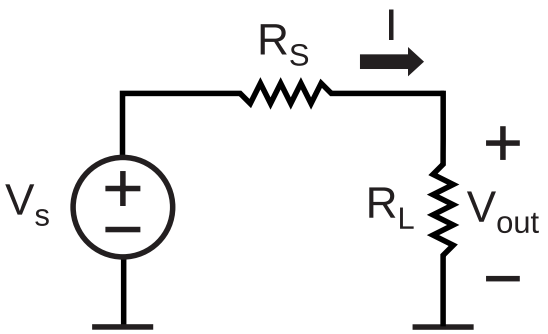
$$R_L \rightarrow \infty \quad \Rightarrow \quad V_{out} \rightarrow V_s$$

- **For maximum power transfer**

$$R_L = R_S \quad \Rightarrow \quad P_{out} = \frac{|V_{out}|^2}{R_L} = \frac{|V_s|^2}{4R_S}$$

Which one do we want?

Note: Maximum Power Transfer Derivation



- **Formulation: R_s is given, R_L is variable**

$$P_{out} = I^2 R_L = \left(\frac{V_s}{R_S + R_L}\right)^2 R_L = \frac{R_L}{(R_S + R_L)^2} V_s^2$$

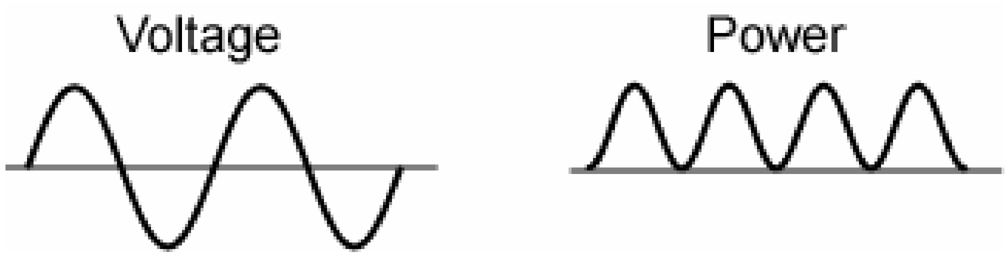
- **Take the derivative and set it to zero**

$$\frac{dP_{out}}{dR_L} = R_L(-2)(R_S + R_L)^{-3} + (R_S + R_L)^{-2} = 0$$

$$\Rightarrow \; 2R_L = R_S + R_L \;\; \Rightarrow \;\; R_L = R_S$$

Voltage Versus Power

- **For most communication circuits, voltage (or current) is the key signal for detection**
  - **Phase information is important**
  - **Power is ambiguous with respect to phase information**
    - Example:

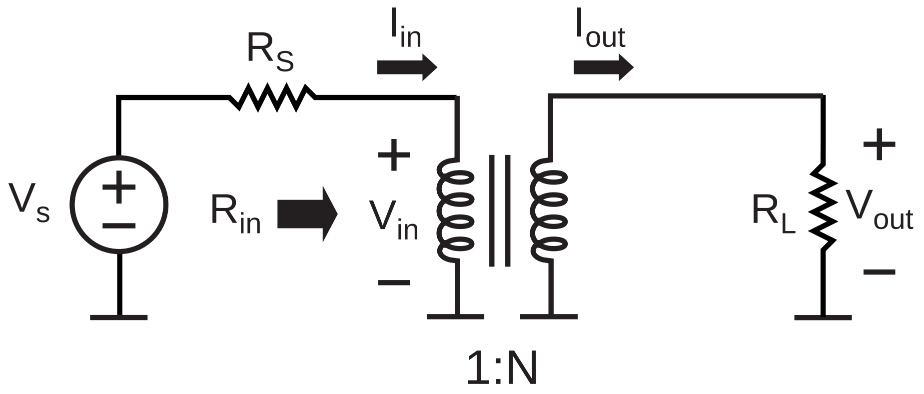


- **For high speed circuits with transmission lines, achieving maximum power transfer is important**
  - **Maximum power transfer coincides with having zero reflections (i.e., $\Gamma_L = 0$)**

Can we ever win on both issues?

Broadband Impedance Transformers

- **Consider placing an ideal transformer between source and load**



- **Transformer basics (passive, zero loss)**

$$1)\ V_{out} = NV_{in} \qquad 2)\ \text{Power In} = \text{Power Out}$$

$$\Rightarrow \quad V_{in}I_{in} = V_{out}I_{out}$$

$$\text{From (1) and (2):}\ V_{in}I_{in} = NV_{in}I_{out} \quad \Rightarrow \quad I_{out} = \frac{I_{in}}{N}$$

- **Transformer input impedance**

$$R_{in} = \frac{V_{in}}{I_{in}} = \frac{V_{out}/N}{NI_{out}} = \frac{1}{N^2}R_L$$

What Value of N Maximizes Voltage Transfer?

- Derive formula for V_{out} versus V_{in} for given N value

$$V_{out} = NV_{in} = N\frac{R_{in}}{R_s + R_{in}}V_s = N\frac{R_L/N^2}{R_s + R_L/N^2}V_s$$

$$= N\frac{R_L}{R_L + N^2 R_s}V_s$$

- Take the derivative and set it to zero

$$\frac{dV_{out}}{dN} = NR_L(-1)(R_L+N^2R_S)^{-2}2NR_s+R_L(R_L+N^2R_S)^{-1} = 0$$

$$\Rightarrow \quad -2N^2R_s(R_L+N^2R_S)^{-2}+(R_L+N^2R_S)^{-1} = 0$$

$$\Rightarrow \quad -2N^2R_s = R_L + N^2R_S \quad \Rightarrow \quad \boxed{N^2 = \frac{R_L}{R_s}}$$

What is the Input Impedance for Max Voltage Transfer?

- **We know from basic transformer theory that input impedance into transformer is**

$$R_{in} = \frac{1}{N^2} R_L$$

- **We just learned that, to maximize voltage transfer, we must set the transformer turns ratio to**

$$N^2 = \frac{R_L}{R_s}$$

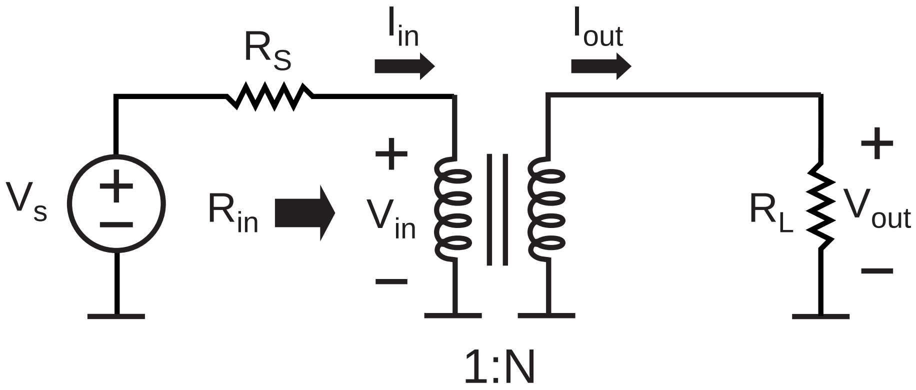
- **Put them together**

$$R_{in} = \frac{1}{N^2} R_L = \frac{1}{R_L/R_s} R_L = R_s \quad !!$$

So, N should be set for max power transfer into transformer to achieve the maximum voltage transfer at the load. This also ensures no reflection.

Benefit of Impedance Matching with Transformers

- **Transformers allow maximum voltage and power transfer relationship to coincide**



- **Turns ratio for max power/voltage transfer**

$$N^2 = \frac{R_L}{R_s}$$

- **Resulting voltage gain (can exceed one!)**

$$V_{out} = NV_{in} = N\left(\frac{1}{2}V_s\right) = \sqrt{\frac{R_L}{R_s}}\left(\frac{1}{2}V_s\right)$$

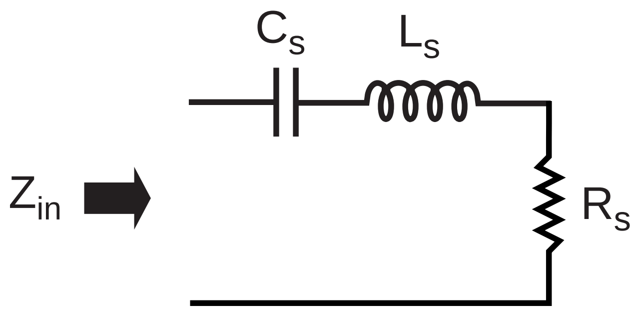
Problems with True Transformers

- It’s difficult to realize a transformer with good performance over a wide frequency range
  - Magnetic materials have limited frequency response
  (both low and high frequency limits)
  - Inductors have self-resonant frequencies, losses, and mediocre coupling to other inductors without magnetic material

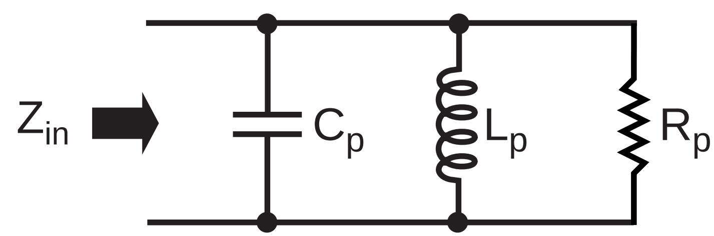
- For wireless applications, we only need transformers that operate over a small frequency range (except UWB)
  - Can we take advantage of this?: use ‘impedance transformer” instead of a true transformer

Consider Resonant Circuits (Chap. 3 (2nd ed.) or 4 (1st ed.) of Text)

Series Resonant Circuit



Parallel Resonant Circuit



$$Z_{in} = \frac{1}{jwC_s} + jwL_s + R_s$$

$$= R_s \text{ for } w = \frac{1}{\sqrt{L_sC_s}} = w_o$$

$$Q = \frac{w_oL_s}{R_s} = \frac{1}{w_oC_sR_s}$$

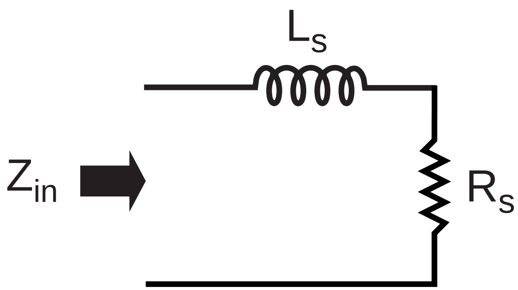
$$Z_{in} = \frac{1}{jwC_p}||jwL_p||R_p$$

$$= R_p \text{ for } w = \frac{1}{\sqrt{L_pC_p}} = w_o$$

$$Q = \frac{R_p}{w_oL_p} = w_oC_pR_p$$

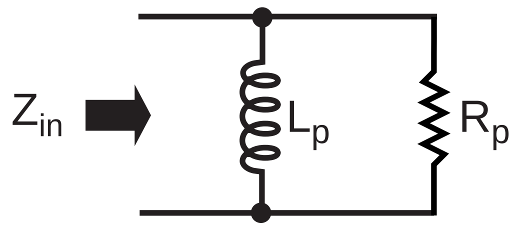
- **Key insight: at resonance Z_{in} becomes purely real despite the presence of reactive elements**

Equivalence of Series and Parallel RL Circuits



$$Q = \frac{w_o L_s}{R_s}$$

$$Z_{in} = j w_o L_s + R_s$$



$$Q = \frac{R_p}{w_o L_p}$$

$$Z_{in} = j w_o L_p || R_p$$

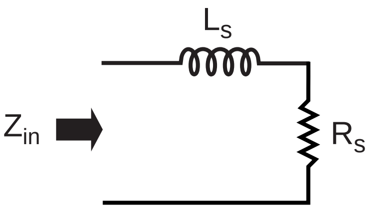
- Equate real and imaginary parts of the left and right expressions (so that Z_{in} is the same for both)
  - Also equate Q values

$$R_p = R_s(Q^2 + 1) \approx R_s Q^2 \ \ (\text{for } Q \gg 1)$$

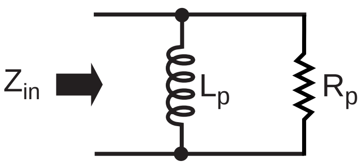
$$L_p = L_s \left(\frac{Q^2 + 1}{Q^2}\right) \approx L_s \ \ (\text{for } Q \gg 1)$$

Series-Parallel Equivalence Analysis

Series RL Circuit



Parallel RL Circuit



$$Z_{inp} = \frac{j\omega_o L_p R_p}{j\omega_o L_p R_p + R_p} = \frac{j\omega_o L_p R_p(-j\omega_o L_p R_p + R_p)}{\omega_o^2 L_p^2 + R_p^2}$$

$$= \frac{\omega_o^2 L_p^2 R_p + j\omega_o L_p^2 R_p^2}{\omega_o^2 L_p^2 + R_p^2}$$

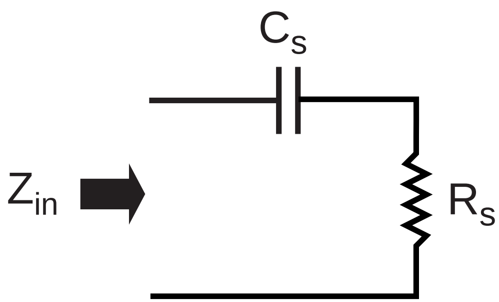
$$R_s = \frac{\omega_o^2 L_p^2 R_p}{\omega_o^2 L_p^2 + R_p^2} = \frac{R_p}{1 + Q_p^2}$$

$$L_s = \frac{L_p^2 R_p^2}{\omega_o^2 L_p^2 + R_p^2} = L_p \frac{1}{\frac{\omega_o^2 L_p^2}{R_p^2} + 1} = L_p \frac{1}{1 + \frac{1}{Q_p^2}}$$

$$Q_s = \frac{\omega_o L_s}{R_s} = \frac{\omega L_p Q_p^2}{R_p} = Q_p = Q$$

Equivalence of Series and Parallel RC Circuits

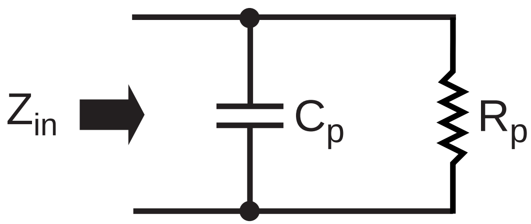
Series RC Circuit



$$Q = \frac{1}{w_o C_s R_s}$$

$$Z_{in} = R_s + \frac{1}{j w_o C_s}$$

Parallel RC Circuit



$$Q = w_o C_p R_p$$

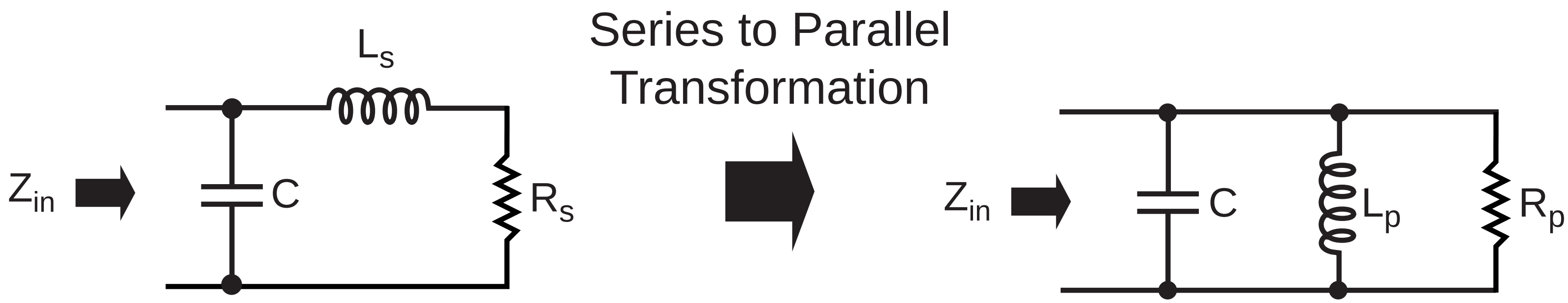
$$Z_{in} = R_p || \frac{1}{j w_o C_p}$$

- **Equate real and imaginary parts of the left and right expressions (so that Z_{in} is the same for both)**
  - **Also equate Q values**

$$R_p = R_s(Q^2 + 1) \approx R_s Q^2 \ \ (\text{for } Q \gg 1)$$

$$C_p = C_s \left(\frac{Q^2}{Q^2 + 1} \right) \approx C_s \ \ (\text{for } Q \gg 1)$$

A Narrowband Impedance Transformer: The L Match



$$R_p = R_s(Q^2 + 1) \approx R_s Q^2 \quad (\text{for } Q \gg 1)$$

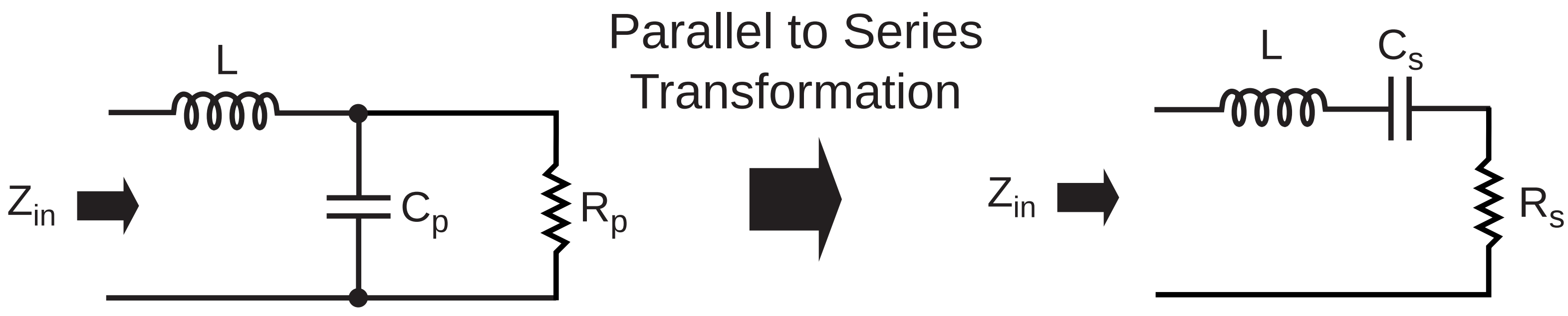
$$L_p = L_s\left(\frac{Q^2+1}{Q^2}\right) \approx L_s \quad (\text{for } Q \gg 1)$$

- **At resonance**

$$Z_{in} = R_p = (1 + Q^2)R_s \approx Q^2 R_s \quad (\text{purely real})$$

- **Transformer steps up impedance!**

Alternate Implementation of L Match



$$R_p = R_s(Q^2+1) \approx R_s Q^2 \quad (\text{for } Q \gg 1)$$

$$C_p = C_s\left(\frac{Q^2}{Q^2+1}\right) \approx C_s \quad (\text{for } Q \gg 1)$$

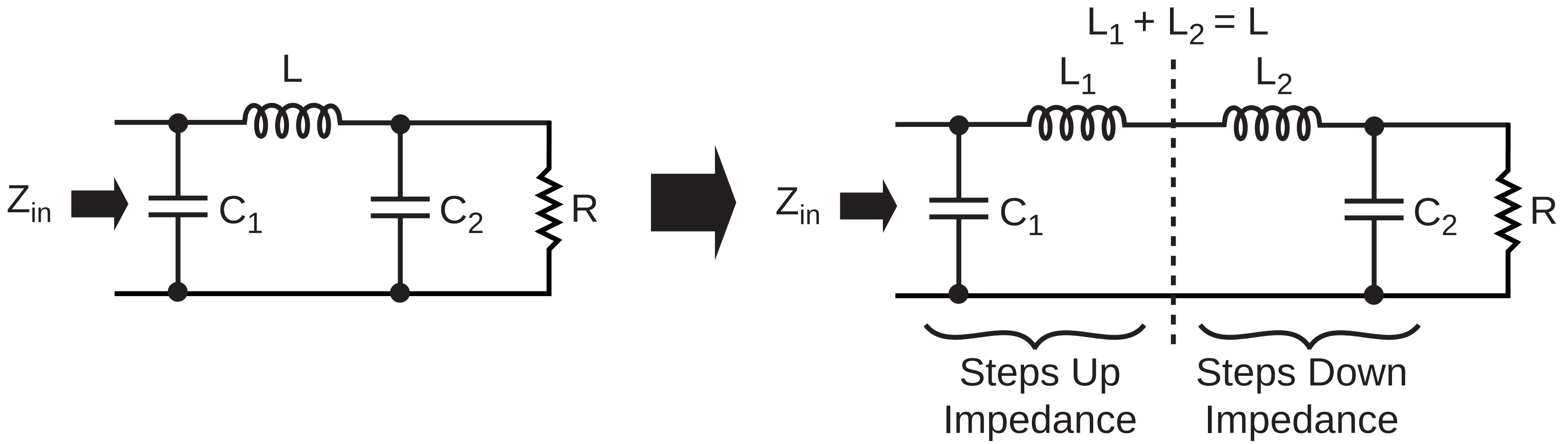
- **At resonance**

$$Z_{in} = R_s = \frac{R_p}{1+Q^2} \approx \frac{R_p}{Q^2} \quad (\text{purely real})$$

- **Transformer steps down impedance!**

The π Match

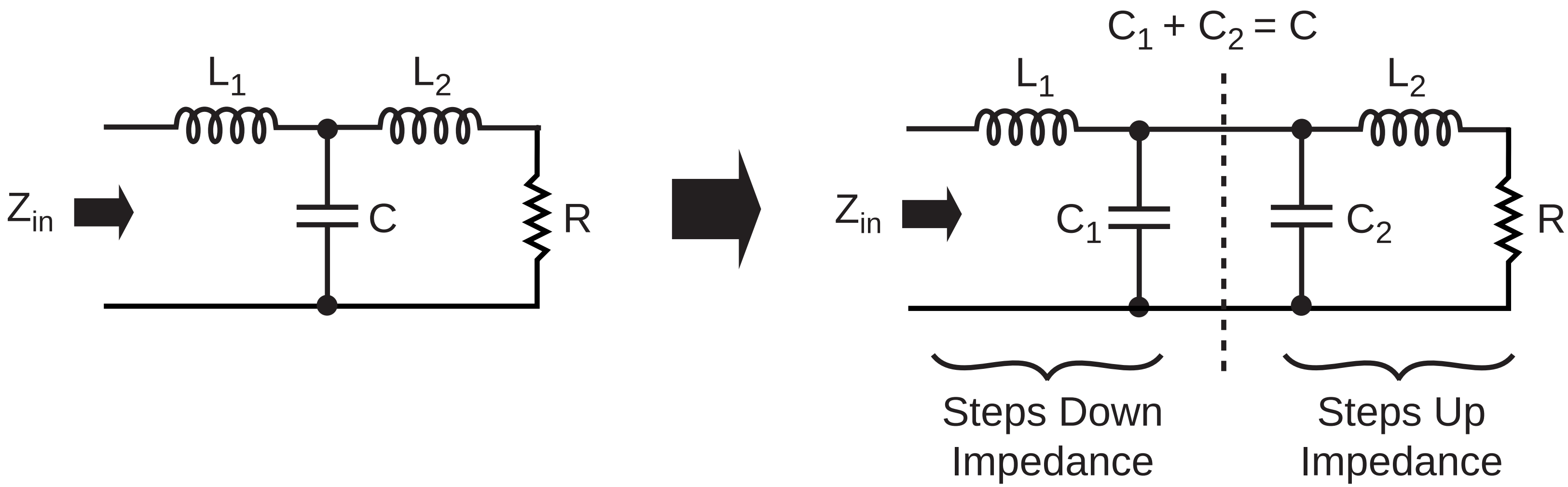
- **Combines two L sections**



- **Provides an extra degree of freedom for choosing component values for a desired transformation ratio**

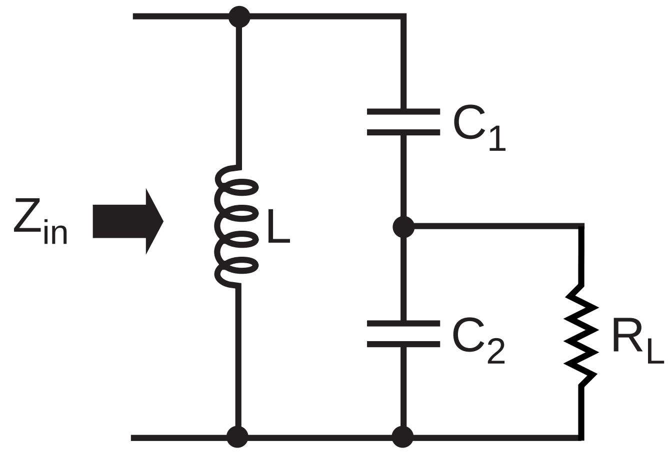
The T Match

- **Also combines two L sections**



- **Again, benefit is in providing an extra degree of freedom in choosing component values**

Tapped Capacitor as a Transformer



- **To first order:**

$$\frac{R_{in}}{R_L} \approx \left(\frac{C_1 + C_2}{C_1}\right)^2$$

- **Useful in VCO design**
- **See Chap. 3 (2nd ed.) or 4 (1st ed.) of Text**