6.728 Applied Quantum and Statistical Physics:

Department of Electrical Engineering and Computer Science
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PROBLEM SET 5

Problem Set Out: 10/4/06   Problem Set Due: 10/11/06 at the beginning of class

PROBLEMS

Problem 5.1 Text Problem 10.3

Problem 5.2 Text Problem 10.4

Problem 5.3 Text Problem 11.2 parts (a) and (b) only.

Problem 5.4 *This problem is NOT to be turned in.* Play with the demos for the oscillating Gaussians in the SHO in the two problems on the reverse of this page. Solutions will be provided for you.

   Quiz 1 will be on Monday October 16th during recitation time 11am-noon. It will cover material up to and including Chapter 12. As a reminder, you will turn in your quiz solutions at the end of class, and then on Wednesday at the beginning of class you will turn in the solutions you have done again outside of class. Your grade will be the average of the two grades.

The following are short demonstrations of the gaussian wave packet in the simple harmonic oscillator potential.

Demo 5.1

Harmonic oscillator with Matlab . Copy the the matlab files. programs `sho.m` and `herm.m`. This program places a gaussian wave packet in a simple harmonic oscillator potential. You can control the width of the initial packet as well as its displacement. The potential is such that the first eigenstate (which is gaussian) has a width of 1nm.

1. Define our initial wave packet width, $L = 1\,\mathrm{nm}$. Let the wave packet be initially centered in the potential. Describe and explain the time evolution. What would a plot of $|C_n|^2$ vs. n, where C_n are the coefficients of the eigenfunction expansion look like?

2. Now, consider our initial gaussian wave packet to be a width of 1nm, and initially displaced 2nm to the right of the well. What, in general terms, will happen to the plot of $|C_n|^2$ vs. n?

3. Plot $\langle x \rangle$ vs. t. Graphically determine the frequency of oscillation. Compare to the results obtained by using Ehrenfest's Theorem.

4. Plot $(\Delta x)^2$ vs. t. Comment on and explain this plot.

Demo 5.2

Now, we want to consider a gaussian wave packet whose width is wider than the first eigenfunction of the system. Define our initial wave packet width, $L = 1.5\,\mathrm{nm}$. Let the initial center of the wavepacket be displaced 3nm to the right of center.

1. Argue what the time evolution of the packet should look like. Explain your reasoning.

2. Plot $\langle x \rangle$ vs. t. Graphically determine the frequency of oscillation. Compare to the results obtained by using Ehrenfest's Theorem, and that of the previous problem . Comment.

3. Plot $(\Delta x)^2$ vs. t. Comment on this plot.

4. Plot 2-D `surf` plot of the probability density as the x vs. t graph. Can you identify the effects shown in parts 2 and 3 in this graph?