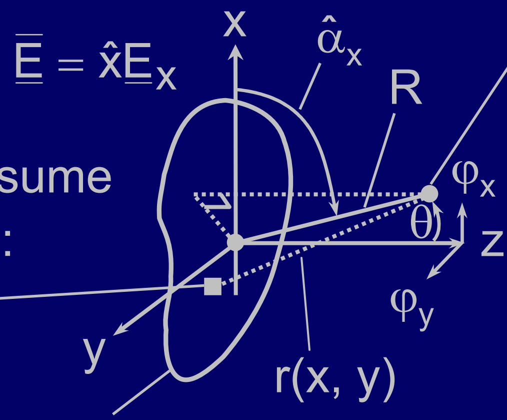
Aperture Radiation: Huygen's Equation

Radiating patch, assume uniform plane wave: $\overline{\underline{E}} \perp \overline{\underline{H}}$, $\underline{E}/\underline{H} = 377\Omega$



Superposition of contributions from radiating patches

Aperture in X-Y plane "A"

$\left.\begin{matrix}\overline{\underline{E}}\\ \overline{\underline{H}}\end{matrix}\right\} \rightarrow \overline{\underline{J}}_S$ surface current $\rightarrow \overline{\underline{E}}_{ff}$ radiated by integral of current elements

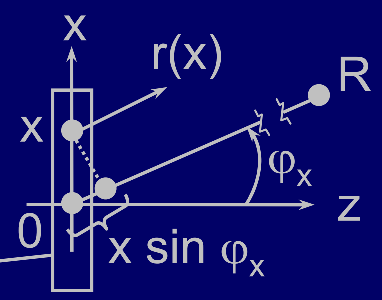
Huygen's superposition integral

$$\overline{\underline{E}}_{eff}(\theta,\phi,R) \cong \frac{j}{2R\lambda}(1+\cos\theta)(-\hat{\alpha}_x)\int_A \underline{E}_x(x,y)\underbrace{e^{-j(2\pi/\lambda)r(x,y)}}_{\text{phase lag }(x,y)}dxdy$$

Huygen's Equation: Geometric approximations

$$\overline{E}_{eff}(\theta,\phi,R) \cong \frac{j}{2R\lambda}(1+\cos\theta)(-\hat{\alpha}_x)\int_A \overline{E}_x(x,y)\underbrace{e^{-j(2\lambda/\pi)r(x,y)}}_{\text{phase lag }(x,y)}dxdy$$

2-D case



Aperture in x-y plane

For $\varphi_x, \varphi_y << 1$: $r(x,y) \cong R - x\sin\varphi_x - y\sin\varphi_y \cong R - x\varphi_x - y\varphi_y$

Thus $$\overline{E}(\theta,\phi,R) \cong \underbrace{\frac{j\,e^{-j\frac{2\pi}{\lambda}R}}{2R\lambda}(1+\cos\theta)\hat{\alpha}_x}_{\overline{K}} \bullet \int_A \overline{E}_x(x,y)e^{+j\frac{2\pi}{\lambda}(x\varphi_x + y\varphi_y)}dxdy$$

Huygen's Equation: Geometric approximations

Thus $$\underline{\underline{E}}(\theta,\phi,R) \cong \underbrace{\frac{j\,e^{-j\frac{2\pi}{\lambda}R}}{2R\lambda}(1+\cos\theta)\hat{\alpha}_x}_{\underline{\overline{K}}} \bullet \int_A \underline{E}_x(x,y)\,e^{+j\frac{2\pi}{\lambda}(x\varphi_x+y\varphi_y)}\,dxdy$$

$$\underline{\overline{E}}(\varphi_x,\varphi_y)\left(vm^{-1}\right) \cong \underline{\overline{K}}\int_A \underline{E}_x(x,y)\,e^{+j\frac{2\pi}{\lambda}(x\varphi_x+y\varphi_y)}\,dxdy$$

$$\hat{x}\underline{E}_x(x,y)\left(vm^{-1}\right) \cong \frac{\hat{x}}{\underline{K}}\int_{2\pi} E_x(\varphi_x,\varphi_y)\,e^{-j\frac{2\pi}{\lambda}(x\varphi_x+y\varphi_y)}\,d\varphi_x d\varphi_y$$

Huygen's Equation: Geometric approximations

$$\underline{\overline{E}}(\varphi_x,\varphi_y)(vm^{-1}) \cong \underline{\overline{K}}\int_A \underline{E}_x(x,y)e^{+j\frac{2\pi}{\lambda}(x\varphi_x+y\varphi_y)}dxdy$$

$$\hat{x}\underline{E}_x(x,y)(vm^{-1}) \cong \frac{\hat{x}}{\underline{K}}\int_{2\pi} \underline{E}_x(\varphi_x,\varphi_y)e^{-j\frac{2\pi}{\lambda}(x\varphi_x+y\varphi_y)}d\varphi_x d\varphi_y$$

$$\text{Let } x_\lambda \triangleq \frac{x}{\lambda},\ y_\lambda = \frac{y}{\lambda},\ 1+\cos\theta \cong 2$$

$$\underline{\overline{E}}(\varphi_x,\varphi_y)(vm^{-1}) \cong \lambda^2\underline{\overline{K}}\int_A \underline{E}_x(x_\lambda,y_\lambda)e^{+j2\pi(x_\lambda\varphi_x+y_\lambda\varphi_y)}dx_\lambda dy_\lambda$$

$$\hat{x}\underline{E}(x_\lambda,y_\lambda)(vm^{-1}) \cong \frac{\hat{x}\lambda^2}{\underline{K}}\int_{2\pi} \underline{E}_x(\varphi_x,\varphi_y)e^{-j2\pi(x_\lambda\varphi_x+y_\lambda\varphi_y)}d\varphi_x d\varphi_y$$

This is a Fourier transform pair

$$\left[\text{Recall } \underline{X}(f) = \int x(t)e^{-j2\pi ft}dt;\ x(t) = \int \underline{X}(f)e^{+j2\pi ft}df\right]$$

Fourier Transform Relations

Thus

Aperture (pulse signal)

$$\underline{\underline{E}}(x,y)\left(\mathrm{Vm}^{-1}\right) \quad \tilde{\leftrightarrow} \quad \underline{\underline{E}}_x\left(\varphi_x,\varphi_y\right)\left[\mathrm{Vm}^{-1}\right] \text{at R}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$R_{\underline{\underline{E}}_x}\left(\tau_{\lambda_x},\tau_{\lambda_y}\right)\left[\mathrm{Vm}^{-1}\right]^2 \quad \tilde{\leftrightarrow} \quad \left|\underline{\underline{E}}_x\left(\varphi_x,\varphi_y\right)\right|^2\left[\mathrm{Vm}^{-1}\right]^2 \text{at R}$$

$$\updownarrow$$

$$\propto S\left(\varphi_x,\varphi_y\right)=\frac{\left|\underline{\underline{E}}_x\left(\varphi_x,\varphi_y\right)\right|^2}{2\eta_o}\left[\mathrm{W\ m}^{-2}\right]$$

Directivity D(θ, ϕ) of an Aperture Antenna

Let P = radiation intensity and
P_{TR} = total power radiated (W)

$$D(\theta,\phi) \triangleq \frac{P(\theta,\phi,f,R)}{P_{TR}/4\pi R^2} \frac{[\mathrm{W\ m^{-2}}]}{[\mathrm{W\ m^{-2}}]}$$

$(\varphi_x, \varphi_y \ll 1)$

$$D \cong \frac{(1+\cos\theta)^2}{2\eta_o (2R\lambda)^2} \frac{\left| \int_A \underline{E}_x(x,y) e^{j\frac{2\pi}{\lambda}(x\varphi_x + y\varphi_y)} dx\,dy \right|^2}{\frac{1}{2\eta_o} \int_A \left| \underline{E}_x(x,y) \right|^2 dx\,dy \Big/ 4\pi R^2}$$

$$D = \frac{\pi(1+\cos\theta)^2}{\lambda^2} \frac{\left| \int_A \underline{E}_x(x,y) e^{j\frac{2\pi}{\lambda}(x\varphi_x + y\varphi_y)} dx\,dy \right|^2}{\int_A \left| \underline{E}_x(x,y) \right|^2 dx\,dy}$$

Directive Gain $D(\theta,\phi)$ of an Aperture Antenna

$$D = \frac{\pi(1+\cos\theta)^2}{\lambda^2} \frac{\left| \int_A \underline{E}_x(x,y) e^{j\frac{2\pi}{\lambda}(x\varphi_x + y\varphi_y)} dx\,dy \right|^2}{\int_A \left|\underline{E}_x(x,y)\right|^2 dx\,dy}$$

Bounds on $D(\varphi_x,\varphi_y)$, $A(\varphi_x,\varphi_y)$

Recall "Schwartz Inequality"

$$\left|\int f\,g\,dx\right|^2 \le \left(\int |f|^2 dx\right)\left(\int |g|^2 dx\right)$$

$A_o(m^2)$ is physical area of aperture

Therefore: $$\left|\int_A \underline{E}_x e^{j[\;]}\,dx\,dy\right|^2 \le \left(\int \left|\underline{E}_x\right|^2 dx\,dy\right)\underbrace{\left(\int_A 1^2\,dx\,dy\right)}_{A_o}$$

$$D(\varphi_x,\varphi_y) \le \frac{4\pi}{\lambda^2} \bullet \frac{\int_A \left|\underline{E}_x\right|^2 dx\,dy \bullet A_o}{\int_A \left|\underline{E}_x\right|^2 dx\,dy} = \frac{4\pi A_o}{\lambda^2}$$

Directive Gain $D(\theta, \phi)$ of an Aperture Antenna

$$D(\varphi_x, \varphi_y) \le \frac{4\pi}{\lambda^2} \bullet \frac{\int_A |\underline{E}_x|^2 \, dx\, dy \bullet A_o}{\int_A |\underline{E}_x|^2 \, dx\, dy} = \frac{4\pi A_o}{\lambda^2}$$

$$\text{But } D = \frac{4\pi}{\lambda^2} \bullet \frac{A_e(\varphi_x, \varphi_y)}{\eta_R} \Rightarrow$$

$$A_e(\varphi_x, \varphi_y) \text{ (effective area)} \le \eta_R A_o$$

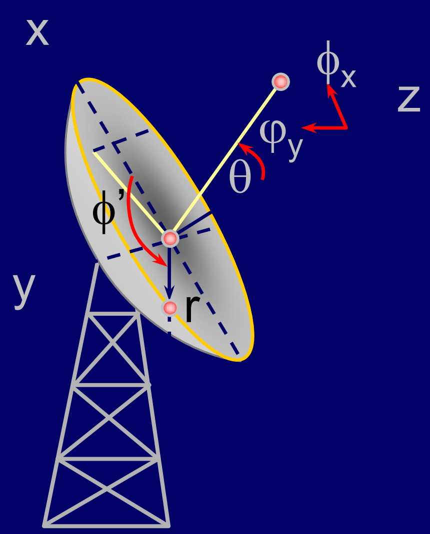
where radiation efficiency $\eta_R \le 1.0$

Define "aperture efficiency" η_A

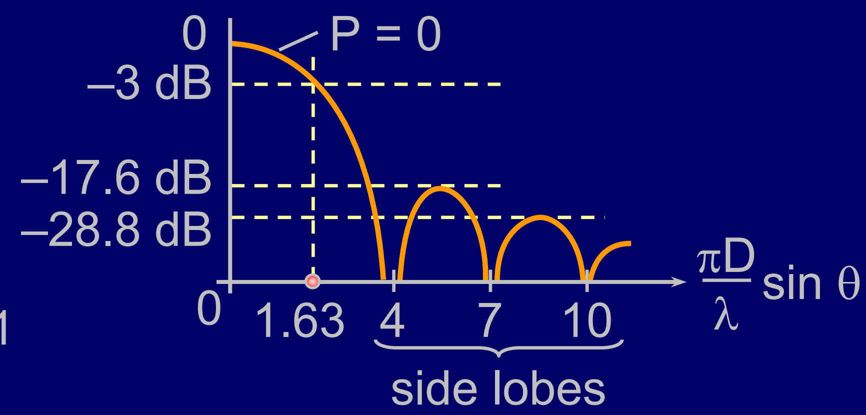
$$\eta_A \triangleq \frac{A_e(\max)}{\eta_R A_o} \cong 0.65 \text{ in practice; } = 1 \text{ for uniform illumination}$$

$$\text{Therefore } A_e = \eta_A \bullet \eta_R A_o$$

Uniformly Illuminated Circular Aperture Antennas



$$D = \frac{\pi(1+\cos\theta)^2}{\lambda^2}\,\frac{\left|\int_A 1\bullet e^{j\frac{2\pi}{\lambda}(x\varphi_x + y\varphi_y)}\, r\,dr\,d\phi'\right|}{\int_A |E|^2\, r\,dr\,d\phi'}$$



Aperture coordinates = r, ϕ'

Source coordinates = φ_x, φ_y for $\theta << 1$

$$D(f,\theta,\phi) = \left[\frac{\pi D}{\lambda}(1+\cos\theta)\right]^2 \Lambda_1^2\left(\frac{\pi D}{\lambda}\sin\theta\right)$$

"Lambda function"

$$= \left(\frac{\pi D}{\lambda}\right)^2 = 4\pi A_o/\lambda^2 \quad \text{at} \quad \theta = 0$$

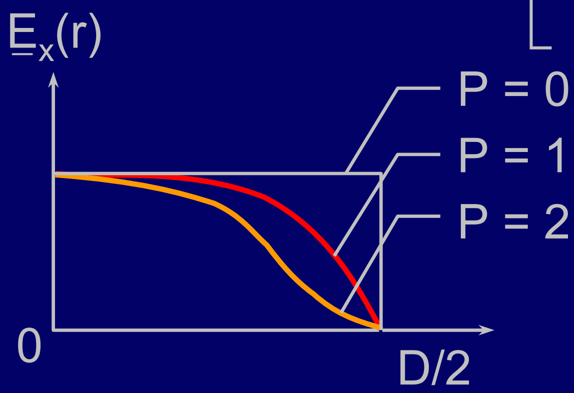
where

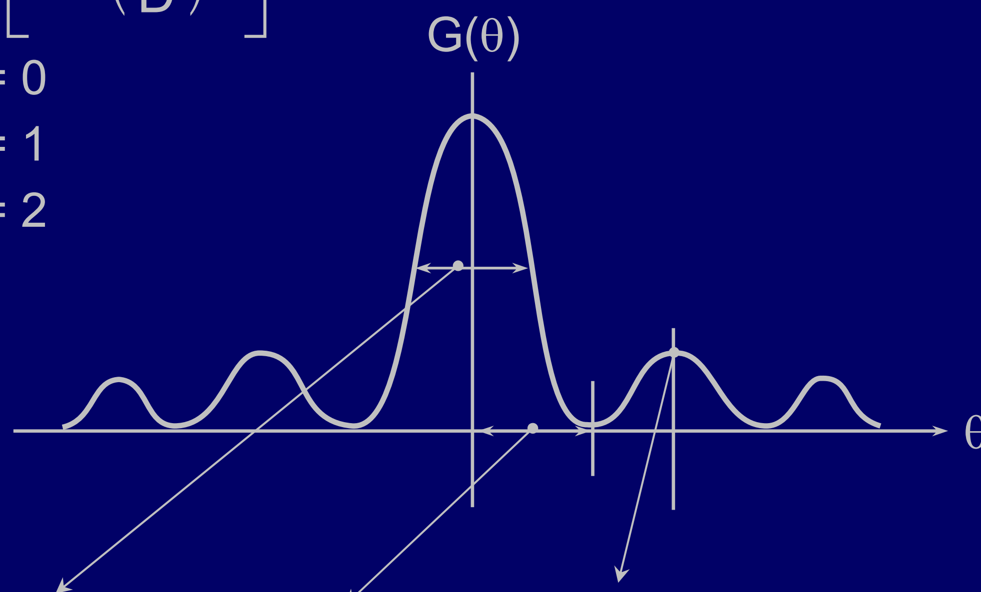
$\Lambda_1(q) = J_1(q)/q$

"Bessel function of first kind"

Non-Uniformly Illuminated Circular Apertures

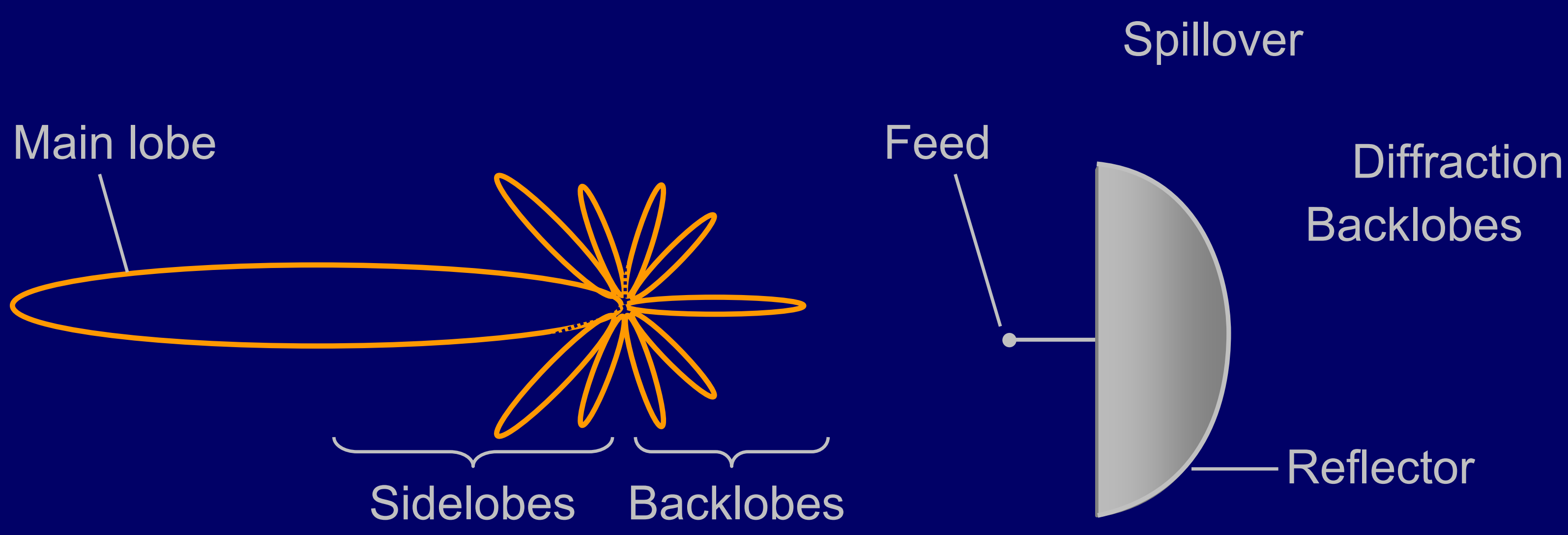
Assume $E_x(r) = \left[1 - \left(\frac{2r}{D}\right)^2\right]^P$





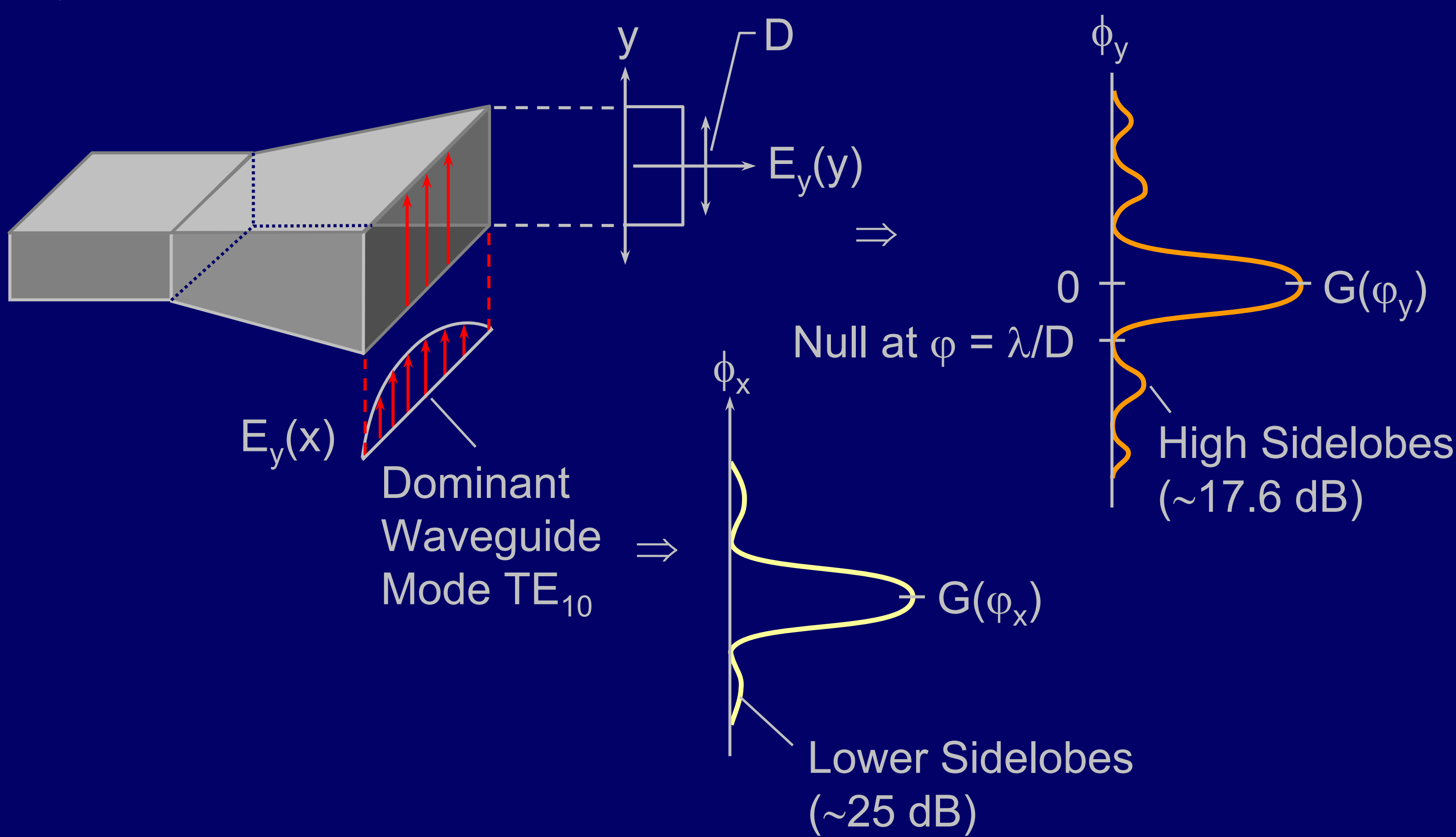
	P	$\theta_{B1/2}$	θ_{NULL} #1	First Side - Lobe	η_A
	0	1.02 λ/D	1.22 λ/D	17.6 dB	1.00
More typical ➡	1	1.27 λ/D	1.63 λ/D	24.6 dB	0.75
	3	1.47 λ/D	2.93 λ/D	30.4 dB	0.56

Sidelobes and Backlobes of Aperture Antennas

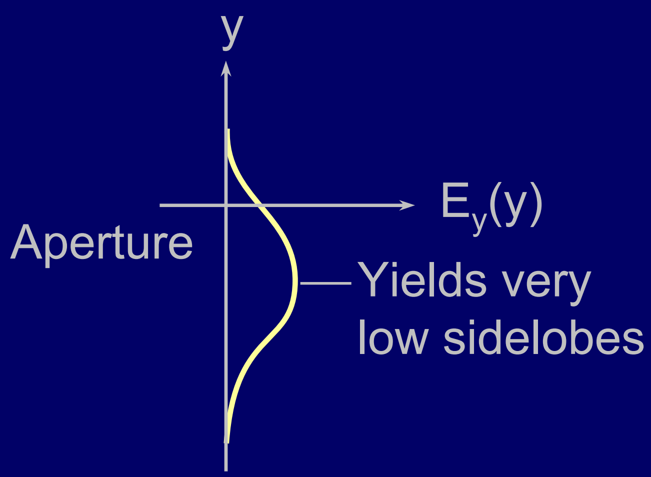


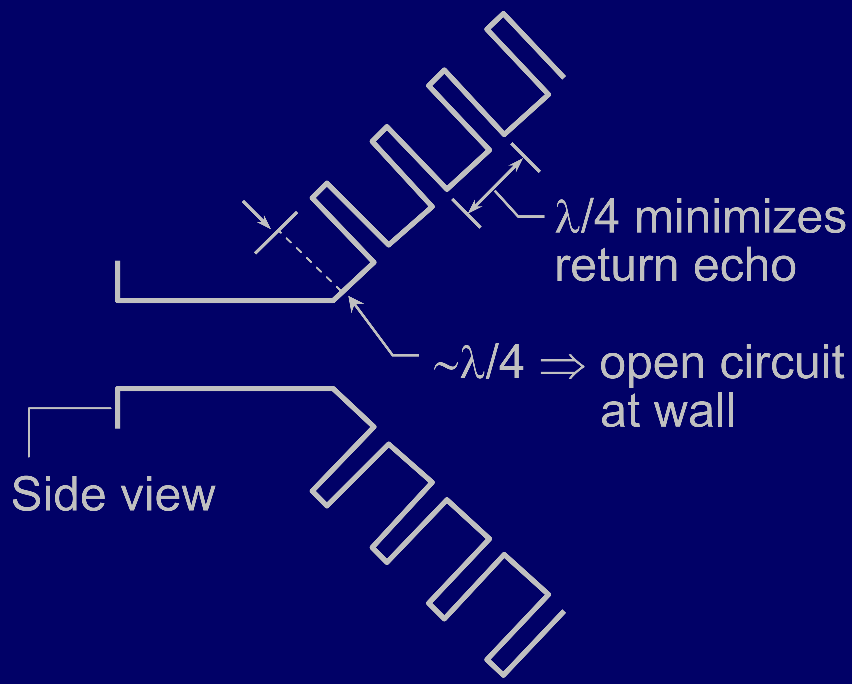
Waveguide Horn Feeds

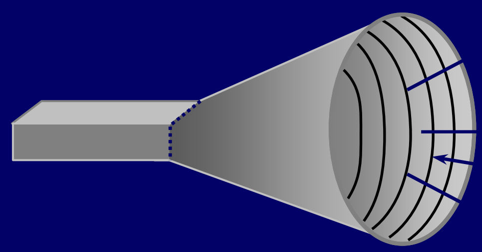
Pyramidal Horn



"Scalar" Feed

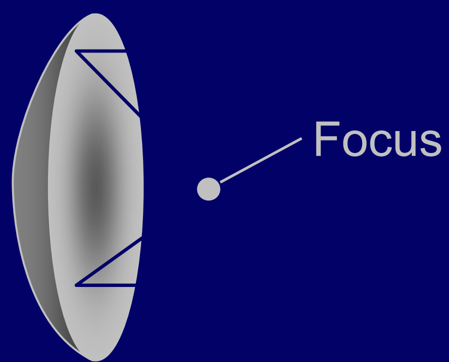




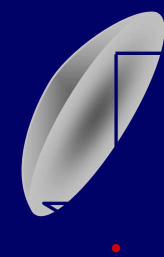


λ/4 grooves cut into wall

Examples of Parabolic Reflector Antennas

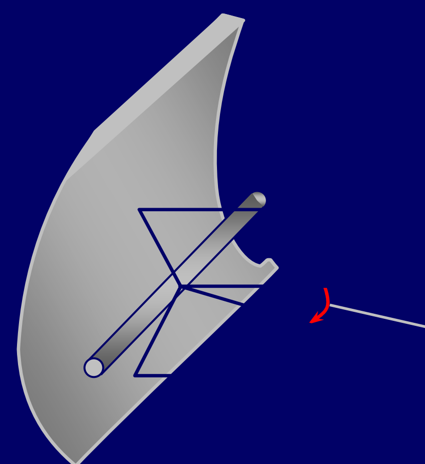


Circularly Symmetric
Parabolic Reflector



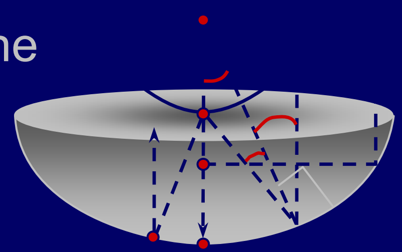
No aperture blockage

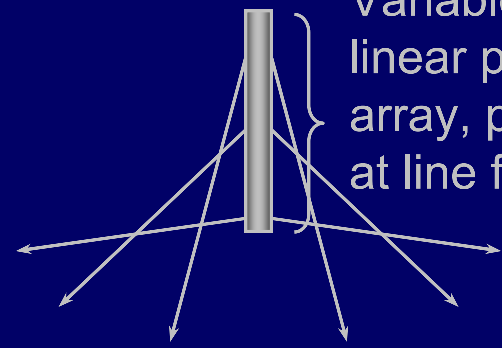
"Off-Axis Paraboloid"

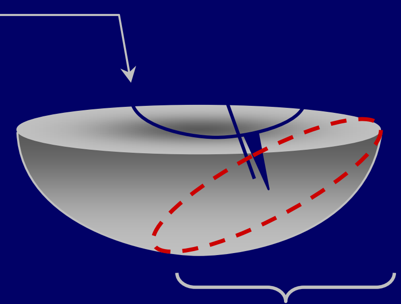


Cylindrical Parabola

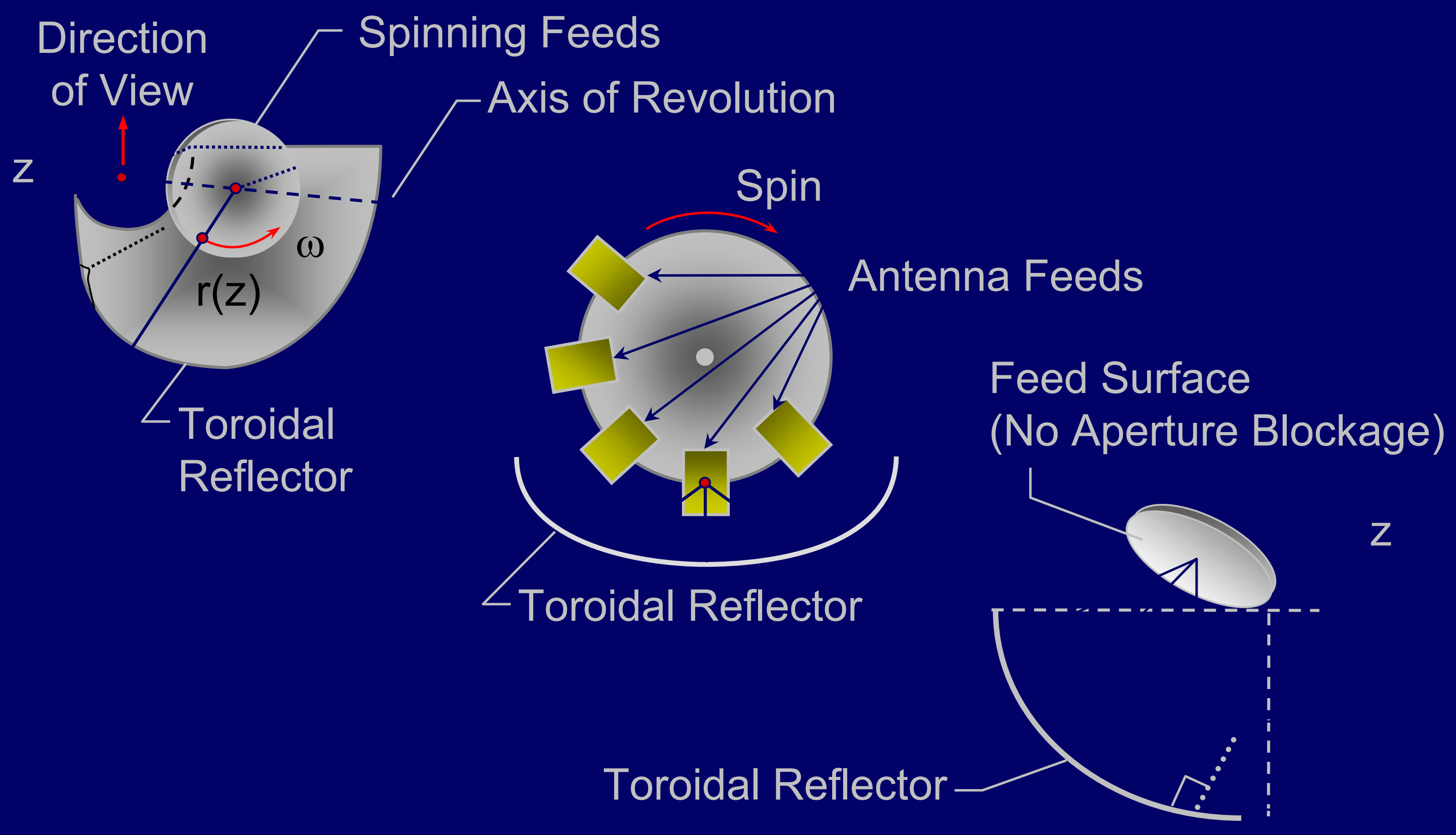
Spherical Reflector Antennas





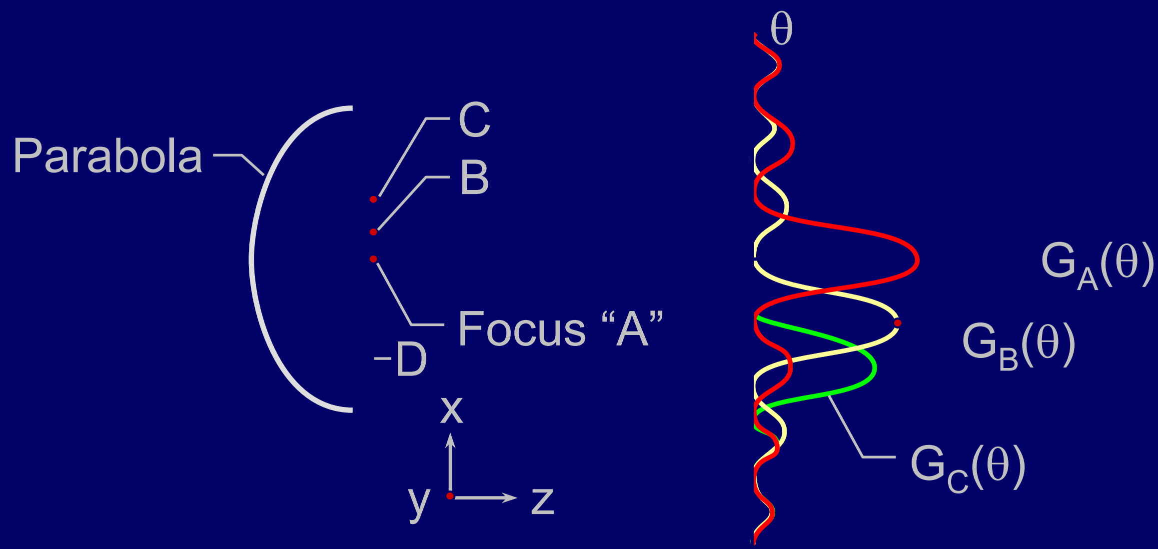


Toroidal Parabolic Reflector Antenna



Advantage: many rapidly scanning spinning feeds

Multifeed Arrays



Focal length $\triangleq$ f

For $f/D = 0.5$, $n \cong 3$ - 5 beams with useable G_o and sidelobes

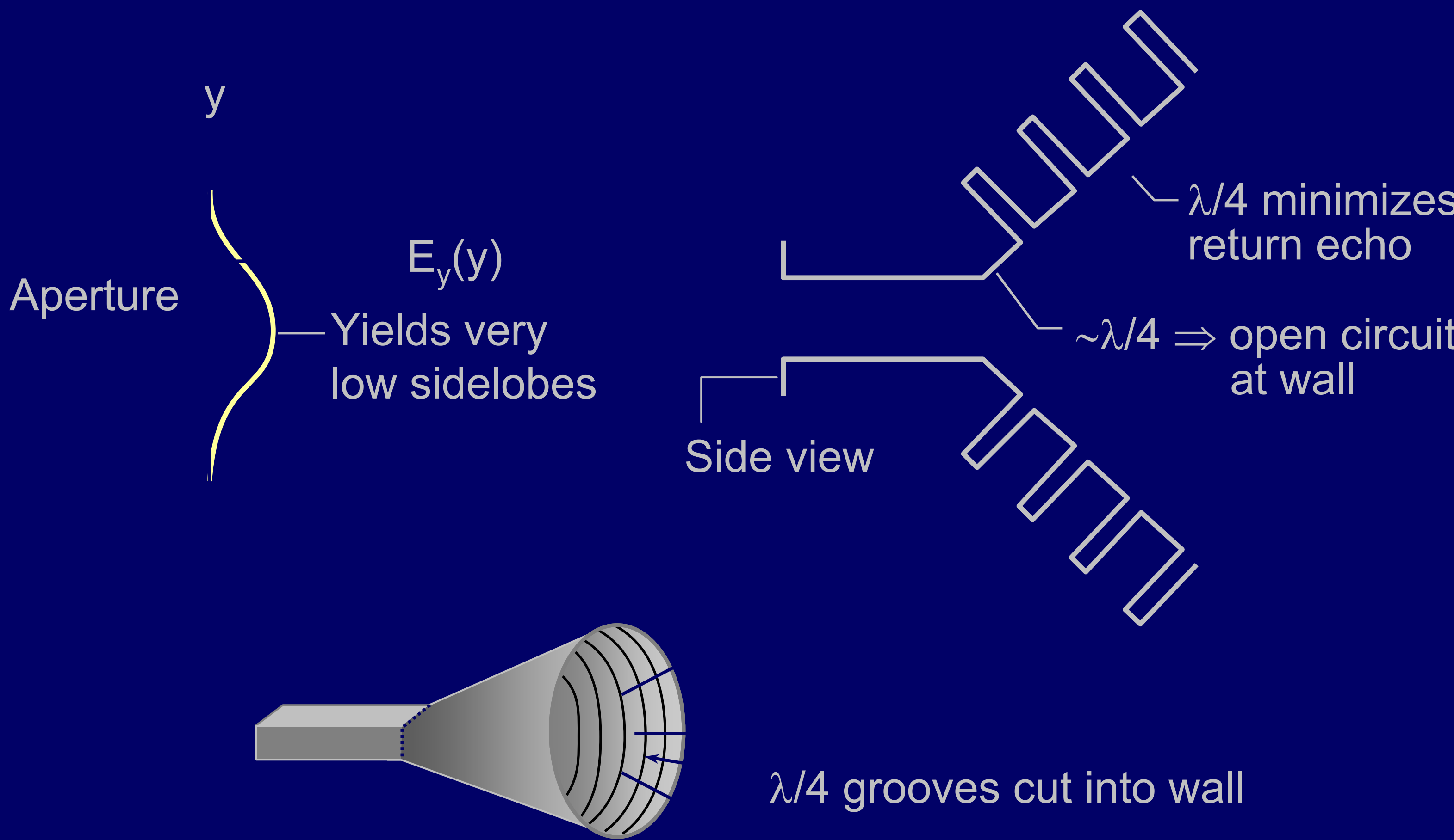
(say ~1 dB gain loss)

$\eta \propto (f/D)^2$ in x - direction

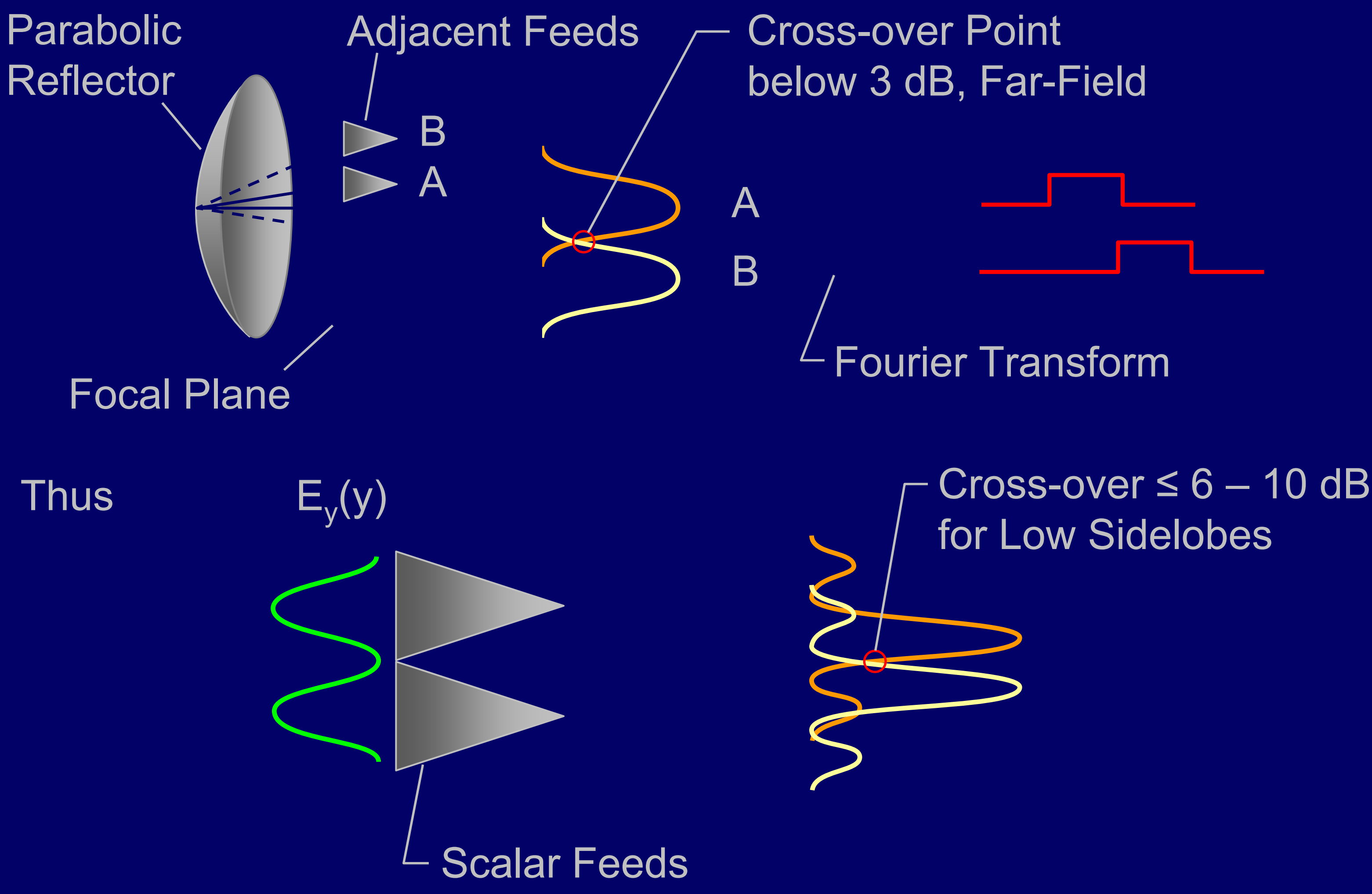
e.g. if $f/D = 7$, $n_x \cong (7/0.5)^2 \bullet 5 \cong 1000$

Can do much better with good lens systems

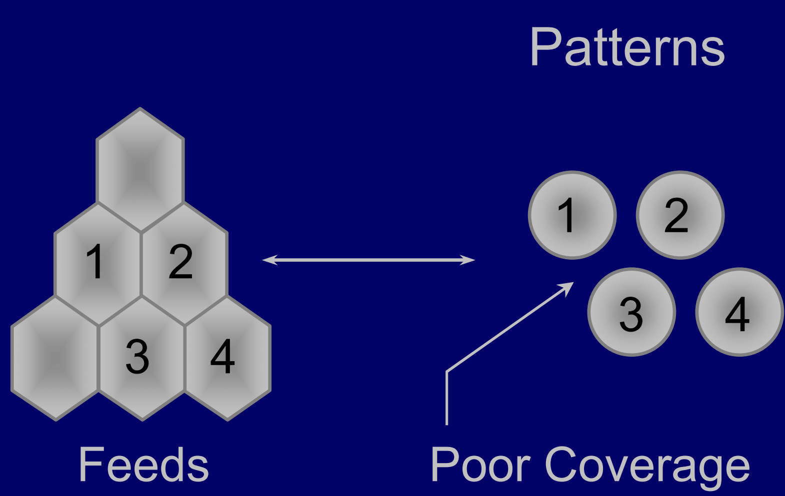
“Scalar” Feed

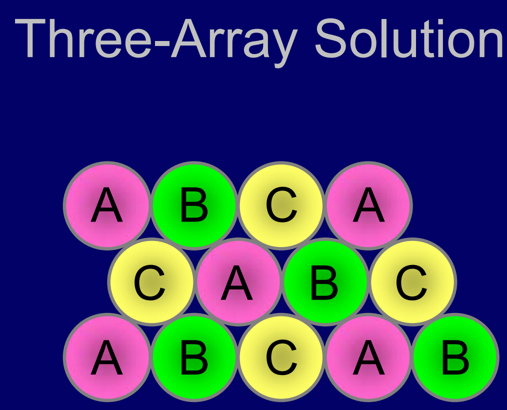

y
Aperture
$E_y(y)$
Yields very low sidelobes
λ/4 minimizes return echo
~λ/4 ⇒ open circuit at wall
Side view
λ/4 grooves cut into wall

Multiple-Horn Feeds

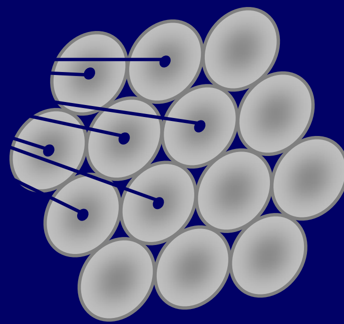


Multiple-Horn Feeds

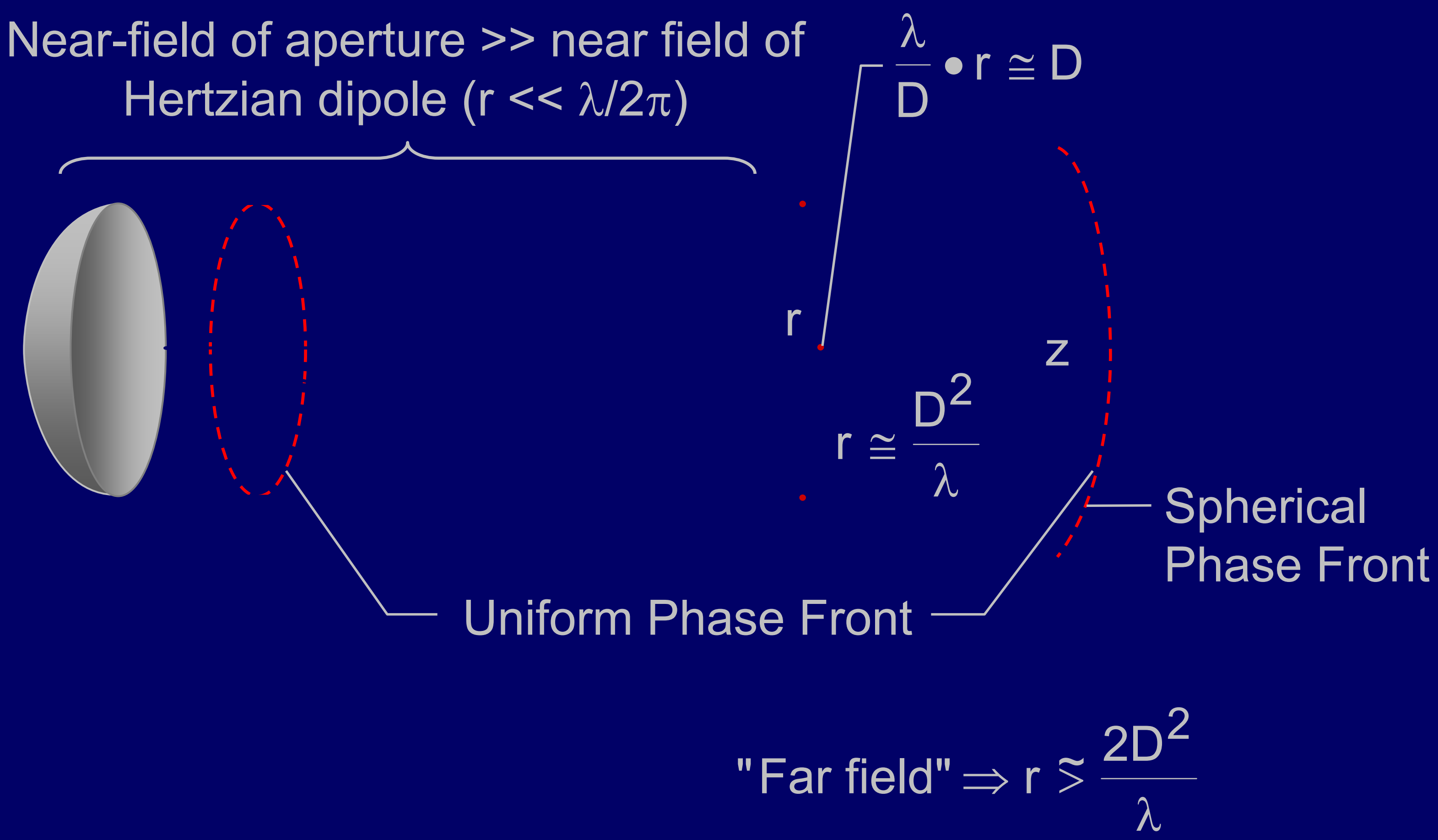




Feed A′ is assembly of excited adjacent feeds

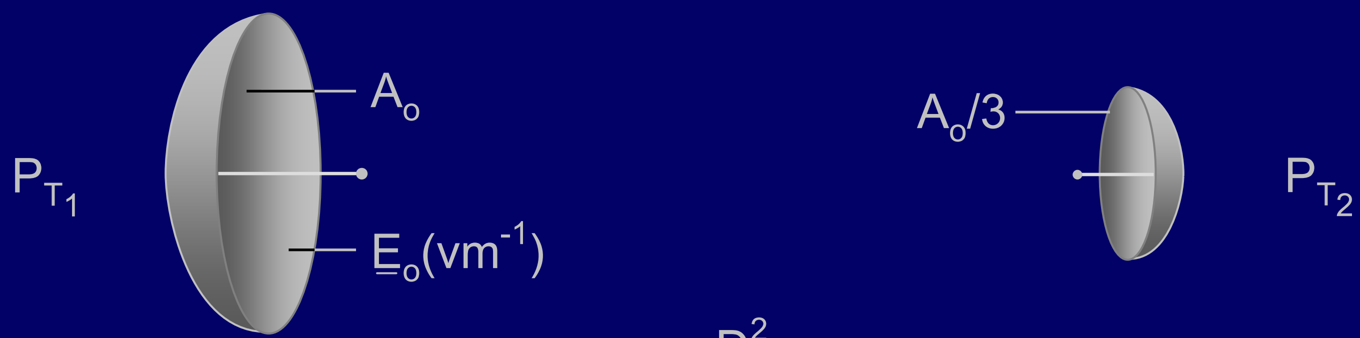


“Near-Field” Antenna Coupling



"Near-Field" Antenna Coupling

Consider near-field link:
Say: uniformly illuminated apertures



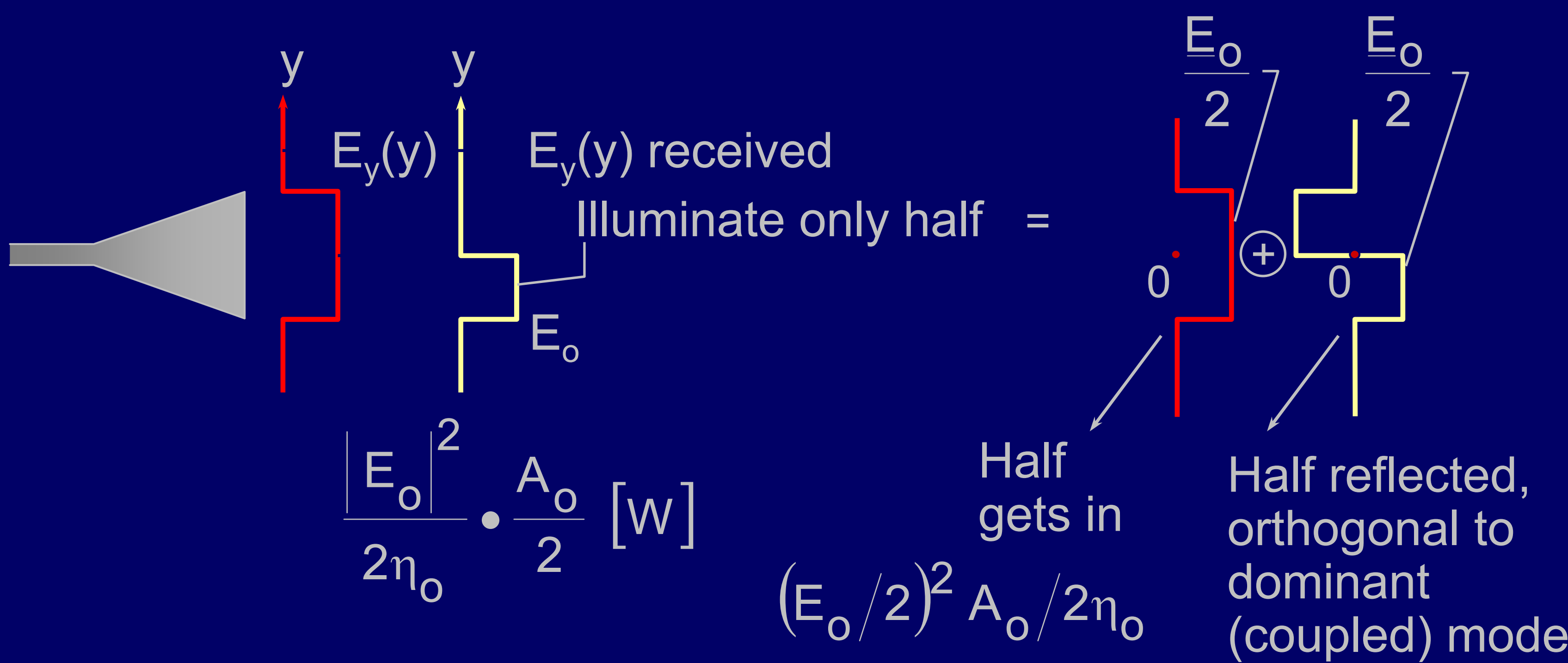
$$r \ll \frac{D^2}{\lambda}$$

$$P_{T_1} = \frac{|E_o|^2}{2\eta_o} \bullet A_o \quad \text{watts} \qquad P_{T_2} = \frac{|E_o|^2}{2\eta_o} \bullet \frac{A_o}{3}$$

$$P_{r_2} = \frac{|E_o|^2}{2\eta_o} \bullet \frac{A_o}{3} = \frac{P_{T_1}}{3}, \qquad P_{r_1} = ?$$

$$\text{Claim } P_{r_1} = \frac{P_{T_2}}{3} \ (\text{reciprocity}) \text{ i.e. } \frac{P_{r_2}}{P_{T_1}} = \frac{1}{3} = \frac{P_{r_1}}{P_{T_2}}$$

"Near-Field" Antenna Coupling, Mode Orthogonality



Only half the power is accepted here!

Waves are not a sum of independent "bullets;" they have phase, modal structure (classic wave/particle issue).