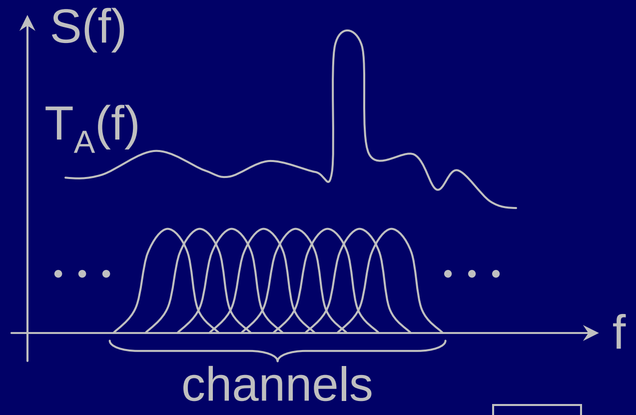
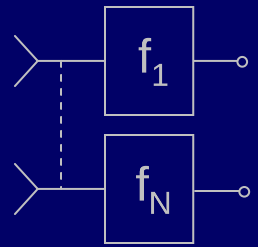
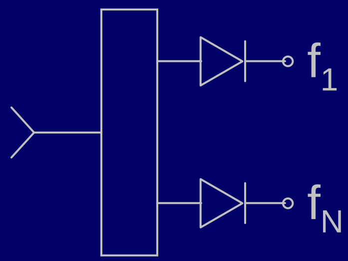
Spectral Measurements



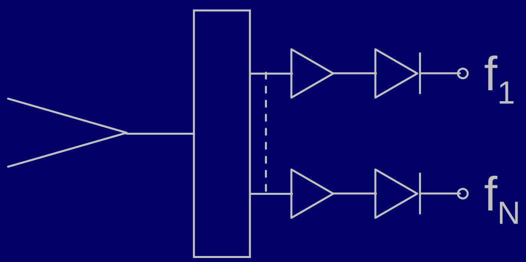
Case A: Bandwidth exceeds that of available amplifiers



1) Extreme bandwidth: use multiple receivers and antennas

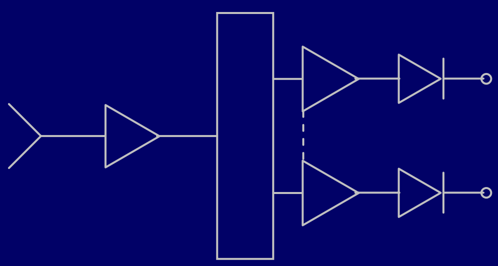


2) If signal large compared to detector noise, detect directly or split frequencies and then detect



3) Use passive frequency splitters before amplification or detection

Spectral Measurements



Case B: Bandwidth permits amplification

1) Amplify before either detection or further frequency splitting

Case C: Bandwidth permits digital spectral analysis

1) If computer resources permit, compute

$$\left\langle \left|\underline{V}(f)\right|_N^2 \right\rangle_M \left(\sim N\log_2 N \text{ multiplys per } N\text{-point transform: average } M \text{ spectra}\right)$$

Resolution $\Delta f \geq 2B/N$

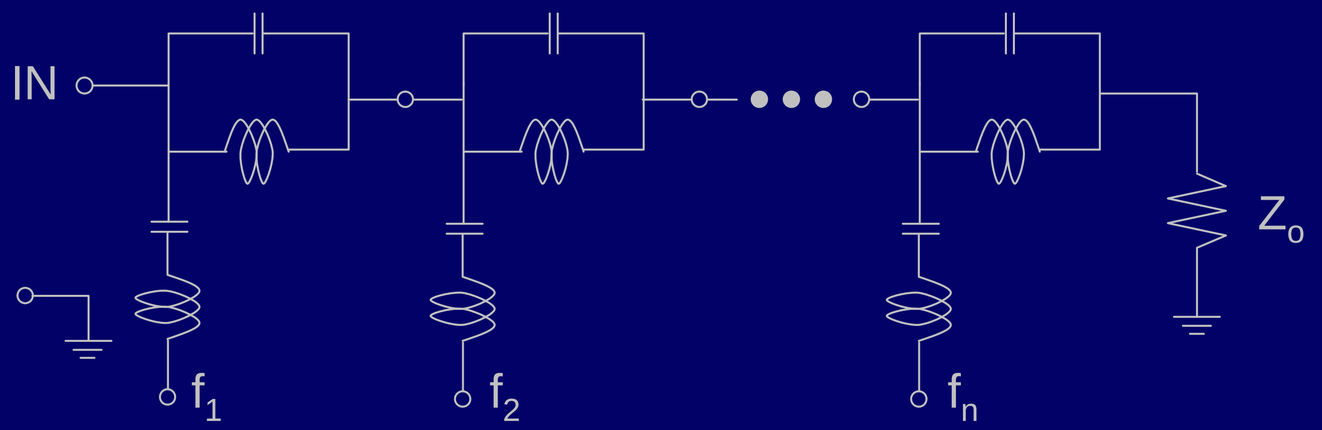
2) Or 1-bit (or n-bit) $\phi_N(\tau) \leftrightarrow \Phi_N(f)$ (N samples)

(Permits ~×100 more B per cm^2 silicon)

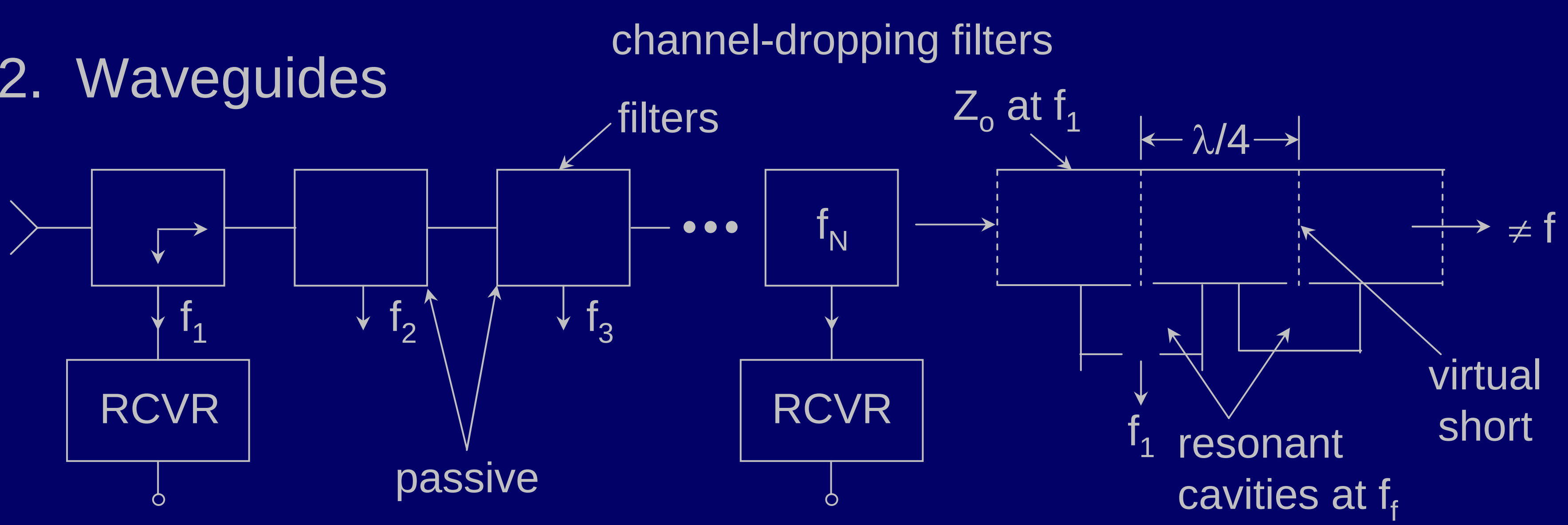
(Reference: Van Vleck and Middleton, *Proc. IEEE,* **54**, (1966)

Examples of Passive Multichannel Filters

1. Circuits

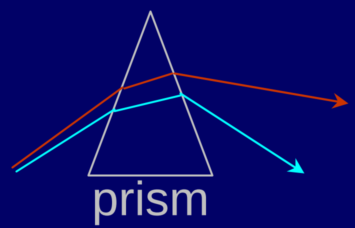


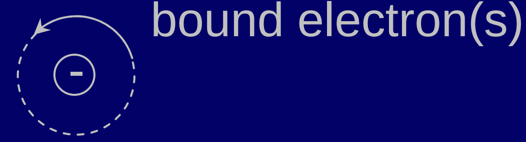
2. Waveguides

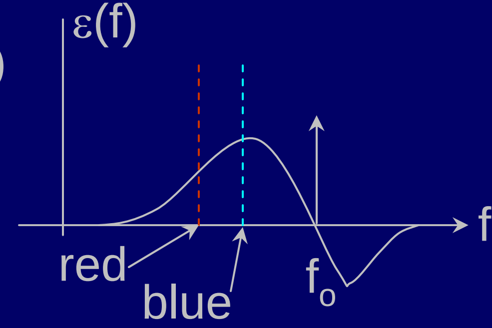


Examples of Passive Multichannel Filters

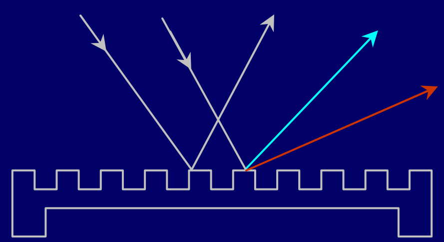
3. Prism



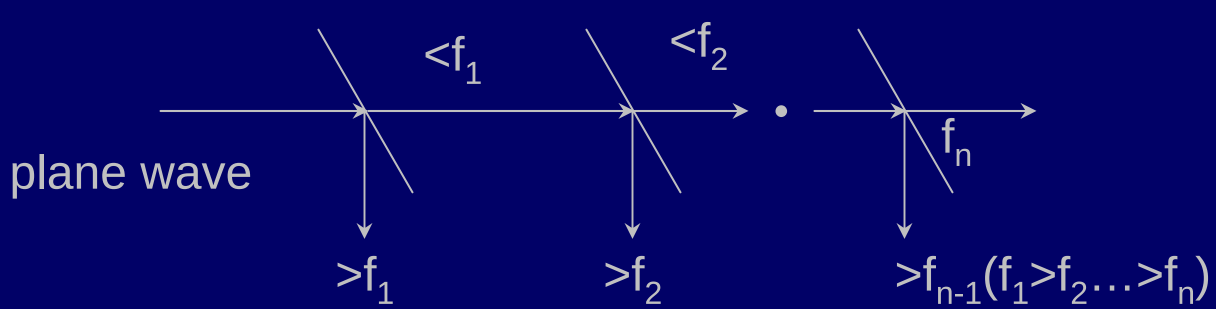




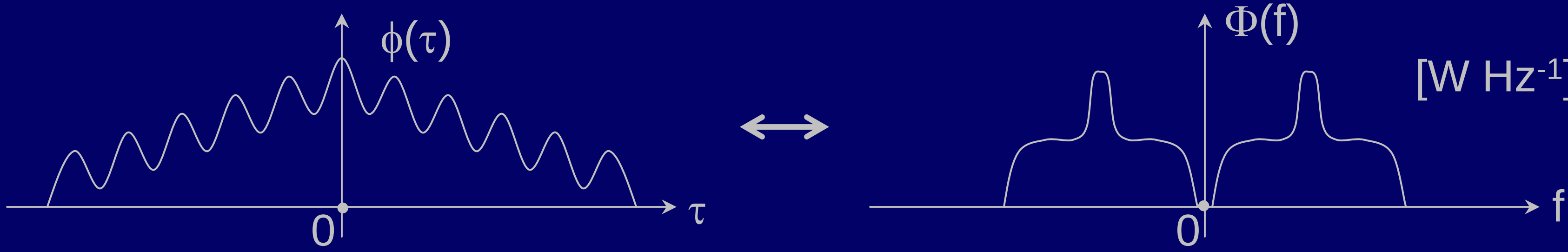
4. Diffraction grating



5. Cascaded Dichroics

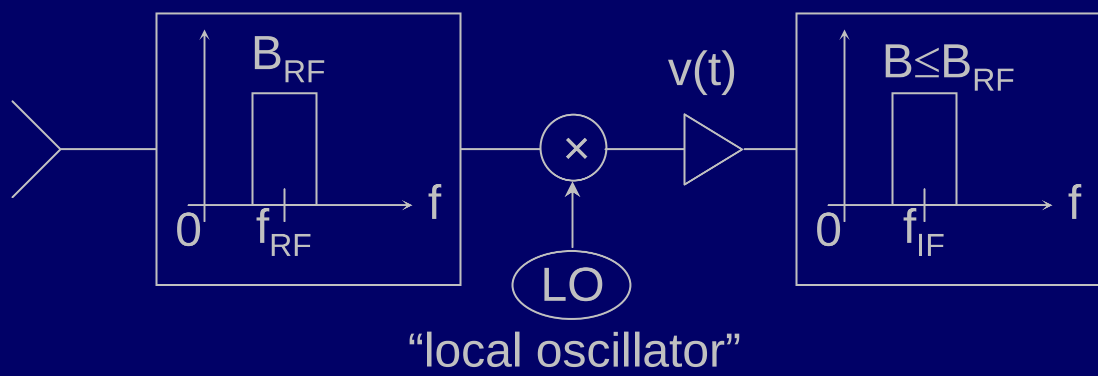


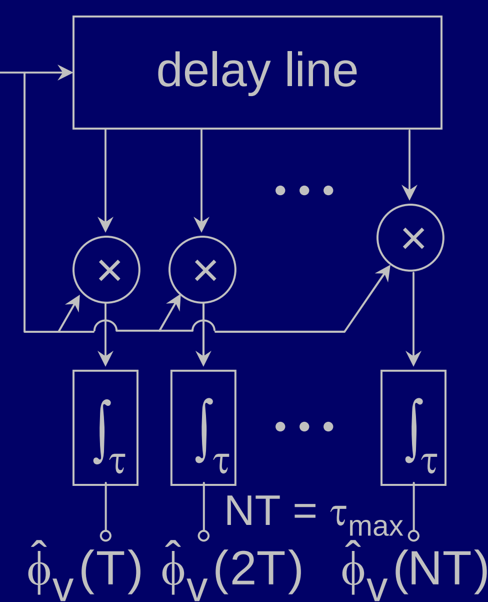
Digital spectral analysis example: autocorrelation



analog signals

Possible analog implementation:





$\hat{\phi}_V(\tau)$ is based on :

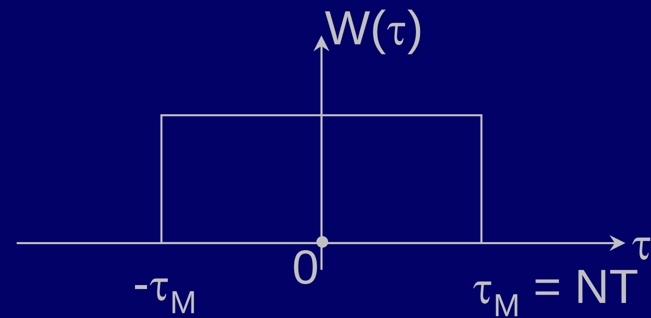
1) max lag = τ_{max} = NT
2) sample lag, T sec
3) finite integration time $\tau >> \tau_{max}$

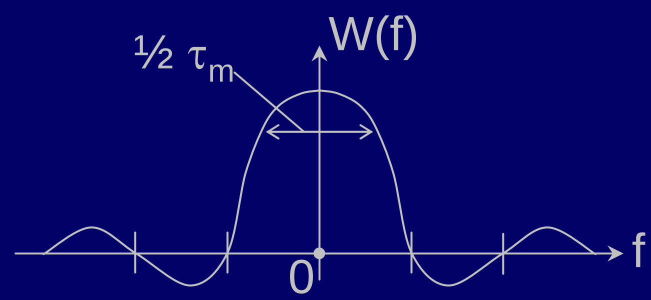
Resolution of autocorrelation analysis

1) $\hat{\phi}_v(\tau) = \phi_y(\tau) \bullet W(\tau)$

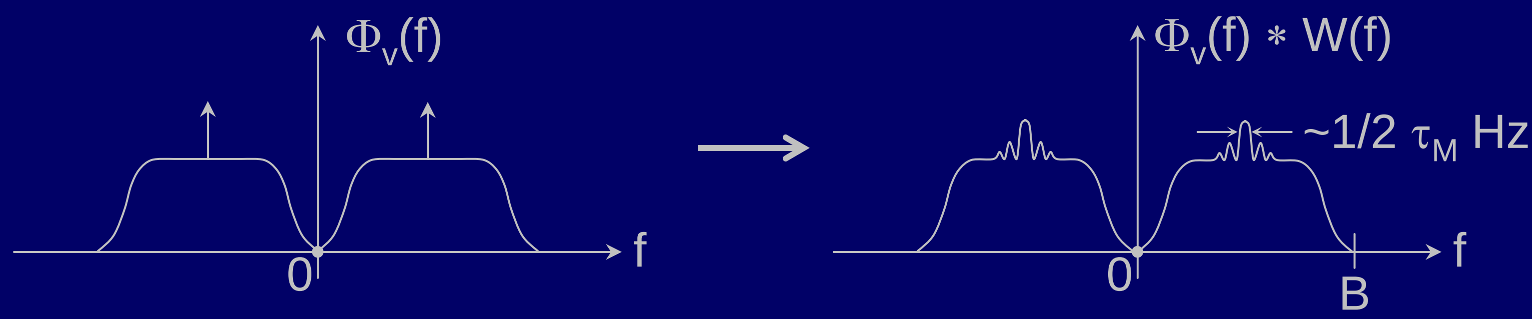
$\Updownarrow \; |\tau| < \tau_M \qquad \Updownarrow \qquad \Updownarrow \qquad \Updownarrow$

$\therefore \hat{\Phi}_v(f) = \Phi_v(f) * W(f)$





Thus

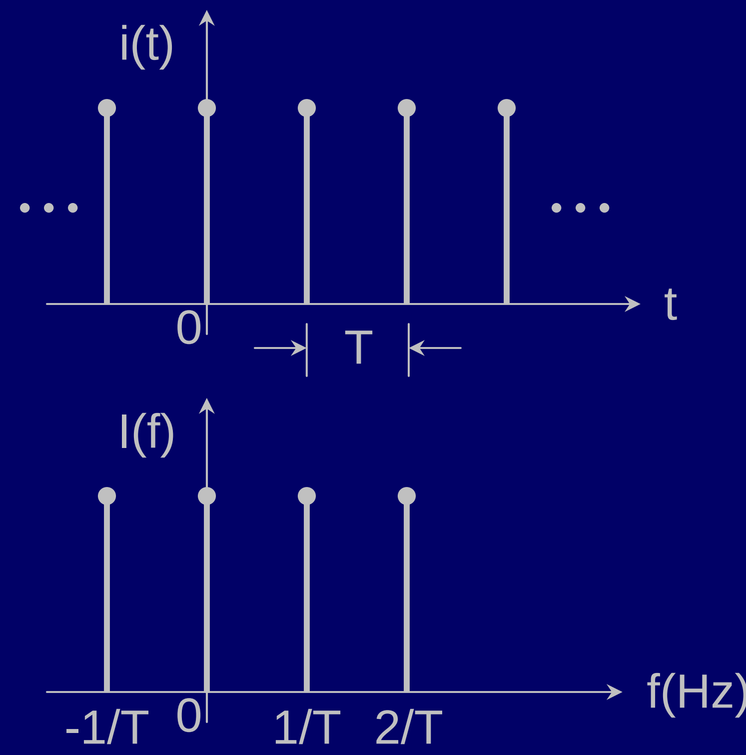


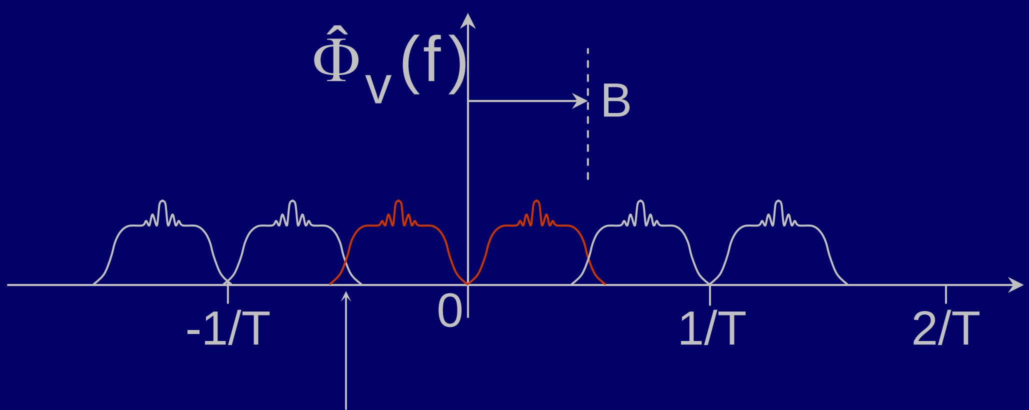
Aliasing in autocorrelation spectrometers

2) $\hat{\phi}_v(\tau) = \phi_v(\tau) \bullet i(t)$

$\quad\updownarrow \qquad\quad \updownarrow \qquad \updownarrow \qquad \updownarrow$

$\hat{\Phi}_v(f) = \Phi_v(f) * I(f)$

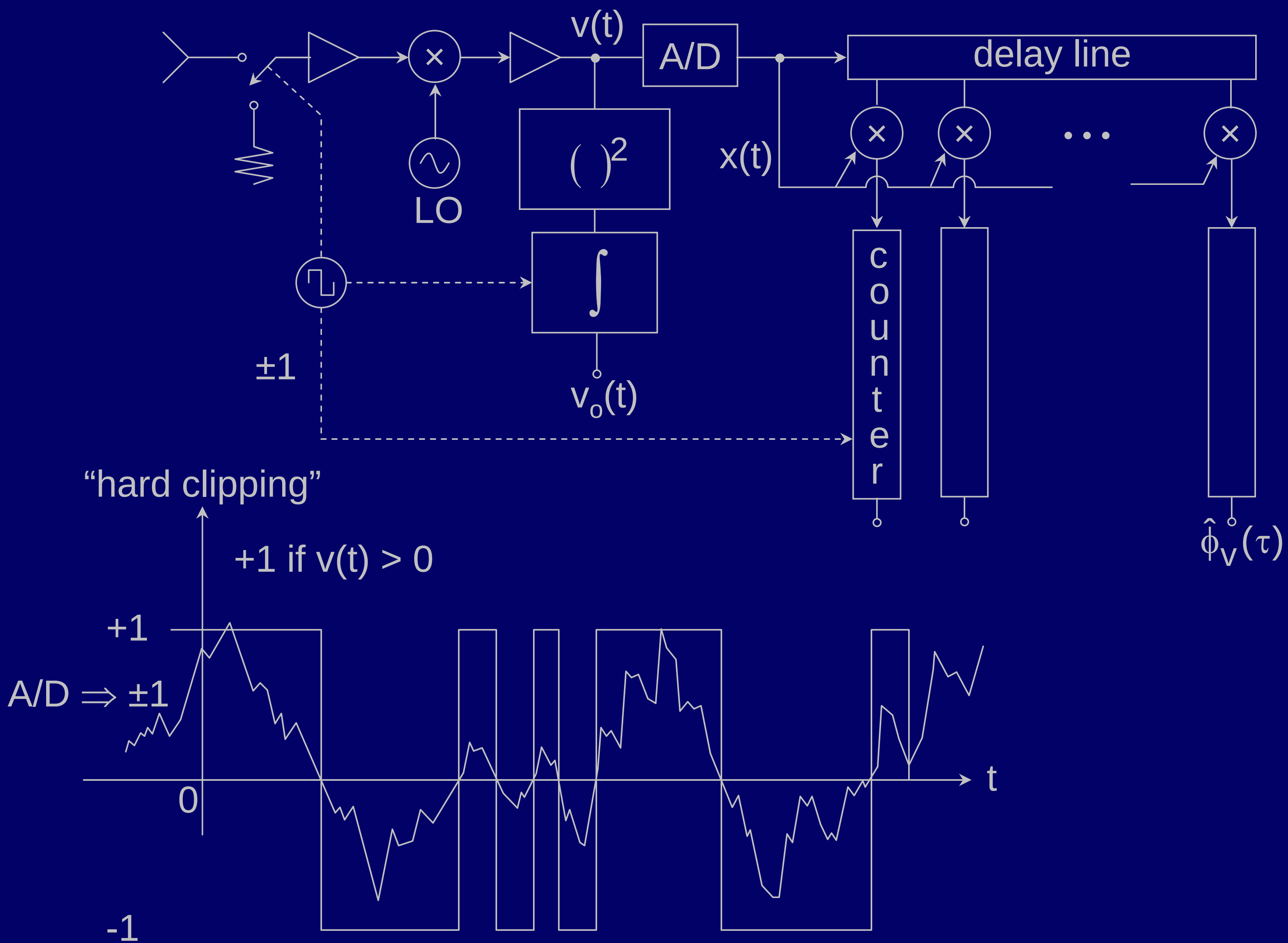




"Aliasing" is spectral overlap

3) Finite averaging time τ adds noise to $\hat{\phi}_v(\tau), \hat{\Phi}_v(f)$

Autocorrelation of hard-clipped signals



Analysis of 1-bit autocorrelation

Let $x(t_1) \triangleq x_1,\ x(t_2) \triangleq x_2,\ \operatorname{sgn} x \triangleq \begin{cases} +1 & x \geq 0 \\ -1 & x < 0 \end{cases}$ where x_1, x_2 are JGRVZM

$$\phi_x(\tau) = E[\operatorname{sgn} x_1 \operatorname{sgn} x_2] =$$

$$\iint_{-\infty}^{\infty} \operatorname{sgn} x_1 \operatorname{sgn} x_2 \left[\frac{1}{2\pi(1-\rho)^{1/2}} e^{-\frac{x_1^2 - 2\rho x_1 x_2 + x_2^2}{2(1-\rho^2)}} \right] dx_1 dx_2$$

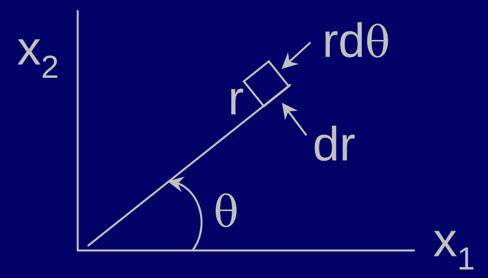
where $\rho(\tau) \triangleq \overline{x_1 x_2} \equiv \phi_v(\tau),\ \tau = t_2 - t_1$

$$\phi_x(\tau) = 2\int_0^{\infty}\!\!\int [p(x_1, x_2)] dx_1 dx_2 - 2\int_{-\infty}^{0}\int_0^{\infty} p(x_1, x_2) dx_1 dx_2$$

$$= 4\int_0^{\infty}\!\!\int p(x_1, x_2) dx_1 dx_2 - 1 \qquad \left\{ \text{Note: } 2\int_0^{\infty}\!\!\int + 2\int_{-\infty}^{0}\int_0^{\infty} = 1 \right\}$$

Power spectrum for 1-bit signal

Change variables



$x_1 = r\cos\theta$
$x_2 = r\sin\theta$
$dx_1 dx_2 = r\,dr\,d\theta$

$$\phi_x(\tau) = 4\int_0^{\pi/2} d\theta \int_0^{\infty} d\left(\frac{r^2}{2}\right)\frac{1}{2\pi\left(1-\rho^2\right)^{1/2}}\, e^{-\left(r^2/2\right)\left(\frac{1-\rho\sin 2\theta}{1-\rho^2}\right)} - 1$$

$$= 4\int_0^{\pi/2} d\theta\, \frac{\left(1-\rho^2\right)^{1/2}}{2\pi\left(1-\rho\sin 2\theta\right)} - 1$$

Power spectrum for 1-bit signal

$$= 4\int_0^{\pi/2} d\theta \, \frac{\left(1-\rho^2\right)^{1/2}}{2\pi\left(1-\rho\sin 2\theta\right)} - 1$$

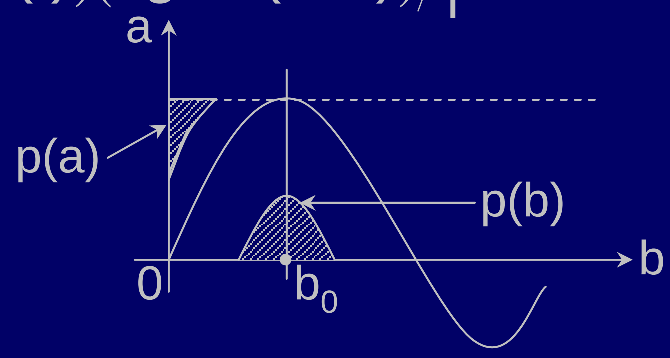
Let $\phi \triangleq 2\theta$

$$\phi_x(\tau) = 4\frac{(1-\rho)^{1/2}}{4\pi}\int_0^{\pi} \frac{1}{1-\rho\sin\phi}\, d\phi - 1 = 4\left\{\frac{1}{2\pi}\left(\frac{\pi}{2} + \sin^{-1}\rho\right)\right\} - 1$$

$$\hat{\phi}_v(\tau) \equiv \hat{\rho} = \sin\left(\frac{\pi}{2}\hat{\phi}_x(\tau)\right)$$

Where $\hat{\phi}_x(\tau) = \left\langle (\mathrm{sgn}\, v(t))(\mathrm{sgn}\, v(t - \tau)) \right\rangle_T$

Note : $\hat{\rho}$ has bias if b not exact



(see Burns & Yao, *Radio Sci.*, **4**(5) p. 431 (1969))

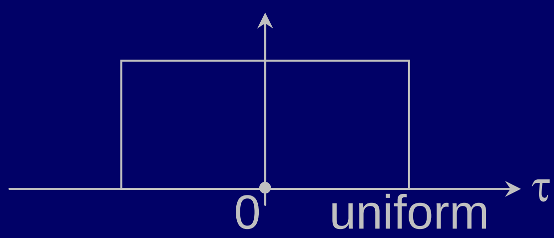
Spectral response & sensitivity: autocorrelation receiver

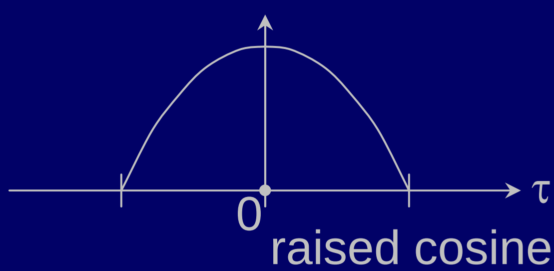
$$\sigma(f)_{rms} \cong \frac{\alpha\beta T_{eff}}{\sqrt{\tau\Delta f}}\sqrt{1-\frac{\Delta f}{B}}\;; \quad \beta \cong 1.6$$

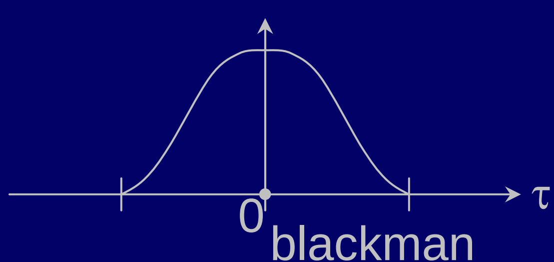
(S. Weinreb empirical result, MIT EE PhD thesis, 1963)

(Δf ← channel bandwidth)

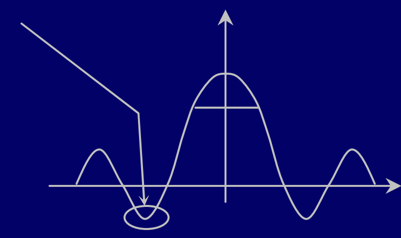
"Apodizing" weighting functions:







Weighting	α	Δf	first sidelobe
uniform	1.099	$\frac{0.60\, f_s}{N}$	-7 dB
raised cosine	0.87	f_s/N	-16 dB
blackman	0.69	$1.13\, f_s/N$	-29 dB



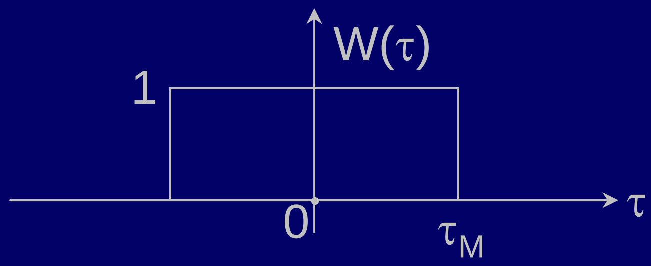
$$\left(f_s \triangleq \frac{1}{T} = \frac{N}{\tau_M}; N = \#\ \text{taps}\right)$$

Note trade between spectral resolution, sidelobes in $\Phi(f)$ and ΔT_{rms}

Spectral response & sensitivity: autocorrelation receiver

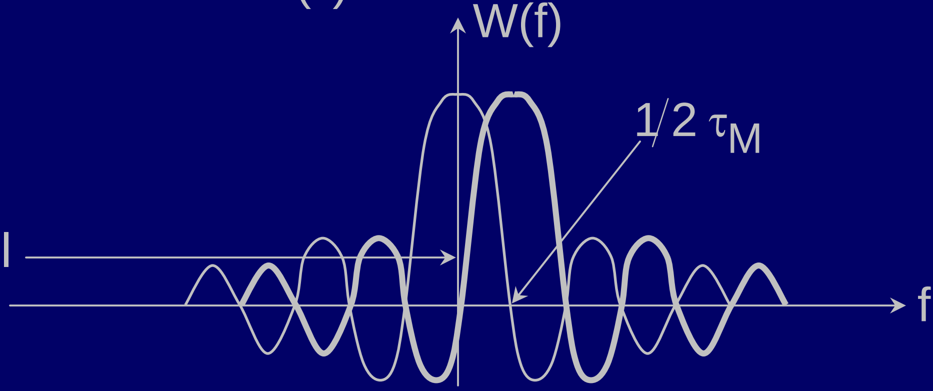
If N delay-line taps, how many spectral samples N_s?

Say uniform weighting of $\phi(\tau)$:



Then $B = N_s \bullet \Delta f = N_s \bullet (1/2\tau_M)$ where spectral resolution $\Delta f \cong 1/2\tau_m$ for orthogonal channels from boxcar $W(\tau)$

W(f) for adjacent channel



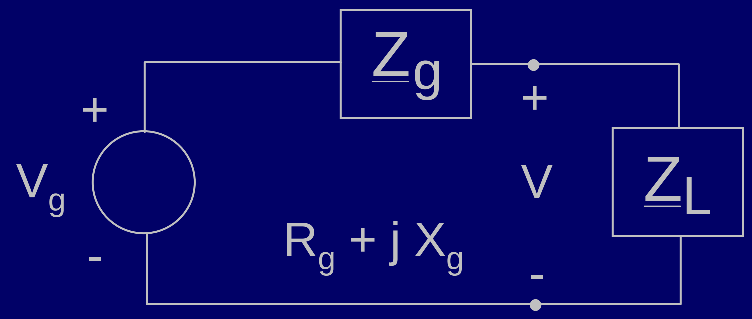
$$\therefore N_s = 2\tau_M B = 2\,NT\,B\,\left(T = 1/2B \text{ at nyquist rate}\right) = N(\#\text{ taps})$$

In practice: raised cosine widens Δf by $1/0.6 \cong 1.7$, so $N_s \cong N/1.7$

Receivers – Gain and Noise Figure

Types of "power"

- Delivered
- Available
- Exchangeable



$$v(t) \triangleq R_e\left\{\underline{V}e^{j\omega t}\right\} = \mathrm{Re}\{\underline{V}\}\cos\omega t + \mathrm{Im}\{\underline{V}\}\sin\omega t$$

$$P_{delivered} \triangleq \frac{1}{2}R_e\left\{\underline{VI}^*\right\}(\triangleq P_D)$$

$$P_{available} \triangleq \max P_D,\ \text{i.e., if } \underline{Z}_L = \underline{Z}_g^*$$

Delivered and Available Power

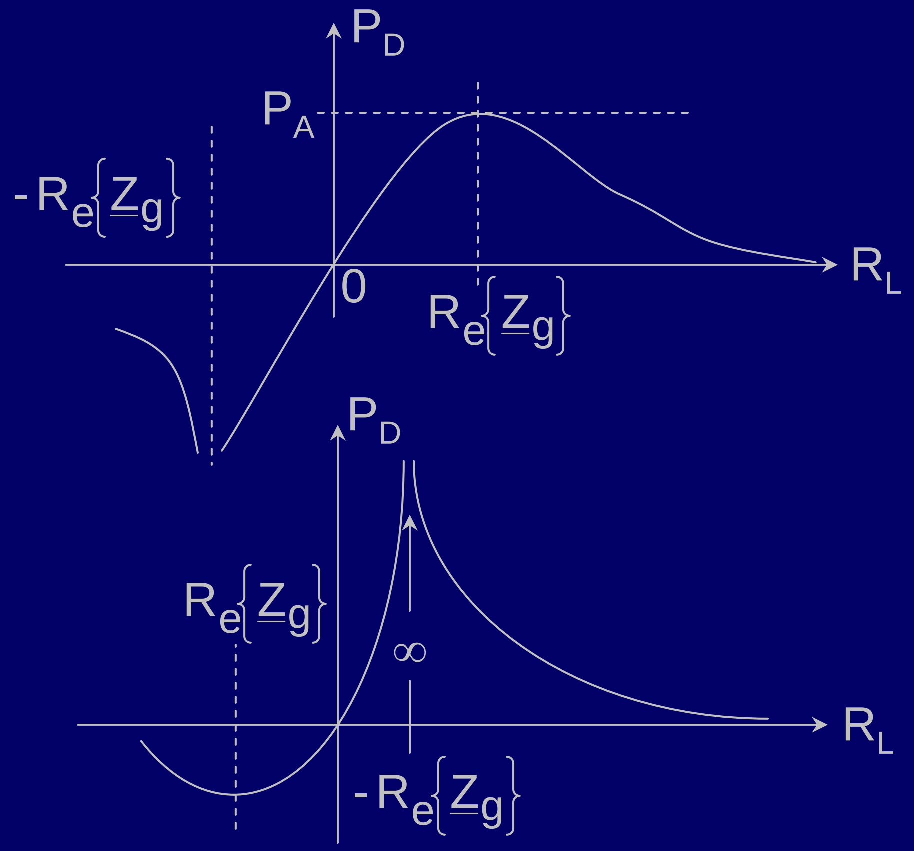
$$P_{delivered} \triangleq \frac{1}{2} R_e\left\{\underline{V}\underline{I}^*\right\} \left(\triangleq P_D\right)$$

$$P_{available} \triangleq \max P_D, \text{ i.e., if } \underline{Z}_L = \underline{Z}_g^*$$

$$\text{If: } R_e\left\{\underline{Z}_g\right\} > 0$$

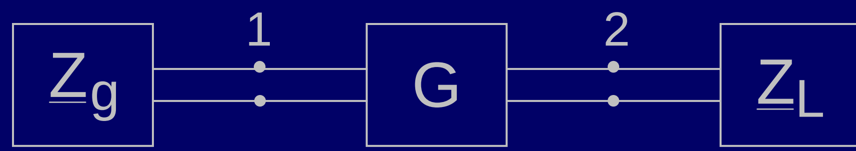
$$Im\left\{\underline{Z}_g\right\} = 0$$

$$\text{If: } R_e\left\{\underline{Z}_g\right\} < 0$$



$$P_{exchangeable} \triangleq \left|P_D\right|_{\underline{Z}_L = \underline{Z}_g^*} \quad (\rightarrow \text{finite - power option})$$

Definition of Gain



G_{power} (= G_p) $\triangleq$ P_{D_2} / P_{D_1}

$G_{available}$ (= G_A) $\triangleq$ P_{A_2} / P_{A_1}

$G_{\substack{transducer\\power}}$ (G_T) $\triangleq$ P_{D_2} / P_{A_1}

$G_{insertion}$ (= G_I) $\triangleq$ $\dfrac{P_{D_2}}{P_{D_1}}$ $\begin{matrix}\text{with amplifier}\\ \text{without amplifier}\end{matrix}$

$G_{exchangeable}$ (=G_E) $\triangleq$ $\dfrac{P_{E_2}}{P_{E_1}}$

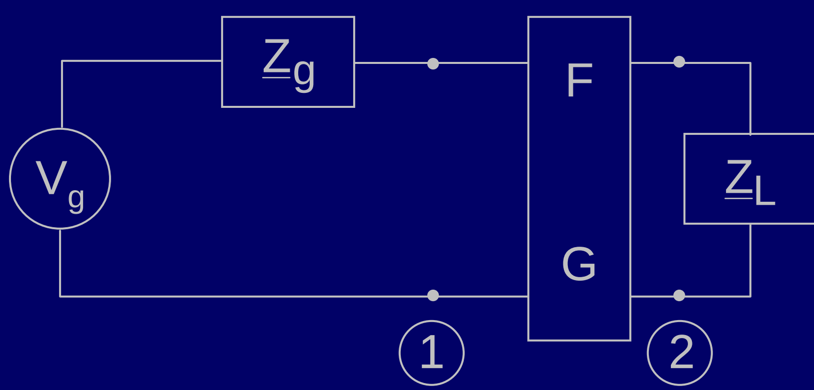
Note: G_A, G_E $\begin{cases}\text{don't depend on } \underline{Z}_L \\ \text{do depend on } \underline{Z}_g \text{ (via } P_{E2}\text{)}\end{cases}$

Definition: Signal-to-Noise Ratio (SNR)

First define:

$$WH_z^{-1}\left\{\begin{array}{lll} N_1 & = & \text{exchangeable noise power spectrum @ Port 1} \\ N_2 & = & \text{same, at 2} \\ S_1 & = & \text{exchangeable signal power spectrum @ Port 1} \\ S_2 & = & \text{same, at 2} \end{array}\right.$$

Recall $G_E = f(\underline{Z}_g)$



$$\text{Define } SNR_1 \triangleq S_1/N_1; \; SNR_2 \triangleq S_2/N_2$$

Definition: Noise Figure F

$$F \triangleq \frac{SNR_1}{SNR_2} \equiv \frac{S_1/N_1}{S_2/N_2}, \text{ where } N_1 \triangleq kT_o, \; T_o \triangleq 290\,K$$

[Ref. *Proc. IRE*, **57**(7), p.52 (7/1957); *Proc. IEEE*, p.436 (3/1963)]

$S_2 = G_E S_1$ (see definition of G_E)

$N_2 = G_E N_1 + N_{2T}$ "transducer noise"

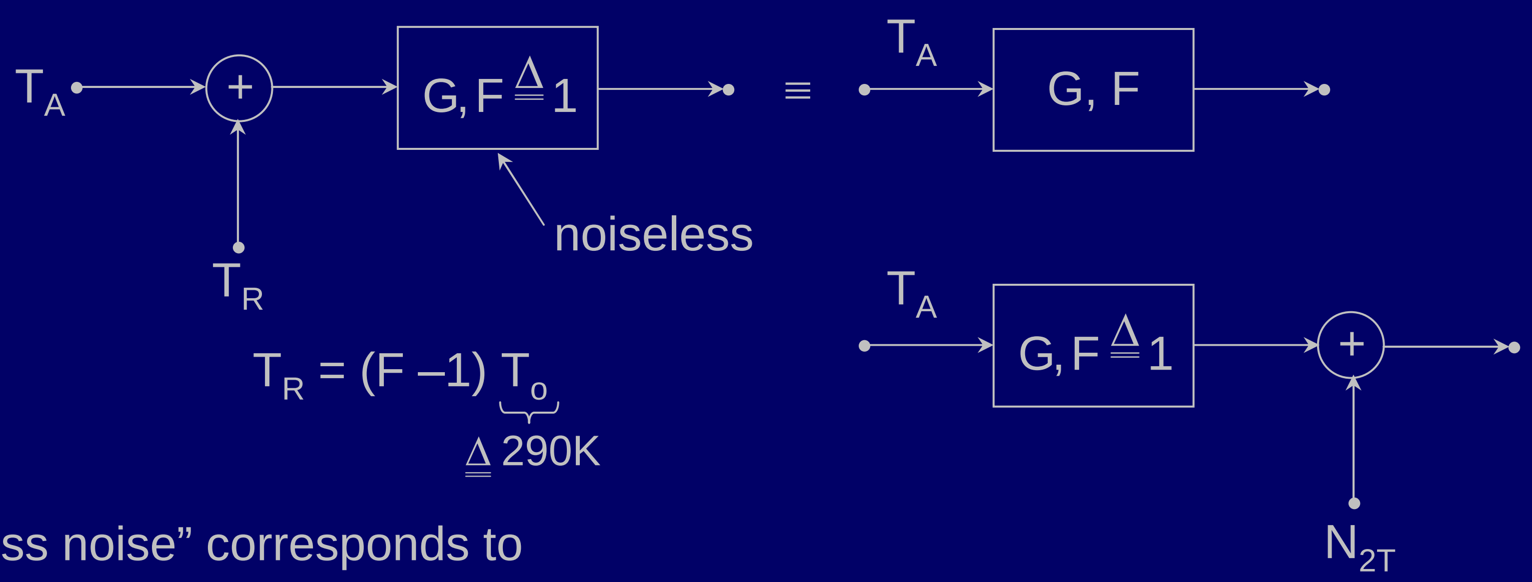
$$\therefore F = \frac{S_1/N_1}{GS_1/(GN_1 + N_{2T})} = 1 + \frac{N_{2T}}{N_1 G} \qquad (\text{let } G \triangleq G_E)$$

$$\therefore \underbrace{F - 1}_{\text{"excess noise figure"}} = \frac{N_{2T}}{N_1 G} \triangleq \frac{kT_R G}{kT_o G} = \frac{T_R}{T_o}$$

T_R: "receiver noise temperature"

"excess noise figure"

Receiver Noise Example



$T_R = (F - 1)\underbrace{T_o}_{\triangleq\, 290K}$

"Excess noise" corresponds to "receiver noise temperature T_R"

Examples:

$T_R = 0°K \quad \Rightarrow \quad F = 1 + \frac{T_R}{T_o} = 1 \quad (F = 0\ dB)$

$T_R = 290°K \quad \Rightarrow \quad F = 2 \quad (F = 3\ dB)$

$T_R = 1500°K \quad \Rightarrow \quad F \cong 6 \quad (F \sim= 7.5\ dB)$