MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Receivers, Antennas, and Signals – 6.661

Hints to Solving Problem Set 1

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General recommendations for using "HINTS" when solving 6.661 homework problems:

It has repeatedly been demonstrated that only by struggling with well-posed homework problems without assistance can one really begin to understand and master typical MIT subject matter—the same is true of 6.661. At the same time, if such a struggle ends in no progress, there again is little learning. These **"Hints"** are an attempt to provide a better balance—to point the way if one is truly stuck on a problem after an hour of earnest effort. After completing a problem set the **"Hints"** should also quickly reveal whether an alternative or simpler approach exists.

Problem 1.1

a)   The Fourier transform of an exponentially decaying unit step can easily be found by direct integration using the definition of Fourier transform. A second approach is to find the Fourier transform of a unit step, and note that the factor $e^{-\alpha t}$ becomes a "frequency" offset in the frequency domain (see class lecture notes).

b)   The autocorrelation function can also be found by straightforward application of the integral that defines that operation. Simple expressions result when positive and negative τ are considered separately. We could also Fourier transform the energy density spectrum, which is calculated in part (c).

c)   The energy density spectrum is easily found from the Fourier transform by simply applying the definition. The energy density spectrum can be found from the autocorrelation function by Fourier transforming it, the transform most easily being performed separately for positive and negative τ. The answer should be identical for the two parts of (c).

Problem 1.2

a)   The power density spectrum of the output of the linear network equals the power density spectrum of the input, times the power spectral transfer function of that network. This can be shown by expressing the output as the convolution of the input and the network impulse response function, and then converting this expressing into the frequency domain, to yield the Fourier transform of the output

in terms of the Fourier transform of the input. The power density spectrum of the output can then readily be found and depends simply on the power spectral transfer characteristics of the network, which is simple related to the Fourier transform of the network impulse response function, which we have found earlier in this problem set.

b)   The standard deviation of y is simple related to the total power emerging from the network, which can be easily calculated by integrating the answer to part (a) over all frequency.

Problem 1.3

The basic notation is defined in Section 1.3 of the course notes. An x-polarized wave moving in the +y direction might be represented as a complex vector $\overline{E} = \hat{x}E_o e^{-jky}$ volts/meter, or in the time domain as $\overline{E}(x, y, z, t) = \hat{x}E_o \cos(\omega t - ky)$ where $k = \omega/c$, ω is the frequency in radians/second, and c is the speed of light $\left(\sim 3 \times 10^8 \text{ m/s}\right)$. To match boundary conditions at the perfect conductor a second (reflected) wave must be present; the total field is the sum of the two, which must cancel $\left(\overline{E}_{parallel} = 0\right)$ at the conductor. For a single uniform plane wave (UPW) $\overline{E}$ and $\overline{H}$ are perpendicular and $\left|\overline{E}\right| = \eta_o \left|\overline{H}\right|$ where the characteristic impedance η_o of free space is $(\mu/\varepsilon)^{0.5} = \sim 377$ ohms. Instantaneous electric and magnetic energy densities are $\varepsilon\left|\overline{E}\right|^2 \big/ 2$ $\left(\text{J}/\text{m}^3\right)$ and $\mu\left|\overline{H}\right|^2 \big/ 2$ $\left(\text{J}/\text{m}^3\right)$.

Problem 1.4

"Cold" is subjective; perhaps pick a cold temperature that simplifies the math. Admiral Byrd once said the coldest place he had ever been was on the Harvard Bridge. Use the Rayleigh-Jeans approximation. Voltage and power are related vie the characteristic impedance of the line, which is its Thevenin source resistance. The text and lecture notes both provide a simple equation that yields the frequency band for which the Rayleigh-Jeans approximation is valid.