6.641 Electromagnetic Fields, Forces, and Motion
Spring 2009

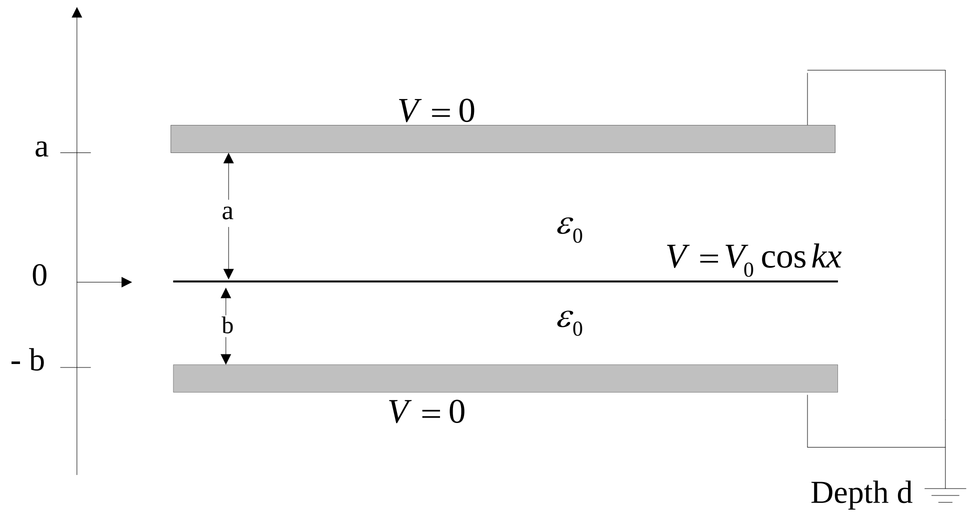
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.641 Electromagnetic Fields, Forces, and Motion

Final Exam

5/15/2008

NOTE: 6.641 Formula Sheet at the end of exam. You are also allowed both sides of two 8½"x11" pages for 6.641 course material that you have prepared yourself.

1. 25 points



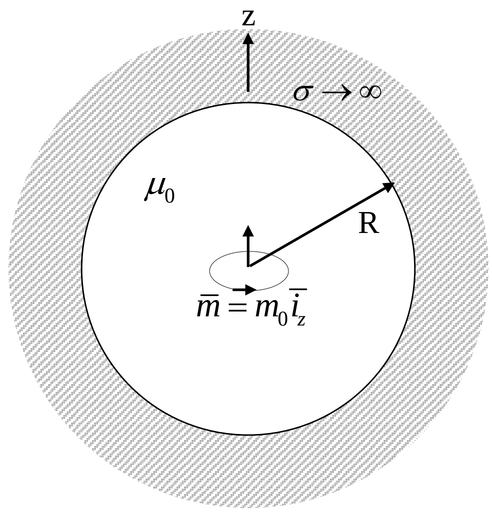
A sheet with electric potential distribution $V(y=0) = V_0 \cos kx$ is placed at $y = 0$, parallel and between two parallel grounded perfect conductors at zero potential at $y = -b$ and $y = a$. The regions above and below the potential sheet have dielectric permittivity of free space ε_0. Neglect fringing field effects.

a) What are the electric potential solutions in the regions $0 \le y \le a$ and $-b \le y \le 0$?
b) What are the electric field distributions in the regions $0 < y < a$ and $-b < y < 0$?
c) What are the free surface charge distrbutions at $y = -b$, $y = 0$, and $y = a$?
d) What are the x and y components of force per unit area on a wavelength of width $2\pi / k$ on the $y = 0$ electric potential sheet?

Hint: $\frac{k}{2\pi}\int_{-\pi/k}^{+\pi/k} \cos^2 kx\, dx = \frac{1}{2}$

$$\frac{k}{2\pi}\int_{-\pi/k}^{+\pi/k} \cos kx \sin kx\, dx = 0$$

2. 25 points



A point magnetic dipole $\overline{m} = m_0 \overline{i}_z$ is placed at the center of a free space spherical cavity of radius R within a perfect conducter ($\sigma \to \infty$).

a) Find the magnitude and direction of magnetic field in the region $0 < r < R$.
   Hint: The scalar magnetic potential of an isolated magnetic dipole with moment $\overline{m} = m_0 \overline{i}_z$ is

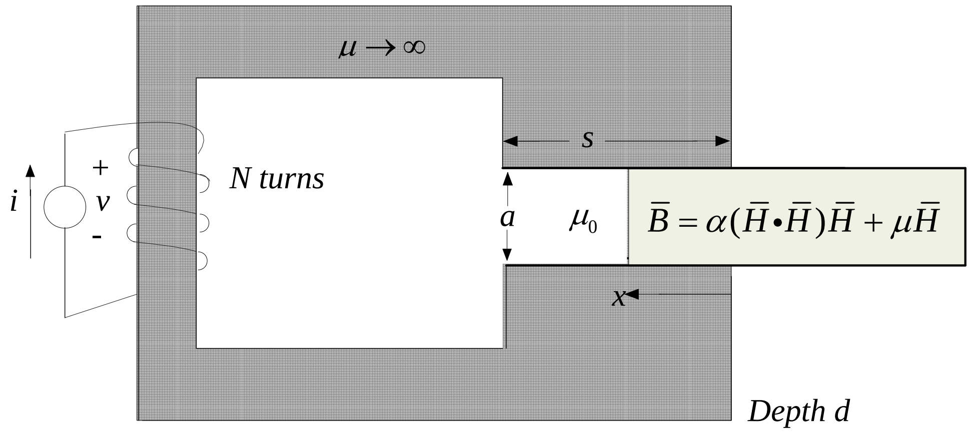
$$\chi(r,\theta) = \frac{m_0}{4\pi r^2}\cos\theta$$

b) What is the surface current on the $r = R$ surface?

c) What is the equation of the magnetic field line that passes through the point ($r = r_0, \theta = \theta_0$)?

Hint:
$$\int \cot\, d\theta = \ell n[\sin\theta]$$
$$\int \frac{2 + (R/r)^3}{r(1-(R/r)^3)} dr = \ell n\left[\frac{(r/R)^3 - 1}{(r/R)}\right]$$

3. 25 points



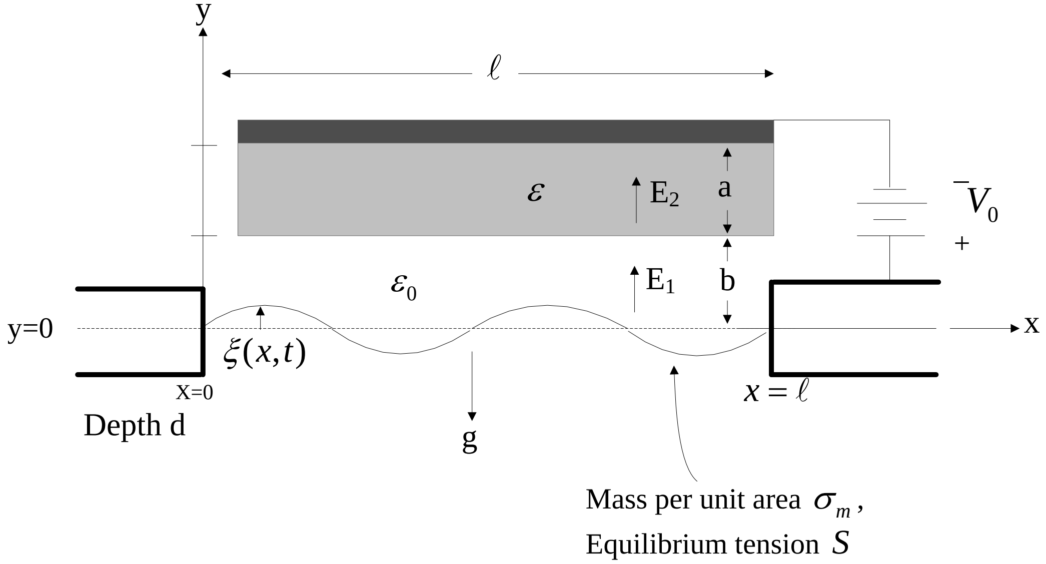
A slab of nonlinear magnetic material with $\bar{B} - \bar{H}$ relationship

$$\bar{B} = \alpha(\bar{H} \bullet \bar{H})\bar{H} + \mu\bar{H}$$

slides a distance x in the free space gap of width a in the above magnetic circuit with $\mu \to \infty$ which is excited by a current i through an N turn coil. The parameters α and μ are constants, μ_0 is the magnetic permeability of free space, s is the gap width, a is the gap thickness, and d is the gap depth.

a) Find the magnetic field $\bar{H}$ in the gap free space region and in the nonlinear magnetic material.
b) What is the total magnetic flux linked by the coil?
c) What is the voltage across the coil terminals when the nonlinear material is stationary, that is, x is constant and $i = I_0 \sin \omega t$?
d) What is the force on the nonlinear magnetic material for any i?

4. 25 points



An electrically conducting membrane with a mass per unit area σ_m is stretched horizontally with equilibrium tension S between two rigid supports at $x=0$ and $x=\ell$. With no other forces acting on the membrane, the downwards gravitational force makes the membrane sag in the middle. We can remove the sag without physical contact with the membrane by placing an electrode above the membrane and applying a voltage V_0 between the membrane and fixed plate electrode. The electrode is attached to a dielectric with permittivity ε and thickness a and the gap distance to the membrane is $b-\xi(x,t)$ where $\xi(x,t)$ is the small signal vertical deflection of the membrane that only depends on coordinate x and time t. Neglect fringing field effects and assume the "long-wave limit" so that E_1 and E_2 can be assumed to only be y directed. There is no free surface charge on the $y=b$ interface.

a) Solve for E_1 and E_2 as a function of $V_0, a, b, \varepsilon, \varepsilon_0$ and $\xi(x,t)$.
b) Linearize E_1 for small deflections in $\xi(x,t)$.
c) Find the electrical force per unit area on the membrane and linearize for small deflections in $\xi(x,t)$.
d) Write the linearized governing force balance equation for the membrane to first order in displacement $\xi(x,t)$ including the inertial, membrane tension, gravity, and electrical forces.
e) What voltage V_0 is required to remove membrane sag in equilibrium so that $\xi(x,t)=0$?
f) For perturbation deflections around the $\xi(x,t)=0$ equilibrium of the form $\xi(x,t)=\mathrm{Re}[\hat{\xi}e^{j(\omega t-kx)}]$, what is the $\omega-k$ dispersion relation?
g) The membrane ends are fixed to the supports at $x=0$ and $x=\ell$ so that $\xi(x=0,t)=\xi(x=\ell,t)=0$. For a given real value of ω what are the allowed values of k that satisfy the boundary conditions?
h) Under what conditions is the equilibrium of part (e) stable?
i) What is the maximum membrane mass density per unit area, σ_m, that the voltage V_0 can stably remove sag so that $\xi(x,t)=0$?