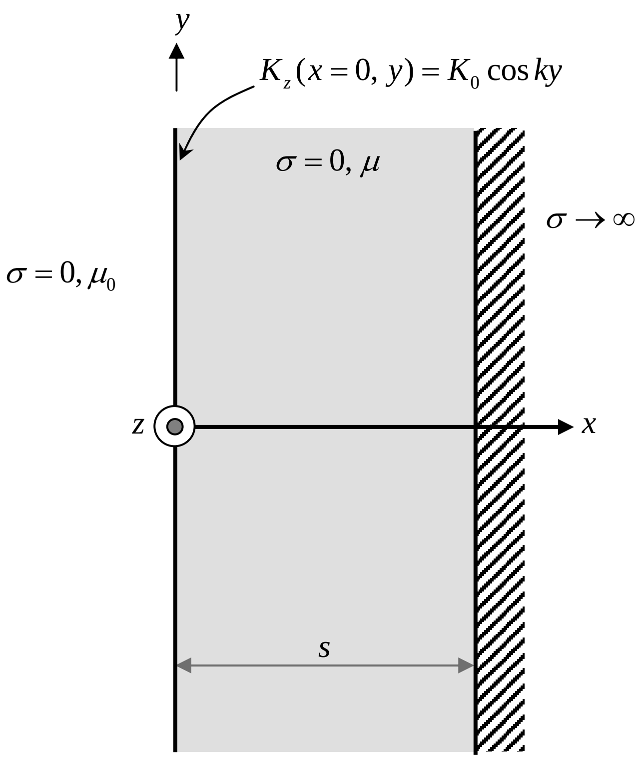
6.641 Electromagnetic Fields, Forces, and Motion
Spring 2009

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.641 Electromagnetic Fields, Forces, and Motion

Final Exam — May 23, 2006
Spring 2006

Final Exam – Tuesday, May 23, 2006, 9 AM – noon.
The 6.641 Formula Sheet is attached. You are also allowed to bring three 8 ½" x 11" sheet of notes (both sides) that **you** prepare.

Problem 1 (25 points)



A sheet of surface current of infinite extent in the y and z directions is placed at $x = 0$ and has distribution $K_z(x=0, y)=K_0 \cos ky$. The surface current flows in the z direction. Free space with no conductivity ($\sigma=0$) and magnetic permeability μ_0 is present for $x < 0$ while for $0 < x < s$ a perfectly insulating medium ($\sigma=0$) with magnetic permeability μ is present. The region for $x > s$ is a grounded perfect conductor so that the magnetic field is zero for x > s. Because there are no volume currents anywhere, $\bar{H}=-\nabla\chi$, where χ is the magnetic scalar potential.

a) What are the boundary conditions necessary to solve for the magnetic fields for $x < 0$ and for $0 < x < s$?

b) What are the magnetic scalar potential and magnetic field distributions for $x < 0$ and $0 < x < s$? **Hint:** The algebra will be greatly reduced if you use one of the following forms of the potential for the region $0 < x < s$

i)	sin(ky)cosh[$k(x$-$s)$]	iii)	sin(ky)sinh[$k(x$-$s)$]
ii)	cos(ky)cosh[$k(x$-$s)$]	iv)	cos(ky)sinh[$k(x$-$s)$]

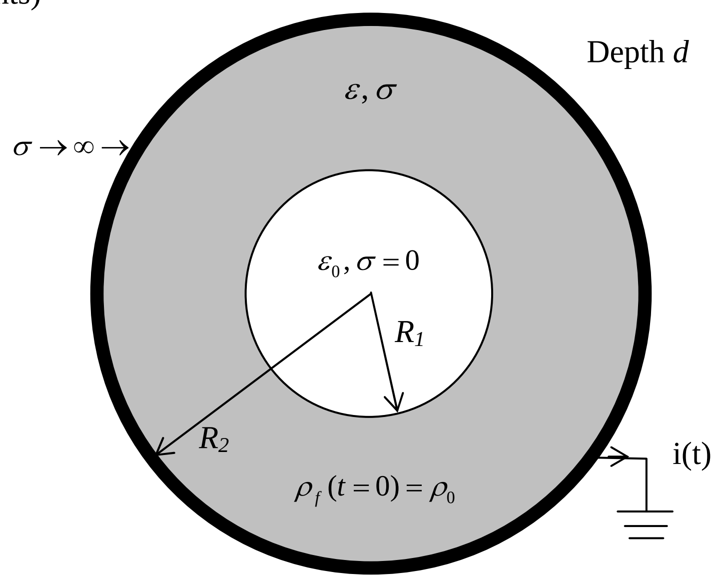
c) What is the surface current distribution on the $x = s$ surface?

d) Use the Maxwell Stress Tensor to find the total force, magnitude and direction, on a section of the perfect conductor at $x = s$ that extends over a wavelength $0< y <\frac{2\pi}{k}$ and $0 < z < D$?

Assume that μ in the region $0 < x < s$ does not depend on density so that $d\mu/d\rho=0$.

Hint: $\int \cos^2(ky)dy=\frac{1}{2}y+\frac{1}{4k}\sin(2ky)$

Problem 2 (25 points)

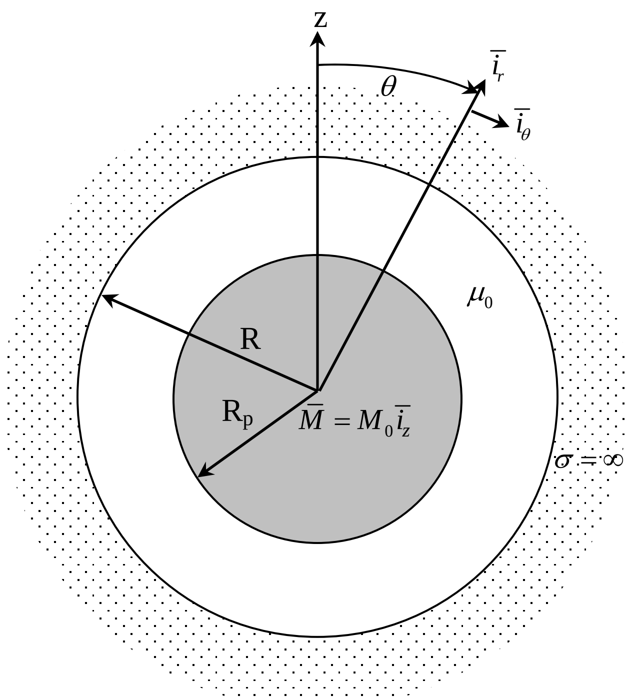


A lossy dielectric cylindrical shell, $R_1 < r < R_2$, having dielectric permittivity ε and ohmic conductivity σ is uniformly charged at time $t=0$ with free volume charge density $\rho_f(t=0) = \rho_0$. The region for $0 < r < R_1$ *is* free space with permittivity ε_0 and zero conductivity ($\sigma = 0$). Assume that the surface charge density at $r = R_1$ is zero for all time, $\sigma_s(r = R_1, t) = 0$.

The $r=R_2$ surface is a grounded perfectly conducting cylinder so that the electric field for $r>R_2$ is zero. The depth d of the cylinder and system is very large so that fringing fields can be neglected.

a) What is the electric field in the free space region, $0 < r < R_1$, as a function of time?
b) What is the volume charge density and electric field within the cylindrical shell, $R_1 < r < R_2$, as a function of radius and time?
c) What is the surface charge density on the interface at $r=R_2$?
d) What is the ground current $i(t)$?

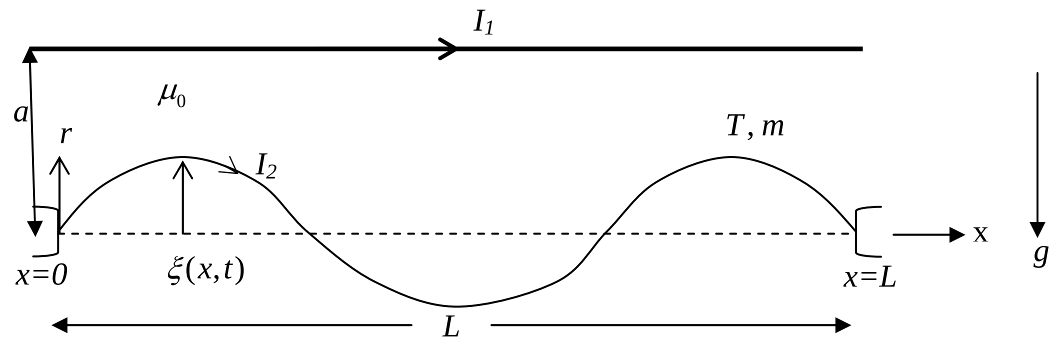
Problem 3 (25 points)



A sphere with radius R_p is comprised of a uniformly permanently magnetized material in the z direction, $M_0 \bar{i}_z = M_0[\bar{i}_r \cos\theta - \bar{i}_\theta \sin\theta]$. The sphere is placed concentric within a free space spherical hole of radius R within a perfect conductor as shown in the cross-sectional drawing in the figure above. The region $R_p < r < R$ is filled with free space with magnetic permeability μ_0. There are no volume free currents anywhere ($\bar{J}_f = 0$) and there is no free surface current on the $r = R_p$ interface.

a) Prove that the magnetic scalar potential χ obeys Laplace's equation for $0 < r < R_p$ and $R_p < r < R$ where $\bar{H} = -\nabla\chi$.
b) What are the boundary conditions required to determine the magnetic field in regions $0 < r < R_p$ and $R_p < r < R$.
c) Find the magnetic field $\bar{H}(r,\theta)$ in regions $0 < r < R_p$ and $R_p < r < R$.
d) Find the free surface current density $\bar{K}$ on the $r = R$ surface.

Problem 4 (25 points)



A conducting string at r=0 having equilibrium tension T and mass per unit length m is stretched horizontally and fixed at two rigid supports a distance L apart. The elastic string carries a current I_2 and can have transverse displacements $\xi(x,t)$. Another rigid wire is placed at r=a and carries a current I_1. The region surrounding the string and wire is free space with magnetic permeability μ_0. Assume that transverse displacements $\xi(x,t)$ of the string centered at $r=0$ depend only on position x and time t. Gravity is downwards with acceleration g.

a) To linear terms in membrane displacement $\xi(x,t)$, find the magnetic force per unit length on the string centered at r=0.
b) What is the governing linearized differential equation of motion of the membrane?
c) What must I_1 be in terms of I_2, m and other relevant parameters so that the membrane is in static equilibrium with $\xi(x,t)=0$.
d) For small membrane deflections of the form $\xi(x,t)=\mathrm{Re}\left[\hat{\xi}e^{j(\omega t-kx)}\right]$ find the $\omega-k$ dispersion relation. Plot the ω-k relationship showing significant intercepts on the axes and slope asymptotes. Assume that k is real and that ω can be pure real or pure imaginary.
e) What are the allowed values of k that satisfy the zero deflection boundary conditions at $x=0$ and $x=L$?
f) Under what conditions will the membrane equilibrium with $\xi(x,t)=0$ first become unstable?