MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.632 Electromagnetic Wave Theory
Quiz No. 2

Problem 1(40%)

Consider the following two Hertzian dipoles as shown in Fig. 1 driven at an angular frequency ω and with the same dipole moment. The first dipole is located at the origin $(0,0,0)$ and oriented in the $\hat{y}$ direction. The second dipole is located at $(0,0,-3\lambda/4)$ and oriented in the $\hat{x}$ direction.

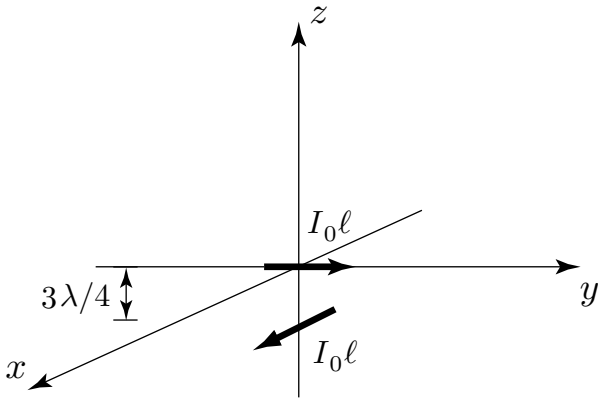

Fig. 1

(a) Give the expression for the vector current moment $\overline{f}(\theta,\phi)$. What are f_θ and f_ϕ?
(b) Show that in the far field the electric field is given by

$$\overline{E} = ik\eta_0 I_0 l \frac{e^{ikr}}{4\pi r}\left[\hat{\theta}\cos\theta(\sin\phi + \cos\phi e^{i\cos\theta\frac{3\pi}{2}}) + \hat{\phi}(\cos\phi - \sin\phi e^{i\cos\theta\frac{3\pi}{2}})\right]$$

(c) What is the polarization of the radiated wave in the $+\hat{z}$ direction?
What is the polarization of the radiated wave in the $-\hat{z}$ direction?
(d) What is the polarization of the radiated wave in the $+\hat{x}$ direction?
What is the polarization of the radiated wave in the $+\hat{y}$ direction?

Problem 2(20%)

In this problem, we consider several equivalent situations for a plane wave propagation in the $\hat{z}$ direction. Let the electric field be $\hat{x}$ directed.

$$\overline{E} = \hat{x}E_0 e^{ikz}, \qquad \overline{H} = \hat{y}\frac{1}{\eta}E_0 e^{ikz}$$

and the region of interest be $z > 0$.

(a) Put an electric current sheet with $\overline{J}_s = A\hat{x}$. What is the value of A so that the same field is preserved in the region of interest?
(b) Put a magnetic current sheet with $\overline{M}_s = B\hat{y}$. What is the value of B so that the same field is preserved in the region of interest?
(c) Replace the region $z < 0$ with a perfect conductor. Place in front of the conductor an electric sheet with $\overline{J}_s = C\hat{x}$ and a magnetic current sheet with $\overline{M}_s = D\hat{y}$. What is the value of C and D so that the same field is preserved in the region of interest?

Problem 3(40%)

An electric dipole antenna with dipole moment $I\ell$ is oriented in the $\hat{z}$ direction and is placed in the front of perfectly conducting wall as shown in the Fig. 2a. The wall is located at $x = 0$ and extends to infinity in both y and z directions. The dipole is located at $(d, 0, 0)$, where $d = \lambda/4$.

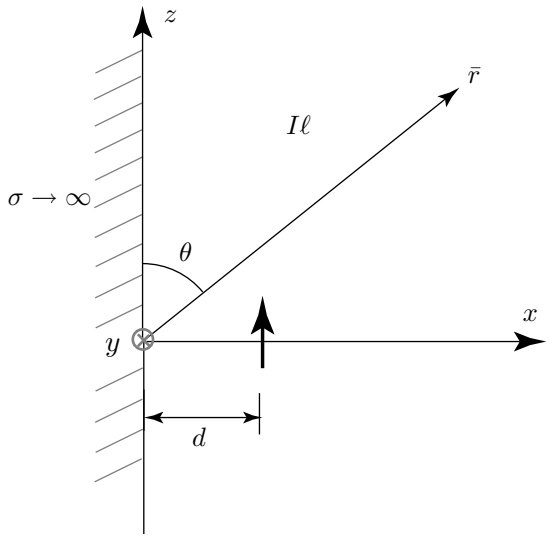

Fig. 2a

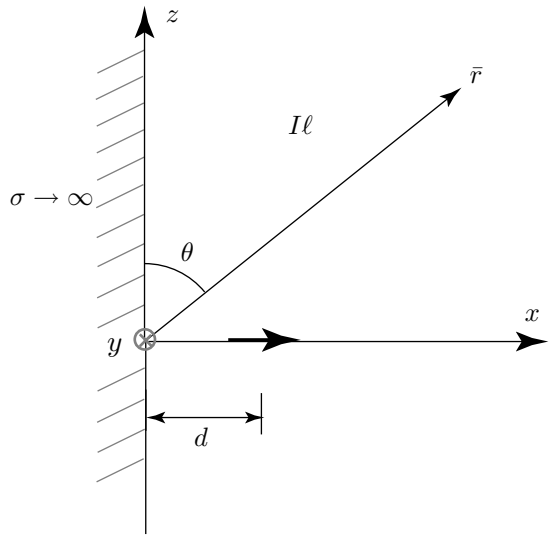

Fig. 2b

(a) Consider the case where the wall is removed in Fig. 2a.
Plot the radiation pattern of the electric field $|\overline{E}|$ in the xz plane.
Plot the radiation pattern of the electric field $|\overline{E}|$ in the xy plane.

(b) The wall is still there as shown in the Fig. 2a. Plot the radiation pattern of the electric field $|\overline{E}|$ in the xz plane.

In part (c), (d), (e), the dipole is oriented in x direction as shown in Fig. 2b.

(c) Consider the case where the wall is removed in Fig. 2b.
Plot the radiation pattern of the electric field $|\overline{E}|$ in the xz plane.

(d) The wall is still there as shown in Fig. 2b. Plot the radiation pattern of the electric field $|\overline{E}|$ in the xz plane.

(e) If the dipole is located at $d = 5\lambda/4$, give all the values of θ $(0 \le \theta \le \pi/2)$ at which the null occurs for the far field.