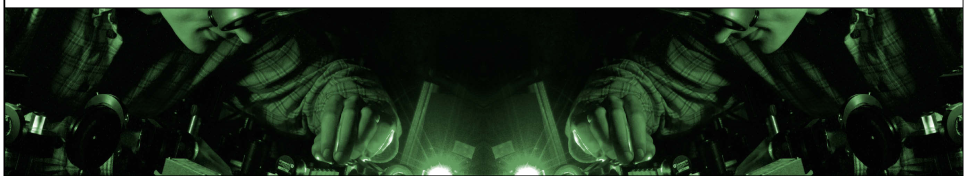


December 8, 2016

6.453 *Quantum Optical Communication* Lecture 23

Jeffrey H. Shapiro

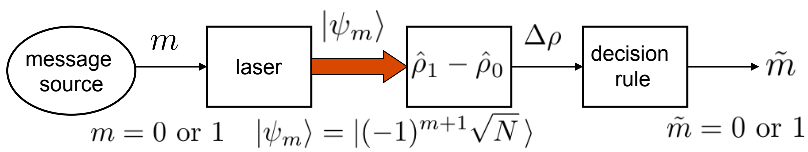
Optical and Quantum Communications Group

6.453 *Quantum Optical Communication* - Lecture 23

- Announcements
  - Pick up lecture notes, slides
  - Term papers are due Tuesday, December 13th
- More Quantum Optical Applications
  - Binary optical communication with squeezed states
  - Phase-sensing interferometry with squeezed states
  - Super-dense coding with entangled states
  - Quantum lithography with "N00N" states

Minimum Probability of Error Binary Communication

- Binary Phase-Shift-Keying with Coherent States

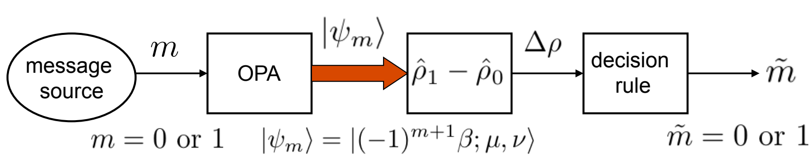


- Optimum Decision Rule and Minimum Probability of Error

$$\Delta\rho \underset{\tilde{m}=0}{\overset{\tilde{m}=1}{\gtrless}} 0. \qquad \Pr(e) \approx \frac{1}{4}e^{-4N}, \text{ for } N \gg 1$$

Minimum Probability of Error Binary Communication

- Binary Phase-Shift-Keying with Squeezed States

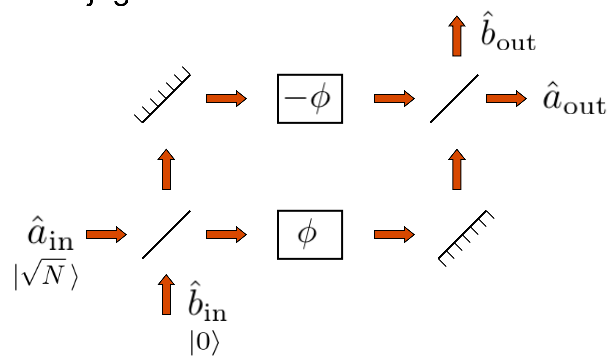


- Optimum Decision Rule and Minimum Probability of Error

$$\Delta\rho \underset{\tilde{m}=0}{\overset{\tilde{m}=1}{\gtrless}} 0. \qquad \Pr(e) \approx \frac{1}{4}e^{-4N^2}, \text{ for } N \gg 1$$

Phase-Sensing Interferometry with Coherent States

- Phase-Conjugate Mach-Zehnder Interferometer: $|\phi| \ll 1$

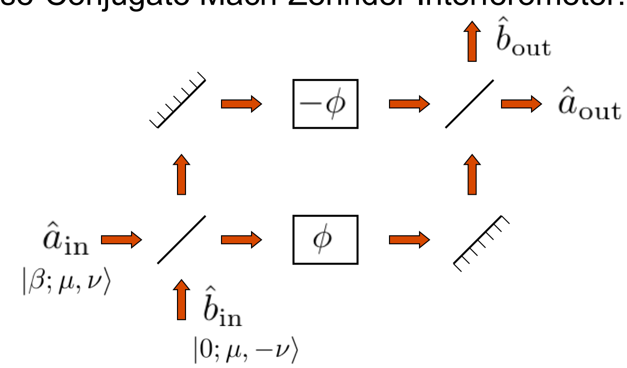


- Homodyne Measurement of $\tilde{\phi} \leftrightarrow -{\rm Im}(\hat{b}_{\rm out})/\sqrt{N}$

$$\langle\tilde{\phi}\rangle = \phi \qquad\qquad \langle\Delta\tilde{\phi}^2\rangle = 1/4N$$

Phase-Sensing Interferometry with Squeezed States

- Phase-Conjugate Mach-Zehnder Interferometer: $|\phi| \ll 1$

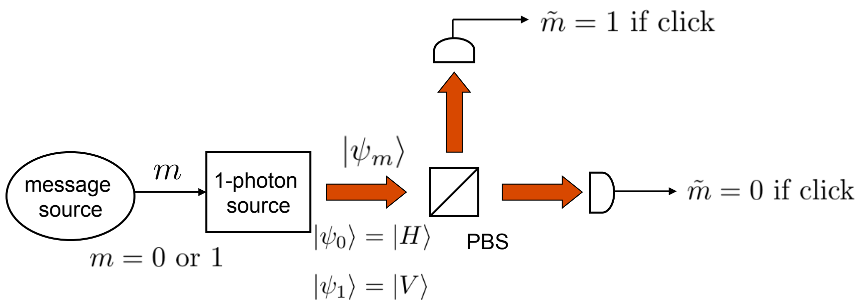


- Homodyne Measurement of $\tilde{\phi} \leftrightarrow -{\rm Im}(\hat{b}_{\rm out})/\sqrt{N-2\nu^2}$

$$\langle\tilde{\phi}\rangle = \phi \qquad\qquad \langle\Delta\tilde{\phi}^2\rangle = 1/2N(N+2)$$

Binary Communication with Single Photons

- Binary Polarization Modulation, Lossless Channel, $\eta = 1$



- One Bit of Information Transmitted per Photon

Super-Dense Coding with Entangled Photons

- Alice and Bob Share a Singlet State of Two Photons:

$$|\psi^-\rangle_{AB} = \frac{|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B}{\sqrt{2}}$$

- Alice Uses Two Classical Bits to Modulate Her Photon:

$$\begin{aligned}
\alpha|H\rangle_A + \beta|V\rangle_A &\longrightarrow \alpha|H\rangle_A + \beta|V\rangle_A, \quad \text{if } m = 00 \\
\alpha|H\rangle_A + \beta|V\rangle_A &\longrightarrow \alpha|H\rangle_A - \beta|V\rangle_A, \quad \text{if } m = 01 \\
\alpha|H\rangle_A + \beta|V\rangle_A &\longrightarrow \alpha|V\rangle_A + \beta|H\rangle_A, \quad \text{if } m = 10 \\
\alpha|H\rangle_A + \beta|V\rangle_A &\longrightarrow \alpha|V\rangle_A - \beta|H\rangle_A, \quad \text{if } m = 11
\end{aligned}$$

Super-Dense Coding with Entangled Photons

- Alice Sends Her Photon to Bob
  - Bob then has a Bell state:

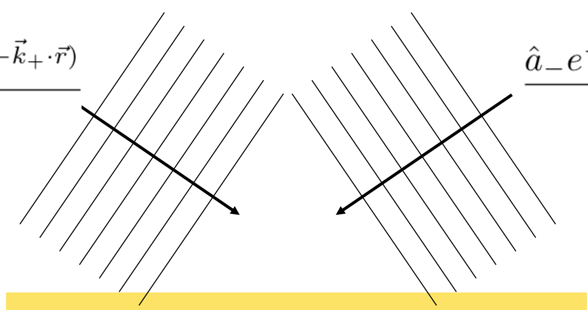
$$
\begin{cases}
\dfrac{|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B}{\sqrt{2}}, & \text{if } m = 00 \\[2ex]
\dfrac{|H\rangle_A|V\rangle_B + |V\rangle_A|H\rangle_B}{\sqrt{2}}, & \text{if } m = 01 \\[2ex]
\dfrac{|H\rangle_A|H\rangle_B - |V\rangle_A|V\rangle_B}{\sqrt{2}}, & \text{if } m = 10 \\[2ex]
\dfrac{|H\rangle_A|H\rangle_B + |V\rangle_A|V\rangle_B}{\sqrt{2}}, & \text{if } m = 11
\end{cases}
$$

- Bob Makes the Bell-Observable Measurement
  - Bob decodes both of Alice's bits without error

Optical Lithography with Coherent States

- Interference Between Plane Waves on a Photoresist

$$\frac{\hat{a}_+ e^{-j(\omega t - \vec{k}_+\cdot\vec{r})}}{\sqrt{T}} \quad |\sqrt{N/2}\rangle$$



$$\frac{\hat{a}_- e^{-j(\omega t - \vec{k}_-\cdot\vec{r})}}{\sqrt{T}} \quad |\sqrt{N/2}\rangle$$

$$\vec{k}_\pm = \pm k\sin(\theta)\vec{i}_x + k\cos(\theta)\vec{i}_z$$

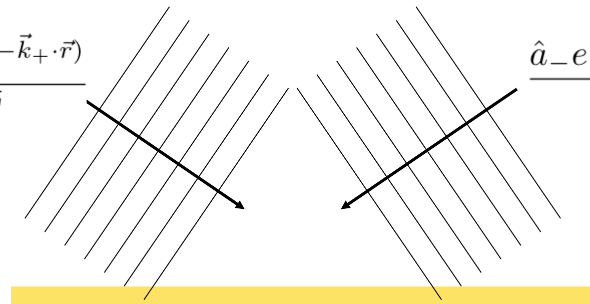
$$\left\langle \int_0^T \mathrm{d}t\, \hat{E}^\dagger(x,t)\hat{E}(x,t) \right\rangle = N[1 + \cos(2k\sin(\theta)x)]$$

Optical Lithography with "N00N" States

- Interference Between Plane Waves on an *N*-Photon Resist

$$\frac{\hat{a}_+ e^{-j(\omega t - \vec{k}_+ \cdot \vec{r})}}{\sqrt{T}}$$

$$\frac{\hat{a}_- e^{-j(\omega t - \vec{k}_- \cdot \vec{r})}}{\sqrt{T}}$$



$$\frac{|N\rangle_+|0\rangle_- + |0\rangle_+|N\rangle_-}{\sqrt{2}}$$

$$\left\langle \int_0^T \mathrm{d}t\, K\hat{E}^{\dagger N}(x,t)\hat{E}^N(x,t) \right\rangle = KN![1+\cos(2kN\sin(\theta)x)]/T^{N-1}$$

Subject Outline Revisited — *We're Done!*

- Quantum Optics
  - Dirac notation quantum mechanics; harmonic oscillator quantization; number states, coherent states, and squeezed states; *P* representation and classical fields.
- Single-Mode and Two-Mode Quantum Systems
  - Direct, homodyne, and heterodyne detection; linear propagation loss; phase insensitive and phase sensitive amplifiers; entanglement and teleportation.
- Multi-Mode Quantum Systems
  - Field quantization; quantum photodetection.
- Nonlinear Optics
  - Phase-matched interactions; optical parametric amplifiers; generation of squeezed states, photon-twin beams, non-classical fourth-order interference, and polarization entanglement.
- Quantum Systems Theory
  - Optimum binary detection; quantum precision measurements; quantum cryptography.